3.6 Graphing Linear Equations in Slope-Intercept Form

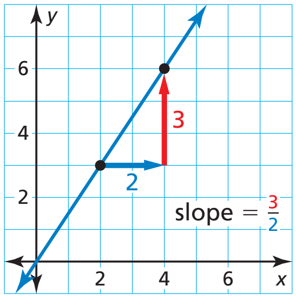
Learning Target: Find the slope of a line and use slope-intercept form.

Success Criteria:
- I can find the slope of a line.
- I can use the slope-intercept form of a linear equation.
- I can solve real-life problems using slopes and *y*-intercepts.

Slope is the rate of change between any two points on a line. It is the measure of the *steepness* of the line.

To find the slope of a line, find the value of the ratio of the change in y (vertical change) to the change in x (horizontal change).

$$\text{slope} = \frac{\text{change in } y}{\text{change in } x}$$



EXPLORE IT! Analyzing Linear Equations

5 MTR USE STRUCTURE
If you complete a similar table for a line with a negative or fractional slope or *y*-intercept, do your results change?

Work with a partner.

a. Complete the table for $y = 2x$. What do you notice about the values in Columns 2 and 4?

x	Change in x	y	Change in y
1	---	2	---
2	1	4	2
3			
4			
5			

b. Complete a similar table for each equation. Interpret your results.

i. $y = 2x + 1$

ii. $y = 4x - 3$

iii. $y = px + q$

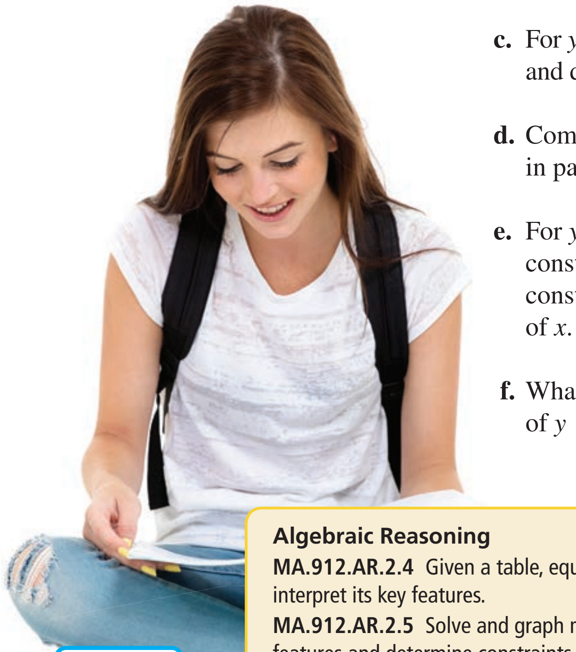
c. For $y = px + q$, when x increases by 1, explain why the change in y is constant and does not depend on the value of x. What does this constant represent?

d. Complete the table for each of the equations in part (a) and part (b). What do you notice?

x	Change in x	y	Change in y
1	---		---
3			
5			
7			

e. For $y = px + q$, when x increases by a constant c, explain why the change in y is constant and does not depend on the value of x. What does this constant represent?

f. What is the relationship between the graph of $y = px + q$ and the values of p and q?



Algebraic Reasoning

MA.912.AR.2.4 Given a table, equation or written description of a linear function, graph that function, and determine and interpret its key features.

MA.912.AR.2.5 Solve and graph mathematical and real-world problems that are modeled with linear functions. Interpret key features and determine constraints in terms of the context.

GO DIGITAL

Vocabulary

AZ VOCAB

slope, *p. 138*
rise, *p. 138*
run, *p. 138*
slope-intercept form, *p. 140*
constant function, *p. 140*

READING

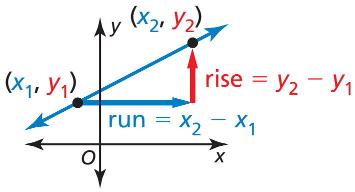
In the slope formula, x_1 is read as "x sub one" and y_2 is read as "y sub two." The numbers 1 and 2 in x_1 and y_2 are called *subscripts*.

The Slope of a Line

KEY IDEA

Slope

The **slope** m of a nonvertical line passing through two points (x_1, y_1) and (x_2, y_2) is the value of the ratio of the **rise** (change in y) to the **run** (change in x).



$$\text{slope} = m = \frac{\text{rise}}{\text{run}} = \frac{\text{change in } y}{\text{change in } x} = \frac{y_2 - y_1}{x_2 - x_1}$$

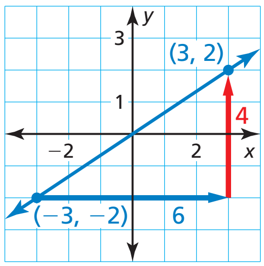
When the line rises from left to right, the slope is positive. When the line falls from left to right, the slope is negative.

EXAMPLE 1 Finding Slopes of Lines

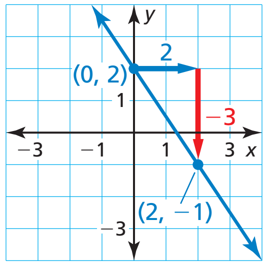
WATCH

Describe the slope of each line. Then find the slope.

a.



b.



SOLUTION

a. The line rises from left to right. So, the slope is positive. Let $(x_1, y_1) = (-3, -2)$ and $(x_2, y_2) = (3, 2)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2 - (-2)}{3 - (-3)} = \frac{4}{6} = \frac{2}{3}$$

b. The line falls from left to right. So, the slope is negative. Let $(x_1, y_1) = (0, 2)$ and $(x_2, y_2) = (2, -1)$.

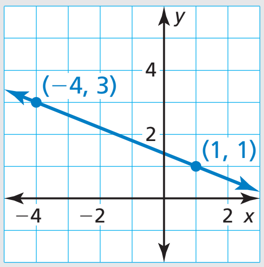
$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-1 - 2}{2 - 0} = \frac{-3}{2} = -\frac{3}{2}$$

SELF-ASSESSMENT

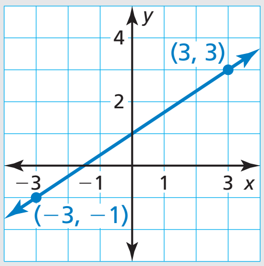
1 I don't understand yet. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

Describe the slope of the line. Then find the slope.

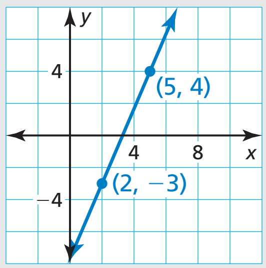
1.



2.



3.



5 MTR

4. **STRUCTURE** When finding slope, can you label either point as (x_1, y_1) and (x_2, y_2)? Explain.

5. **WRITING** When the graph of a line is not horizontal or vertical, how can you tell whether the graph has a positive or a negative slope?

6. **REASONING** Line p has a slope of -4. Line q has a slope of $\frac{7}{4}$. Which line is steeper? Explain your reasoning.

GO DIGITAL

EXAMPLE 2 Finding Slopes from Tables

WATCH

The points represented by each table lie on a line. How can you find the slope of each line from the table? What is the slope of each line?

a.

x	y
4	20
7	14
10	8
13	2

b.

x	y
-1	2
1	2
3	2
5	2

c.

x	y
-3	-3
-3	0
-3	6
-3	9

SOLUTION

Choose any two points from the table and use the slope formula.

a. Let $(x_1, y_1) = (4, 20)$ and $(x_2, y_2) = (7, 14)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{14 - 20}{7 - 4} = \frac{-6}{3}, \text{ or } -2$$

▶ The slope is -2.

b. Let $(x_1, y_1) = (-1, 2)$ and $(x_2, y_2) = (5, 2)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2 - 2}{5 - (-1)} = \frac{0}{6}, \text{ or } 0$$

The change in y is 0.

▶ The slope is 0.

c. Let $(x_1, y_1) = (-3, 0)$ and $(x_2, y_2) = (-3, 6)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{6 - 0}{-3 - (-3)} = \frac{6}{0} \ ✗$$

The change in x is 0.

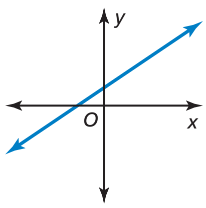
▶ Because division by zero is undefined, the slope of the line is undefined.

6 MTR ASSESS REASONABLENESS
Plot the points in the table to show that your answer in part (a) is reasonable.

CONCEPT SUMMARY

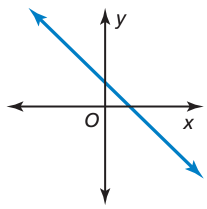
Slope

Positive slope



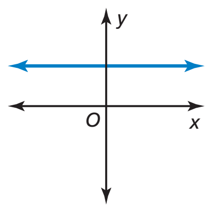
The line rises from left to right.

Negative slope



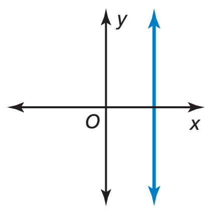
The line falls from left to right.

Slope of 0



The line is horizontal.

Undefined slope



The line is vertical.

SELF-ASSESSMENT

1 I don't understand yet. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

The points represented by each table lie on a line. How can you find the slope of each line from the table? What is the slope of each line?

7.

x	2	4	6	8
y	10	15	20	25

8.

x	5	5	5	5
y	-12	-9	-6	-3

GO DIGITAL

Using the Slope-Intercept Form of a Linear Equation

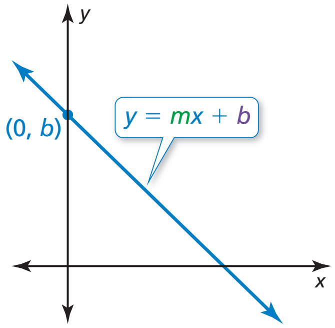
KEY IDEA

Slope-Intercept Form

Words A linear equation written in the form $y = mx + b$ is in **slope-intercept form**. The slope of the line is m, and the y-intercept of the line is b.

Algebra $y = mx + b$

slope (m) y-intercept (b)



A linear equation written in the form $y = 0x + b$, or $y = b$, is a **constant function**. The graph of a constant function is a horizontal line.

EXAMPLE 3 Identifying Slopes and y-Intercepts

WATCH

Find the slope and the y-intercept of the graph of each linear equation.

a. $y = 3x - 4$ **b.** $y = 6.5$ **c.** $-5x - y = -2$

SOLUTION

a. $y = mx + b$ Write the slope-intercept form.

slope y-intercept

$y = 3x + (-4)$ Rewrite the original equation in slope-intercept form.

▶ The slope is 3, and the y-intercept is -4.

b. The equation represents a constant function. The equation can also be written as $y = 0x + 6.5$.

▶ The slope is 0, and the y-intercept is 6.5.

c. Rewrite the equation in slope-intercept form by solving for y.

$-5x - y = -2$ Write the original equation.

$-y = 5x - 2$ Add $5x$ to each side.

$y = -5x + 2$ Divide each side by -1.

▶ The slope is -5, and the y-intercept is 2.

STUDY TIP

For a constant function, every input has the same output. For instance, in Example 3(b), every input has an output of 6.5.

STUDY TIP

When you rewrite a linear equation in slope-intercept form, you are expressing y as a function of x.

SELF-ASSESSMENT

1 I don't understand yet. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

Find the slope and the y-intercept of the graph of the linear equation.

9. $y = -6x + 1$ **10.** $y = -\frac{1}{2}$ **11.** $x + 4y = -10$

4 MTR **12. WHICH ONE DOESN'T BELONG?** Which equation does *not* belong with the other three? Explain your reasoning.

$y = -5x - 1$	$2x - y = 8$	$y = x + 4$	$y = -3x + 13$

GO DIGITAL

EXAMPLE 4 Using Slope-Intercept Form to Graph an Equation

WATCH

Graph $2x + y = 4$. Identify the x-intercept.

SOLUTION

Step 1 Rewrite the equation in slope-intercept form.

$$y = -2x + 4$$

Step 2 Find the slope and the y-intercept.

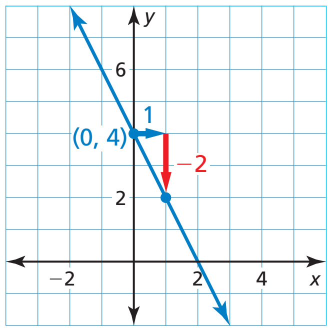
$$m = -2 \text{ and } b = 4$$

Step 3 The y-intercept is 4. So, plot (0, 4).

Step 4 Use the slope to find another point on the line.

$$\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{-2}{1}$$

Plot the point that is 1 unit right and 2 units down from (0, 4). Draw a line through the two points.



▶ The line appears to intersect the x-axis at (2, 0). So, the x-intercept is 2.

STUDY TIP

You can use the slope to find points on a line in either direction. In Example 4, note that the slope can be written as $\frac{2}{-1}$. So, you can move 1 unit left and 2 units up from (0, 4) to find the point (−1, 6).

2 MTR USE ANOTHER METHOD

Show how you can find the x-intercept by substituting 0 for y in the equation.

EXAMPLE 5 Graphing from a Verbal Description

WATCH

A linear function g models a relationship in which the dependent variable increases 3 units for every 1 unit the independent variable increases. Graph g when $g(0) = 3$. Identify the slope and the intercepts of the graph.

SOLUTION

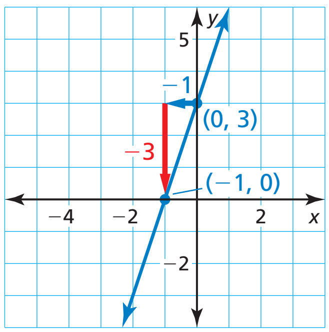
Because the function g is linear, it has a constant rate of change. Let x represent the independent variable and y represent the dependent variable.

Step 1 Find the slope. When the dependent variable increases by 3, the change in y is +3. When the independent variable increases by 1, the change in x is +1.

So, the slope is $\frac{3}{1}$, or 3.

Step 2 Find the y-intercept. The statement $g(0) = 3$ indicates that when $x = 0$, $y = 3$. So, the y-intercept is 3. Plot (0, 3).

Step 3 Use the slope to find another point on the line. A slope of 3 can be written as $\frac{-3}{-1}$. Plot the point that is 1 unit left and 3 units down from (0, 3). Draw a line through the two points. The line crosses the x-axis at (−1, 0). So, the x-intercept is −1.



▶ The slope is 3, the y-intercept is 3, and the x-intercept is −1.

SELF-ASSESSMENT

1 I don't understand yet. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

Graph the linear equation. Identify the x-intercept.

13. $y = 4x - 4$

14. $3x + y = -3$

15. $x + 2y = 6$

16. A linear function h models a relationship in which the dependent variable decreases 2 units for every 5 units the independent variable increases. Graph h when $h(0) = 4$. Identify the slope and the intercepts of the graph.

GO DIGITAL

Solving Real-Life Problems

In most real-life problems, slope indicates a rate, such as miles per hour, dollars per hour, or people per year.

7 MTR

EXAMPLE 6 **Modeling Real Life** WATCH

The function $h(t) = -75t - 4000$ represents the elevation $h(t)$ (in feet) of a submersible t minutes after it begins to descend. The ocean floor is at a depth of 13,000 feet.

a. Find the domain and range in this context. Then graph the function.

b. Interpret the slope and the intercepts of the graph.

SOLUTION

1. Understand the Problem You know the function that models the elevation. You are asked to find the domain and range in this context and to graph the function. Then you are asked to interpret the slope and intercepts of the graph.

2. Make a Plan Use the equation to find the domain and range in this context. Then graph the function. Examine the graph to interpret the slope and the intercepts.

3. Solve and Check

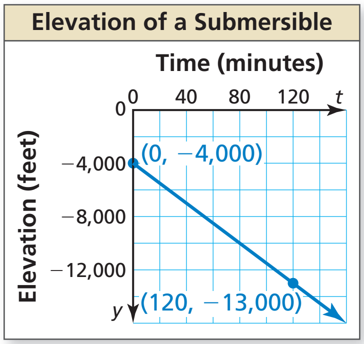
a. Because the ocean floor is at a depth of 13,000 feet, $h(t) \geq -13{,}000$. Find the value of t when $h(t) = -13{,}000$.

$h(t) = -75t - 4000$ Write the equation.

$-13{,}000 = -75t - 4000$ Substitute –13,000 for $h(t)$.

$120 = t$ Solve for t.

The time t must be greater than or equal to 0, and the y-intercept is -4000. So, the domain is $\{t \mid 0 \leq t \leq 120\}$, and the range is $\{h(t) \mid -13{,}000 \leq h(t) \leq -4{,}000\}$. Use the slope of -75 and the y-intercept of -4000 to graph the function in this context.



b. The slope is -75. So, the elevation changes at a rate of -75 feet per minute. The term -4000 is the y-intercept. So, the elevation of the submersible when the descent begins is -4000 feet. There is no t-intercept in this context.

In 1975, scientists used submersibles to discover the Oculina coral reefs off Florida's east coast. The discovery led to the world's first Marine Protected Area for deepwater coral.

Look Back Show that the slope between the points $(0, -4000)$ and $(120, -13{,}000)$ is -75.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{-13{,}000 - (-4000)}{120 - 0}$$

$$= -75 \checkmark$$

SELF-ASSESSMENT

1 I don't understand yet. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

17. The table shows the elevation $p(t)$ (in meters) of a submersible t minutes after it begins to descend. The ocean floor is at a depth of 3810 meters.

a. Does this submersible descend faster than the submersible in Example 6? Use a graph to justify your answer.

b. Both submersibles begin to descend at the same time. Which one reaches the ocean floor first? Explain.

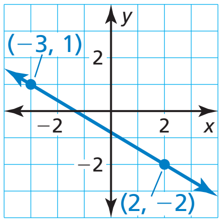
Time (minutes), t	Elevation (meters), $p(t)$
0	−1800
5	−1900
10	−2000

GO DIGITAL

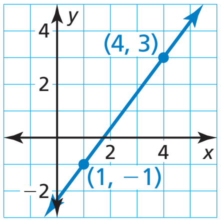
3.6 Practice WITH CalcChat® AND CalcView®

In Exercises 1–4, describe the slope of the line. Then find the slope. *(See Example 1.)*

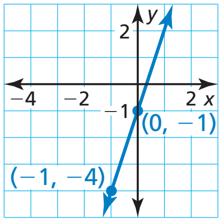
▶ **1.**



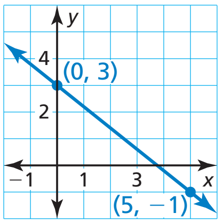
2.



3.



4.



In Exercises 5 and 6, find the slope of the line that passes through the given points.

5. $(1, 4), (3, -6)$ **6.** $(2, -2), (-7, -5)$

In Exercises 7–10, the points represented by the table lie on a line. Find the slope of the line. *(See Example 2.)*

▶ **7.**

x	−9	−5	−1	3
y	−2	0	2	4

8.

x	−1	2	5	8
y	−6	−6	−6	−6

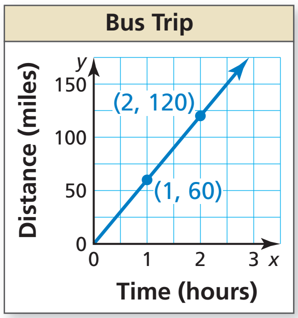
9.

x	0	0	0	0
y	−4	0	4	8

10.

x	−4	−3	−2	−1
y	2	−5	−12	−19

11. ANALYZING A GRAPH The graph shows the distance y (in miles) that a bus travels in x hours. Find and interpret the slope of the line.



12. ANALYZING A TABLE The table shows the amount x (in hours) of time you spend at a theme park and the admission fee y (in dollars) to the park. The points represented by the table lie on a line. Find and interpret the slope of the line.

Time (hours), *x*	Admission (dollars), *y*
6	54.99
7	54.99
8	54.99

In Exercises 13–20, find the slope and the *y*-intercept of the graph of the linear equation. *(See Example 3.)*

13. $y = -3x + 2$ **14.** $y = 4x - 7$

15. $y = 6x$ **16.** $y = -1$

▶ **17.** $-0.75x + y = 4$ **18.** $x + y = -6\frac{1}{2}$

19. $\frac{1}{6}x = \frac{1}{3} - y$ **20.** $0 = 4.5 - 2y + 4.8x$

4 MTR **ERROR ANALYSIS** **In Exercises 21 and 22, describe and correct the error in finding the slope and the *y*-intercept of the graph of the equation.**

21.

✗ $x = -4y$
The slope is −4, and the y-intercept is 0.

22.

✗ $y = 3x - 6$
The slope is 3, and the y-intercept is 6.

In Exercises 23–30, graph the linear equation. Identify the *x*-intercept. *(See Example 4.)*

23. $y = -x + 7$ **24.** $y = \frac{1}{2}x + 3$

25. $y = 2x$ **26.** $y = -x$

▶ **27.** $3x + y = -1$ **28.** $x + 4y = 8$

29. $-y + \frac{3}{5}x = 0$ **30.** $2.5x - y - 7.5 = 0$

GO DIGITAL

In Exercises 31 and 32, graph the function with the given description. Identify the slope and the intercepts of the graph. *(See Example 5.)*

31. A linear function f models a relationship in which the dependent variable decreases 4 units for every 2 units the independent variable increases, and $f(0) = -2$.

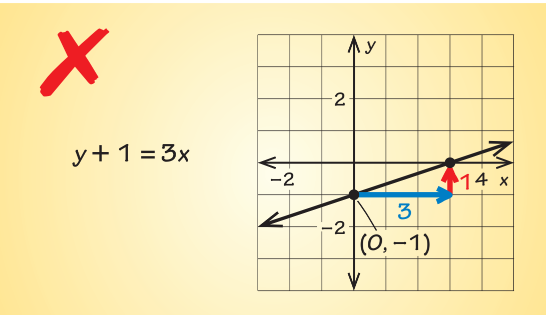
32. A linear function h models a relationship in which the dependent variable increases 1 unit for every 5 units the independent variable decreases, and $h(0) = 3$.

33. MODELING REAL LIFE A linear function r models the growth of your right index fingernail. The length of the fingernail increases 0.7 millimeter every week. Graph r when $r(0) = 12$. Identify the slope and interpret the y-intercept of the graph.

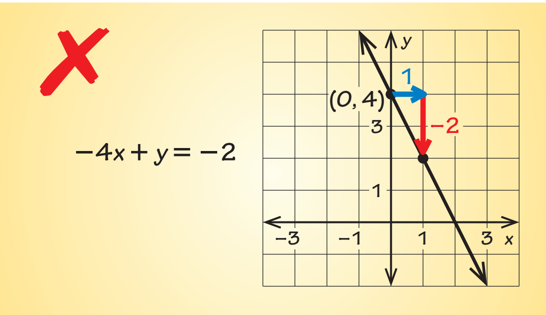
34. MODELING REAL LIFE A linear function m models the amount of milk sold by a farm per month. The amount decreases 500 gallons for every \$1 increase in price. Graph m when $m(0) = 3000$. Identify the slope and interpret the intercepts of the graph.

ERROR ANALYSIS In Exercises 35 and 36, describe and correct the error in graphing the function.

35.



36.



37. MODELING REAL LIFE The function $d(t) = \frac{1}{2}t + 2$ represents the amount (in inches) of water in a rain barrel for six rainy days, where t is the time (in days) since the rainfall began. *(See Example 6.)*

a. Find the domain and range in this context. Then graph the function.

b. Interpret the slope and intercepts of the graph.

38. MODELING REAL LIFE The function $f(x) = -200x + 1000$ represents the altitude (in feet) of a paraglider x minutes from the time the paraglider begins a descent to a landing site located 100 feet above sea level.

a. Find the domain and range in this context. Then graph the function.

b. Interpret the slope and intercepts of the graph.

c. The function $g(x) = -150x + 900$ represents the altitude (in feet) of a second paraglider x minutes from the time the paraglider begins a descent to the same landing site. Both paragliders start their descent at the same time. Who reaches an altitude of 100 feet first? Explain.

39. CHOOSE A METHOD Describe two ways to graph the equation $4x - 6y = 18$. Which method do you prefer? Explain.

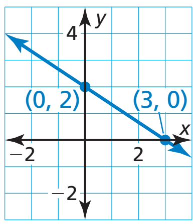
40. PROBLEM SOLVING A linear function represents the cost of catering an event from a Puerto Rican food truck. The table shows the cost y (in dollars) for x dishes. The function $c(x) = 9x + 600$ represents the cost (in dollars) of catering an event from a Dominican food truck, where x is the number of dishes ordered. Graph each function in terms of the context. Which truck charges a greater initial fee? Which truck can cater the greatest number of dishes for \$695?

Dishes, x	Cost (dollars), y
0	500
5	550
10	600

The national dish of Puerto Rico is arroz con gandules, which features rice and pigeon peas seasoned with sofrito.

41. B.E.S.T. TEST PREP Which of the following linear functions has a slope of $-\frac{2}{3}$ and a y-intercept of 2? Select all that apply.

Ⓐ



Ⓑ

x	y
−2	6
0	3
2	0
4	−3

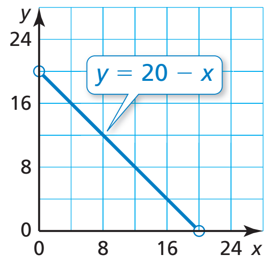
Ⓒ $f(x)$ decreases by 3 units for every 2 units x increases, and $f(0) = 2$.

Ⓓ $-y + 2 = \frac{2}{3}x$

GO DIGITAL

42. WRITING Describe the end behavior of the function $y = mx + b$ when (a) $m > 0$ and (b) $m < 0$.

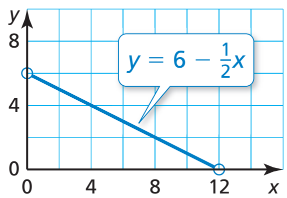
5 MTR **43. CONNECTING CONCEPTS** The graph shows the relationship between the width y (in inches) and the length x (in inches) of a rectangle. The perimeter of a second rectangle is 10 inches less than the perimeter of the first rectangle.



a. Graph the relationship between the width and length of the second rectangle.

b. How does your graph in part (a) compare to the graph shown?

5 MTR **44. CONNECTING CONCEPTS** The graph shows the relationship between the base length x (in meters) and the lengths y (in meters) of the two equal sides of an isosceles triangle. The perimeter of a second isosceles triangle is 8 meters more than the perimeter of the first triangle.



a. Graph the relationship between the base length and the side lengths of the second triangle.

b. How does your graph in part (a) compare to the graph shown?

5 MTR **45. CONNECTING CONCEPTS** Graph the equations in the same coordinate plane. What is the area of the enclosed figure?

$3y = -9$

$2y - 14 = 4x$

$-4x + 5 - y = 0$

$y - 1 = 0$

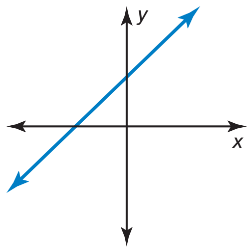
4 MTR **46. MAKING AN ARGUMENT** Your friend says that you can write the equation of any line in slope-intercept form. Is your friend correct? Explain your reasoning.

GO DIGITAL

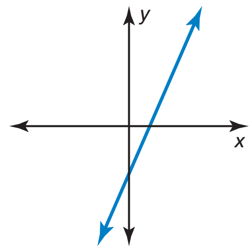
47. ANALYZING EQUATIONS Which equations could be represented by each graph? (The graphs are not drawn to scale.)

$y = -3x + 8$	$y = -x - \frac{4}{3}$
$y = -7x$	$y = 2x - 4$
$y = \frac{7}{4}x - \frac{1}{4}$	$y = \frac{1}{3}x + 5$
$y = -4x - 9$	$y = 6$

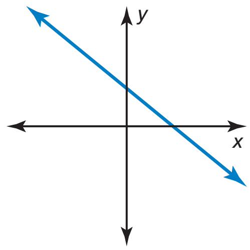
a.



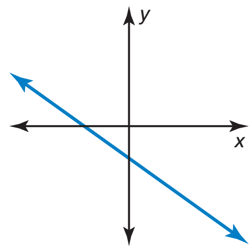
b.



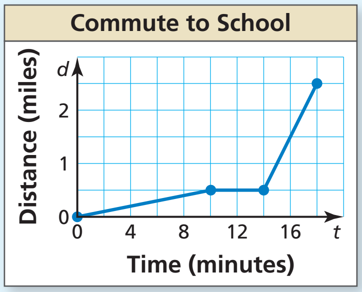
c.



d.



48. HOW DO YOU SEE IT?
You commute to school by walking and by riding a bus. The graph represents your commute.



a. Describe your commute in words.

b. Calculate and interpret the slopes of the different parts of the graph.

PROBLEM SOLVING In Exercises 49 and 50, find the value of k so that the graph of the equation has the given slope or y-intercept.

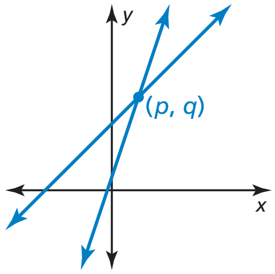
49. $16kx - 4y = 20;\ m = \frac{1}{2}$

50. $\frac{2}{3}x + 2y - \frac{5}{3}k = 0;\ b = -10$

51. **DIG DEEPER** To show that the slope of a line is constant, let (x_1, y_1) and (x_2, y_2) be any two points on the line $y = mx + b$. Use the equation of the line to express y_1 in terms of x_1 and y_2 in terms of x_2. Then show that the slope between the points is m.

52. **THOUGHT PROVOKING**
Your family goes on vacation to a beach 300 miles from your house. You stop along the way and reach your destination 6 hours after departing. Draw a graph that describes your trip. Explain what each part of your graph represents.

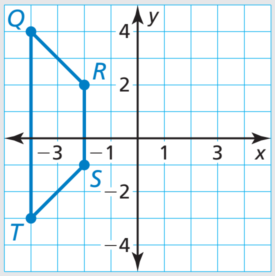
53. **DIG DEEPER** The graphs of the functions $g(x) = 6x + a$ and $h(x) = 2x + b$, where a and b are constants, are shown. They intersect at the point (p, q). Which is greater, $g(p + 2)$ or $h(p + 2)$? How much greater? Explain your reasoning.



REVIEW & REFRESH

WATCH

In Exercises 54–56, find the coordinates of the figure after the transformation.



54. Translate the trapezoid 3 units right.

55. Dilate the trapezoid with respect to the origin using a scale factor of $\frac{1}{2}$.

56. Reflect the trapezoid in the y-axis.

In Exercises 57–60, solve the equation.

57. $-\frac{x}{7} = 2.5$

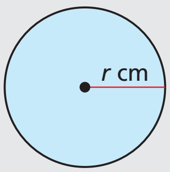
58. $\frac{1}{3}n - 4 = -4 + \frac{1}{3}n$

59. $-4(7 - a) + 10 = -12$

60. $5(2q - 1) = -3(q + 6)$

61. Find the slope and y-intercept of the graph of $7x - 4y = 10$.

62. The circumference of the circle is at least 14π centimeters. Find the possible values of r.



2 MTR **63.** **MAKE A CONNECTION** Let f be a function. Use each statement to write a point on the graph of f.

a. $f(3)$ is equal to -8.

b. A solution of the equation $f(x) = \frac{3}{4}$ is -1.

64. Graph $y = -\frac{3}{2}x - 6$. Identify the x-intercept.

In Exercises 65 and 66, determine whether the table or equation represents a *linear* or *nonlinear* function. Explain.

65.

x	−2	−1	0	1
y	−4	−1	3	8

66. $\frac{x}{4} + \frac{y}{12} = 1$

7 MTR **67.** **MODELING REAL LIFE** The table shows the prices of four video games. You have a coupon for 20% off one game, and you want to spend a total of \$30 with an absolute deviation of at most \$5. Which video games can you buy? Use an absolute value inequality to justify your answer.

Video game	Price
A	\$44.99
B	\$41.99
C	\$49.99
D	\$39.99

68. Use intercepts to graph the equation $-3x + 2y = 9$. Label the points corresponding to the intercepts.

In Exercises 69 and 70, graph the inequality.

69. $h < -4$

70. $\frac{9}{2} \leq t$

GO DIGITAL