3.8 Graphing Absolute Value Functions

Learning Target: Graph absolute value functions.

Success Criteria:
- I can identify characteristics of absolute value functions.
- I can graph absolute value functions.
- I can describe transformations of graphs of absolute value functions.

EXPLORE IT! Understanding Graphs of Absolute Value Functions

Work with a partner.

a. Think about the absolute value of a number. What characteristics might you observe in the graph of an *absolute value function*?

b. Graph $y = |x|$. Make several observations about the graph.

c. Let f be the parent absolute value function $f(x) = |x|$. Which of the following functions have a graph with one x-intercept? two x-intercepts? no x-intercept? Explain your reasoning.

$g(x) = f(x) - 1$ $\quad h(x) = f(x - 1)$ $\quad p(x) = -f(x)$ $\quad q(x) = f(x) + 1$

5 MTR RELATE CONCEPTS
How can you use what you know about transformations of linear functions to make conclusions about transformations of absolute value functions?

d. Match each absolute value function with its graph. Explain your reasoning. Then use technology to check your answers.

i. $g(x) = -|x - 2|$ $\quad$ **ii.** $g(x) = |x - 2| + 2$

iii. $g(x) = |x - 2| - 2$ $\quad$ **iv.** $g(x) = 2|x - 2|$

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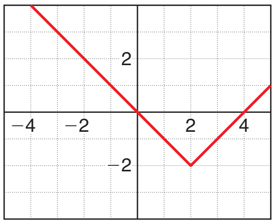

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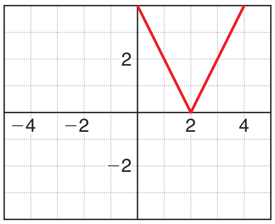

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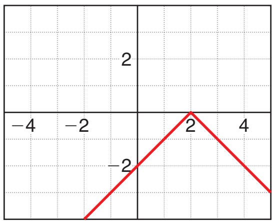

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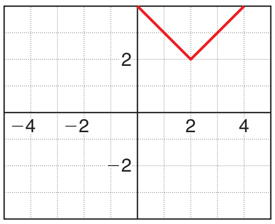

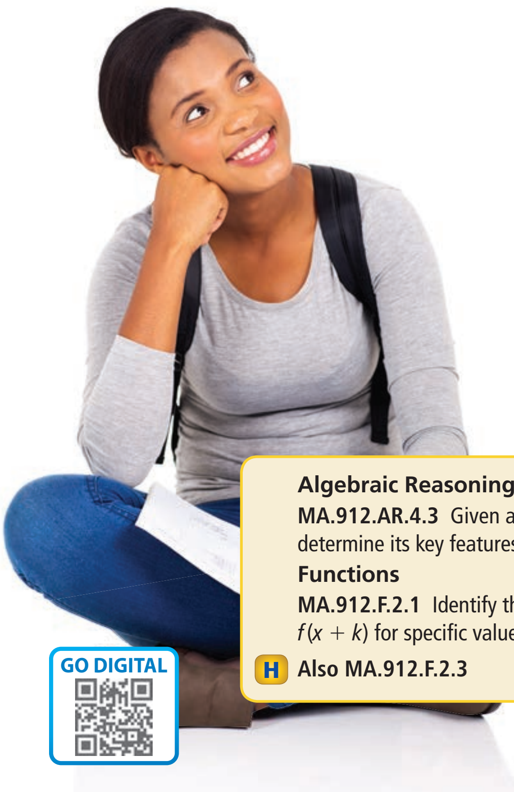

Algebraic Reasoning
MA.912.AR.4.3 Given a table, equation or written description of an absolute value function, graph that function and determine its key features.
Functions
MA.912.F.2.1 Identify the effect on the graph or table of a given function after replacing $f(x)$ by $f(x) + k$, $kf(x)$, $f(kx)$ and $f(x + k)$ for specific values of k.
H Also MA.912.F.2.3

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Vocabulary

absolute value function, *p. 158*
vertex, *p. 158*
vertex form, *p. 161*

WORDS AND MATH

In geometry, a *vertex* is the point where the sides meet for an angle, polygon, or solid.

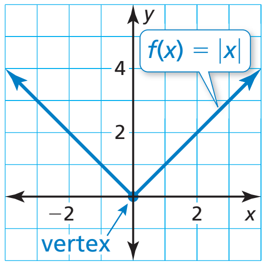
Characteristics of Absolute Value Functions

KEY IDEA

Absolute Value Function

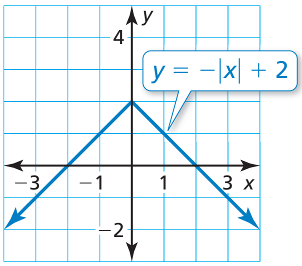
An **absolute value function** is a function that contains an absolute value expression. The parent absolute value function is $f(x) = |x|$. The graph of $f(x) = |x|$ is V-shaped and symmetric about the y-axis. The **vertex** is the point where the graph changes direction. The vertex of the graph of $f(x) = |x|$ is (0, 0).

The domain of $f(x) = |x|$ is all real numbers. The range is $y \geq 0$.



EXAMPLE 1 Describing Characteristics

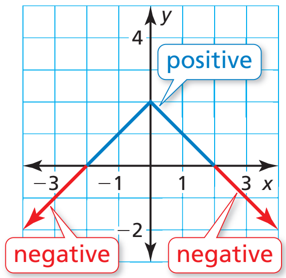
Determine when the function $y = -|x| + 2$ is positive, negative, increasing, or decreasing. Then describe the end behavior of the function.



SOLUTION

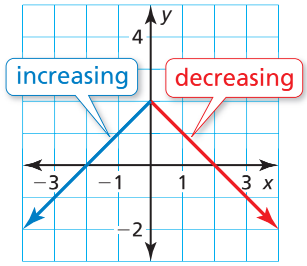
Positive and Negative:

The x-intercepts are ± 2. The function is negative when $x < -2$, positive when $-2 < x < 2$, and negative when $x > 2$.



Increasing and Decreasing:

The vertex is (0, 2). The function is increasing when $x < 0$ and decreasing when $x > 0$.



End behavior: The graph shows that the function values decrease as x approaches both positive and negative infinity. So, $y \to -\infty$ as $x \to -\infty$ and $y \to -\infty$ as $x \to +\infty$.

SELF-ASSESSMENT

1 I don't understand yet. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

Determine when the function is positive, negative, increasing, or decreasing. Then describe the end behavior of the function.

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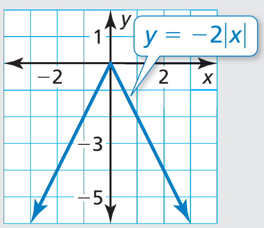

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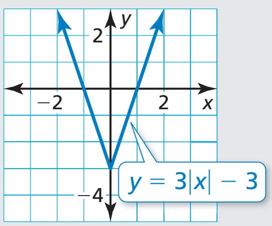

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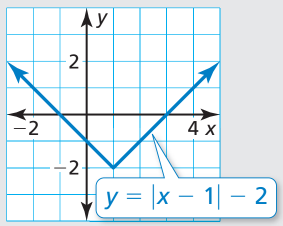

4. HELP A CLASSMATE Explain to a classmate how to use the vertex and x-intercepts to determine when an absolute value function is positive, negative, increasing, or decreasing.

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Transforming Graphs of Absolute Value Functions

The graphs of all other absolute value functions are transformations of the graph of the parent function $f(x) = |x|$. The transformations presented in the previous section also apply to absolute value functions.

EXAMPLE 2 **Graphing Absolute Value Functions** WATCH

Graph each function. Compare each graph to the graph of $f(x) = |x|$. Find the domain and range.

a. $q(x) = 2|x|$ **b.** $m(x) = |x - 2|$

STUDY TIP

A vertical stretch of the graph of $f(x) = |x|$ is *narrower* than the graph of $f(x) = |x|$.

A vertical shrink of the graph of $f(x) = |x|$ is *wider* than the graph of $f(x) = |x|$.

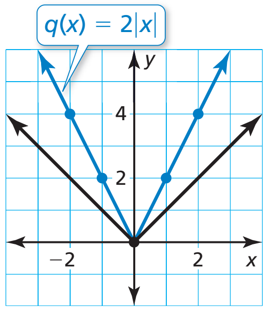
SOLUTION

a. Step 1 Make a table of values.

x	−2	−1	0	1	2
$q(x)$	4	2	0	2	4

Step 2 Plot the ordered pairs.

Step 3 Draw the graph.



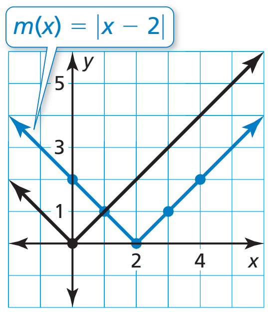
▶ The function q is of the form $y = a \cdot f(x)$, where $a = 2$. So, the graph of q is a vertical stretch of the graph of f by a factor of 2. The domain is all real numbers. The range is $y \geq 0$.

b. Step 1 Make a table of values.

x	0	1	2	3	4
$m(x)$	2	1	0	1	2

Step 2 Plot the ordered pairs.

Step 3 Draw the graph.



▶ The function m is of the form $y = f(x - h)$, where $h = 2$. So, the graph of m is a horizontal translation 2 units right of the graph of f. The domain is all real numbers. The range is $y \geq 0$.

1 MTR HELP A CLASSMATE

Explain to a classmate why you can also describe the transformation in part (a) as a horizontal shrink.

SELF-ASSESSMENT

1 I don't understand yet. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

Graph the function. Compare the graph to the graph of $f(x) = |x|$. Find the domain and range.

5. $h(x) = |x| - 1$

6. $n(x) = |x + 4|$

7. $t(x) = -3|x|$

8. $v(x) = \frac{1}{4}|x|$

9. REASONING Without graphing, explain how the graph of $g(x) = |x| + 5$ compares to the graph of $f(x) = |x|$.

5 MTR **10. USE STRUCTURE** How do you know whether the graph of $g(x) = a|x|$ is a vertical stretch or a vertical shrink of the graph of $f(x) = b|x|$?

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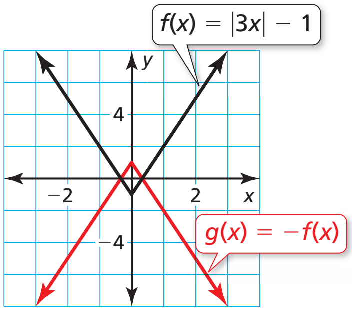
Describing Transformations

EXAMPLE 3 Describing a Transformation

Let $f(x) = |3x| - 1$. Graph $g(x) = -f(x)$. Describe the transformation from the graph of f to the graph of g.



SOLUTION

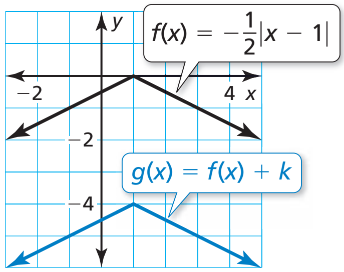
To find the outputs of g, multiply the outputs of f by -1. The graph of g consists of the points $(x, -f(x))$.

x	-2	-1	0	1	2
$f(x)$	5	2	-1	2	5
$-f(x)$	-5	-2	1	-2	-5

▶ The graph of g is a reflection in the x-axis of the graph of f.

EXAMPLE 4 Describing a Transformation

Describe the transformation from the graph of f to the graph of g.



SOLUTION

Compare f and g to find the value of k. The function $g(x) = f(x) + k$ indicates that the graph of g is a vertical translation of the graph of f. The graphs of f and g show that for any input, the output of g is 4 less than the output of f. For example, $g(1) = -4$ and $f(1) = 0$. So, $k = -4$.

▶ The graph of g is a translation 4 units down of the graph of f.

SELF-ASSESSMENT

1 I don't understand yet. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

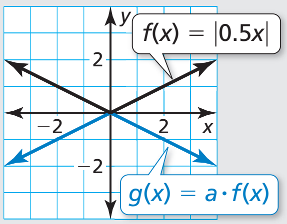
Using f, graph (a) g and (b) h. Describe the transformations from the graph of f to the graphs of g and h.

11. $f(x) = |x + 3|$; $g(x) = f(x) - 4$; $h(x) = f(x - 4)$

12. $f(x) = -|x|$; $g(x) = 2f(x)$; $h(x) = f(2x)$

Describe the transformation from the graph of f to the graph of g.

13.



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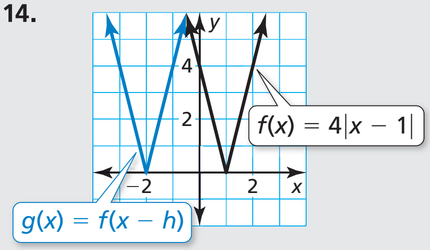

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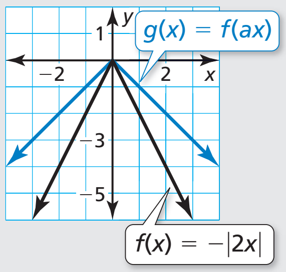

16. Graph the absolute value functions f and g represented by the table. Describe the transformation from the graph of f to the graph of g.

x	-2	-1	1	2
$f(x)$	2	4	4	2
$g(x)$	1	2	2	1

17. OPEN-ENDED Write an absolute value function that is a transformation of the parent absolute value function. Describe the end behavior of the function.

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Using Vertex Form

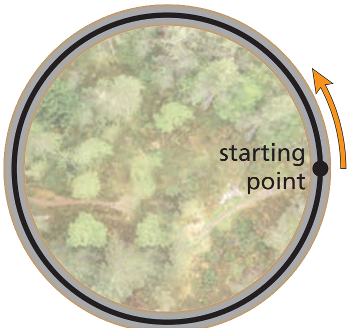
KEY IDEA

Vertex Form of an Absolute Value Function

An absolute value function written in the form $g(x) = a|x - h| + k$, where $a \neq 0$, is in **vertex form**. The vertex of the graph of g is (h, k).

Any absolute value function can be written in vertex form, and its graph is symmetric about the line $x = h$.

7 MTR **EXAMPLE 5** Modeling Real Life WATCH



You ride your bicycle around a circular trail one time. The function $f(x) = -\frac{1}{3}|x - 4.5| + 1.5$ represents the shortest distance (in miles) along the trail between you and your starting point after x minutes. (a) Graph the function. Find the domain and range in this context. (b) Interpret the intercepts and the vertex.

SOLUTION

a. The function is in vertex form, $g(x) = a|x - h| + k$, where $h = 4.5$ and $k = 1.5$. So, the vertex is $(h, k) = (4.5, 1.5)$. Substitute 0 for $f(x)$ to find any x-intercepts.

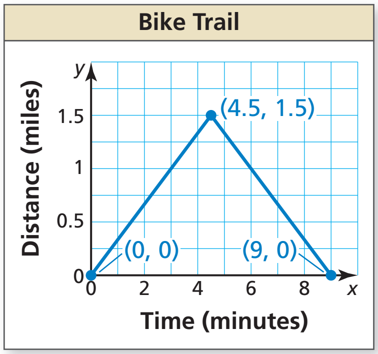
$$0 = -\tfrac{1}{3}|x - 4.5| + 1.5$$

$$4.5 = |x - 4.5|$$

$$x - 4.5 = 4.5 \quad \text{or} \quad x - 4.5 = -4.5$$

$$x = 9 \qquad\qquad x = 0$$

Graph the function using the points (0, 0), (4.5, 1.5), and (9, 0). The time x and distance $f(x)$ must be greater than or equal to 0 in this context. So, the domain is $\{x \mid 0 \leq x \leq 9\}$ and the range is $\{f(x) \mid 0 \leq f(x) \leq 1.5\}$.



1 MTR **ANALYZE A PROBLEM**
Explain why it makes sense that the graph begins at (0, 0). How can you use symmetry to find the other intercept?

b. The intercepts 0 and 9 indicate that it takes you $9 - 0 = 9$ minutes to ride around the trail. The vertex (4.5, 1.5) indicates that you are 1.5 miles from your starting point after 4.5 minutes. You can also determine that you are halfway around the trail at that time.

SELF-ASSESSMENT

1 I don't understand yet. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

18. The function $d(x) = \frac{1}{3}|x - 15| - 5$ represents the elevation (in meters) of a scuba diver x seconds after descending from the surface.

a. Graph the function. Find the domain and range in this context.

b. Interpret the intercepts and the vertex.

19. WHAT IF? The function $f(x) = -0.25|x - 8| + 2$ models the situation in Example 5. What is the radius of the trail? At what speed did you ride your bicycle? Explain your reasoning.

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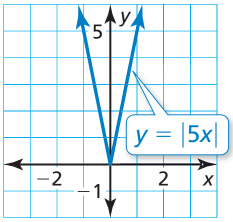
3.8 Practice WITH CalcChat® AND CalcView®

In Exercises 1–6, determine when the function is positive, negative, increasing, or decreasing. Then describe the end behavior of the function. *(See Example 1.)*

▶ **1.**



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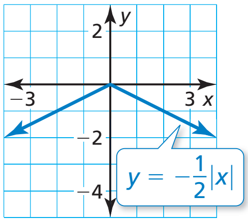

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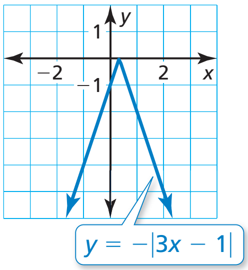

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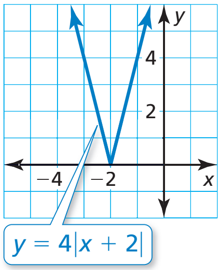

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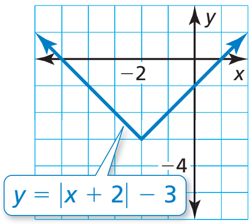

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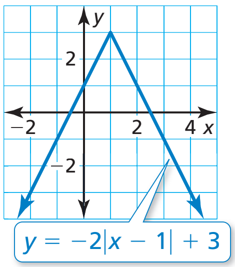


In Exercises 7–10, use the graphs of *f* and *h* to describe the transformation from the graph of *f* to the graph of *h*.

7. $f(x) = |x - 6|$; $h(x) = f(x) - 2$

8. $f(x) = -2|x + 4|$; $h(x) = f(-x)$

9. $f(x) = -8x$; $h(x) = f\left(\frac{3}{4}x\right)$

10. $f(x) = \frac{1}{7}|x| + 1$; $h(x) = f(6x)$

In Exercises 11–20, graph the function. Compare the graph to the graph of $f(x) = |x|$. Find the domain and range. *(See Example 2.)*

▶ **11.** $d(x) = |x| - 4$

12. $r(x) = |x| + 5$

13. $m(x) = |x + 1|$

14. $v(x) = |x - 3|$

15. $m(x) = -|x|$

16. $v(x) = |-x|$

17. $p(x) = \frac{1}{3}|x|$

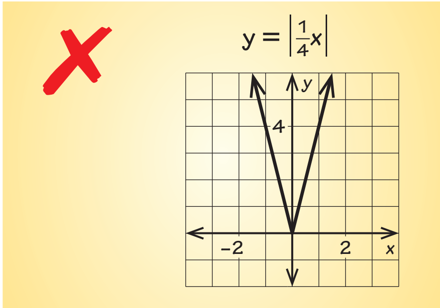
18. $j(x) = 3|x|$

▶ **19.** $a(x) = |5x|$

20. $q(x) = \left|\frac{3}{2}x\right|$

4 MTR **ERROR ANALYSIS** **In Exercises 21 and 22, describe and correct the error in graphing the function.**

21.



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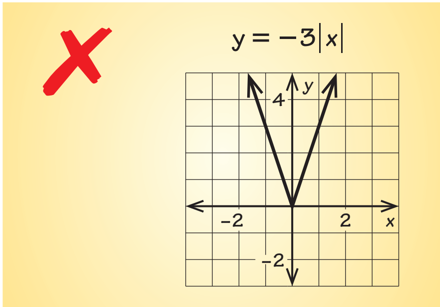

In Exercises 23–28, write an equation that represents the given transformation(s) of the graph of $g(x) = |x|$.

23. vertical translation 7 units down

24. horizontal translation 10 units left

25. reflection in the x-axis

26. reflection in the y-axis

27. vertical shrink by a factor of $\frac{1}{4}$

28. horizontal stretch by a factor of 3

In Exercises 29–32, using *f*, graph (a) *g* and (b) *h*. Describe the transformations from the graph of *f* to the graphs of *g* and *h*. *(See Example 3.)*

▶ **29.** $f(x) = -|x + 7|$; $g(x) = f(x) + 4$; $h(x) = f(x + 4)$

30. $f(x) = 2|x| + 11$; $g(x) = -f(x)$; $h(x) = f(-x)$

31. $f(x) = \frac{1}{6}|x + 5|$; $g(x) = 3f(x)$; $h(x) = f(3x)$

32. $f(x) = 3|x| - 4$; $g(x) = \frac{1}{5}f(x)$; $h(x) = f\left(\frac{1}{5}x\right)$

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In Exercises 33–42, describe the transformation from the graph of f to the graph of g. *(See Example 4.)*

33.

x	−4	−3	−2	−1	0
$f(x)$	−1	0	−1	−2	−3
$g(x) = f(x) + k$	−6	−5	−6	−7	−8

34.

x	−5	−3	−1	1	3
$f(x)$	2	0	−2	0	2
$g(x) = a \cdot f(x)$	1	0	−1	0	1

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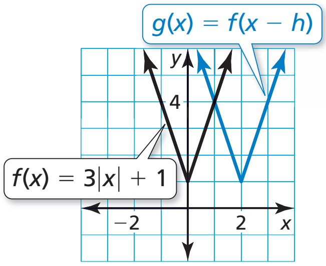

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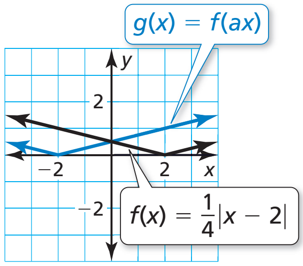


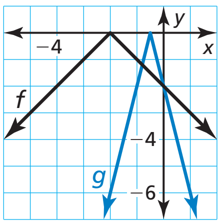
37.

x	1	2	3	4	5
$f(x)$	3	2	1	0	1
$g(x)$	9	6	3	0	3

38.

x	−4	−3	−2	−1	0
$f(x)$	−5	−6	−7	−6	−5
$g(x)$	5	6	7	6	5

39.



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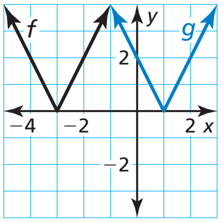

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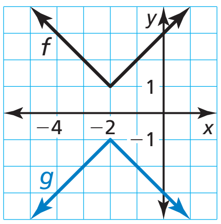

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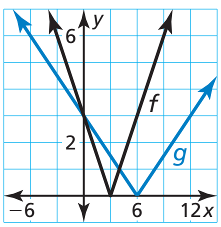


43. MODELING REAL LIFE You are running a ten-mile race. The function $d(t) = \frac{1}{8}|t - 40|$ represents the distance (in miles) you are from a water stop after t minutes. *(See Example 5.)*

a. Graph the function. Find the domain and range in this context.

b. Interpret the intercepts and the vertex. When is the function decreasing? increasing? Explain what each represents in this context.

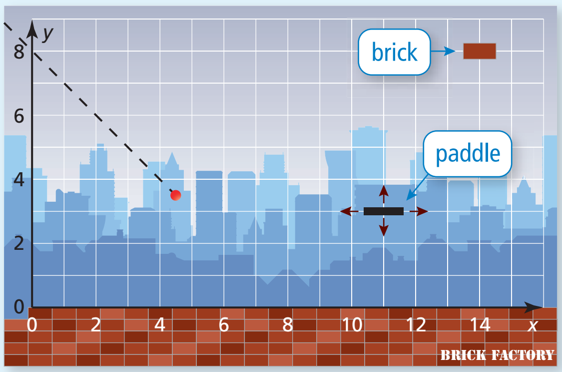
44. MODELING REAL LIFE A traveler is driving from Georgia to Florida. The function $d(t) = 60|t - 2.5|$ represents the distance (in miles) the car is from the state line after t hours.

a. Graph the function. Find the domain and range in this context.

b. Interpret the intercepts and the vertex. When is the function decreasing? increasing? Explain what each represents in this context.

45. REASONING Is it possible for an absolute value function to always be increasing? decreasing? Explain your reasoning.

46. HOW DO YOU SEE IT?
The object of a computer game is to break bricks by deflecting a ball toward them using a paddle. The graph shows the current path of the ball and the location of the last brick.



a. You can move the paddle up, down, left, and right. At what coordinates should you place the paddle to break the last brick? Assume the ball deflects at a right angle.

b. You move the paddle to the coordinates in part (a), and the ball is deflected. How can you write an absolute value function that describes the path of the ball?

GO DIGITAL

47. B.E.S.T. TEST PREP Which of the following functions have the same end behavior?

Ⓐ $a(x) = 4|x - 2| + 3$

Ⓑ $b(x) = -\frac{3}{2}|x + 5|$

Ⓒ $c(x) = |-2x - 6| - 1$

Ⓓ $d(x) = \left|9 - \frac{1}{2}x\right|$

48. THOUGHT PROVOKING
Graph an absolute value function f that represents the route a wide receiver runs in a football game. Let the x-axis represent distance (in yards) across the field horizontally. Let the y-axis represent distance (in yards) down the field. Limit the domain so the route is realistic.

49. USING TOOLS Graph $y = 2|x + 2| - 6$ and $y = -2$ in the same coordinate plane. Use the graph to solve the equation $2|x + 2| - 6 = -2$. Use technology to check your solutions.

50. DIG DEEPER Write the vertex of the absolute value function $f(x) = |ax - h| + k$ in terms of a, h, and k.

4 MTR **51. MAKING AN ARGUMENT** Let p be a positive constant, where the graph of $y = |x| + p$ is a vertical translation in the positive direction of the graph of $y = |x|$. Does this mean that the graph of $y = |x + p|$ is a horizontal translation in the positive direction of the graph of $y = |x|$? Explain.

REVIEW & REFRESH

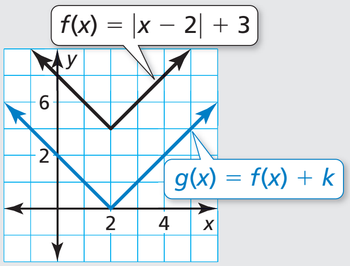
WATCH

In Exercises 52 and 53, solve the equation.

52. $-4|2x - 3| + 12 = -8$

53. $|x + 4| = |5x + 2|$

54. Describe the transformation from the graph of f to the graph of g.



In Exercises 55–58, solve the inequality. Graph the solution, if possible.

55. $2a - 7 \leq -2$

56. $-3(2p + 4) > -6p - 5$

57. $4(3h + 1.5) \geq 6(2h - 2)$

58. $-4(x + 6) < 2(2x - 9)$

In Exercises 59 and 60, use the graphs of f and g to describe the transformation from the graph of f to the graph of g.

59. $f(x) = -\frac{1}{2}x$; $g(x) = f(x + 2)$

60. $f(x) = 3x - 1$; $g(x) = -f(x)$

7 MTR **61. MODELING REAL LIFE** You have \$15 to purchase pecans and walnuts. The equation $12x + 7.5y = 15$ models this situation, where x is the number of pounds of pecans and y is the number of pounds of walnuts.

a. Interpret the terms and coefficients in the equation.

b. Graph the equation. Interpret the intercepts.

62. Let $f(t)$ be the outside temperature (in degrees Celsius) t hours after 9 A.M. Explain the meaning of each statement.

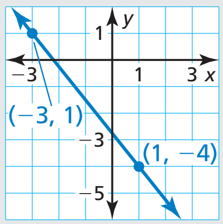
a. $f(4) = 30$ **b.** $f(m) = 28.9$

c. $f(2) = f(9)$ **d.** $f(6.5) > f(0)$

63. Solve $y = 3x - 7$ for x.

64. OPEN-ENDED Draw a graph that does *not* represent a function.

65. Find the slope of the line.



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