3.5 Graphing Linear Equations in Standard Form

Learning Target: Graph and interpret linear equations written in standard form.

Success Criteria:
- I can graph equations of horizontal and vertical lines.
- I can graph linear equations written in standard form using intercepts.
- I can solve real-life problems using linear equations in standard form.

EXPLORE IT! Analyzing and Graphing a Linear Equation

Work with a partner. You sold a total of \$80 in tickets to a fundraiser. You lost track of how many of each type of ticket you sold. Adult tickets are \$4 each. Child tickets are \$2 each. The equation $4x + 2y = 80$ represents this situation, where x is the number of adult tickets and y is the number of child tickets.

a. If you sold a large quantity of adult tickets, does that mean you also sold a large quantity of child tickets? Explain.

b. Construct a table of values to show different combinations of tickets you might have sold. Then plot the points and describe any patterns you notice.

x					
y					

c. Graph the equation. Find the intercepts. Explain the meanings of the intercepts in the context of the problem.

d. Use technology to check your results in parts (b) and (c). Describe the characteristics of the graph.

e. If you know how many adult tickets you sold, can you determine how many child tickets you sold? Explain your reasoning.

f. Determine whether each statement is correct. Explain your reasoning.

i. As the value of x increases, the value of $2y$ decreases.

ii. As the value of y decreases, the value of $4x$ decreases.

iii. For $x < 10$, $y > 20$.

iv. $x = 20$ makes the equation true.

2 MTR MODEL A PROBLEM
How many different ways did you model the problem? Describe the benefits of each representation.

Algebraic Reasoning
MA.912.AR.2.4 Given a table, equation or written description of a linear function, graph that function, and determine and interpret its key features.
MA.912.AR.2.5 Solve and graph mathematical and real-world problems that are modeled with linear functions. Interpret key features and determine constraints in terms of the context.

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Horizontal and Vertical Lines

Vocabulary

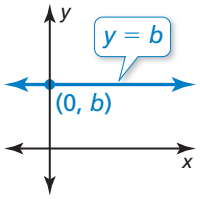
standard form, *p. 132*

The **standard form** of a linear equation is $Ax + By = C$, where A, B, and C are real numbers and A and B are not both zero.

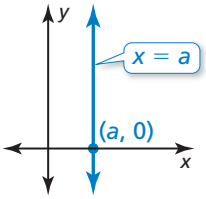
Consider what happens when $A = 0$ or when $B = 0$. When $A = 0$, the equation becomes $By = C$, or $y = \frac{C}{B}$. Because $\frac{C}{B}$ is a constant, you can write $y = b$. Similarly, when $B = 0$, the equation becomes $Ax = C$, or $x = \frac{C}{A}$, and you can write $x = a$.

KEY IDEAS

Horizontal and Vertical Lines



The graph of $y = b$ is a horizontal line. The line passes through the point $(0, b)$.



The graph of $x = a$ is a vertical line. The line passes through the point $(a, 0)$.

STUDY TIP

For a horizontal line, notice that for every value of x, the value of y is b.

For a vertical line, notice that for every value of y, the value of x is a.

EXAMPLE 1 Graphing Horizontal and Vertical Lines

WATCH

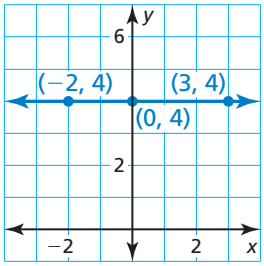
Graph each linear equation.

a. $y = 4$

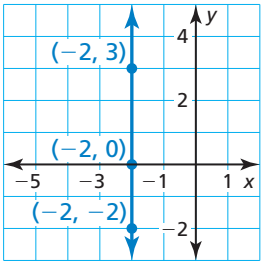
b. $x = -2$

SOLUTION

a. For every value of x, the value of y is 4. The graph of the equation $y = 4$ is a horizontal line 4 units above the x-axis.



b. For every value of y, the value of x is -2. The graph of the equation $x = -2$ is a vertical line 2 units to the left of the y-axis.



STUDY TIP

For every value of x, the ordered pair $(x, 4)$ is a solution of $y = 4$.

SELF-ASSESSMENT

1 I don't understand yet. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

Graph the linear equation.

1. $y = -2.5$ **2.** $x = 5$ **3.** $x = -\frac{4}{3}$ **4.** $y = 18$

5. WRITING Describe the x- and y-intercepts of the horizontal line that passes through the origin and the vertical line that passes through the origin.

6. REASONING Graph $x = -1$ and $y = -1$. Does each graph represent a function? If so, find the domain and range.

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Using Intercepts to Graph Linear Equations

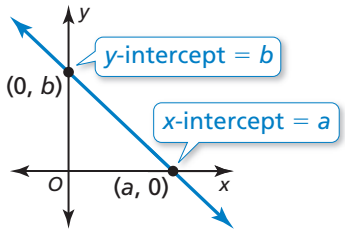
You can use the fact that two points determine a line to graph a linear equation. Two convenient points are the x- and y-intercepts.

KEY IDEA

Using Intercepts to Graph Equations

To graph the linear equation $Ax + By = C$ using intercepts, find the intercepts and draw the line that passes through them.

- To find the x-intercept, let $y = 0$ and solve for x.
- To find the y-intercept, let $x = 0$ and solve for y.



EXAMPLE 2 Using Intercepts to Graph a Linear Equation

Use intercepts to graph the equation $3x + 4y = 12$.

WATCH

SOLUTION

Step 1 Find the intercepts.

To find the x-intercept, substitute 0 for y and solve for x.

$3x + 4y = 12$ Write the original equation.

$3x + 4(0) = 12$ Substitute 0 for y.

$x = 4$ Solve for x.

To find the y-intercept, substitute 0 for x and solve for y.

$3x + 4y = 12$ Write the original equation.

$3(0) + 4y = 12$ Substitute 0 for x.

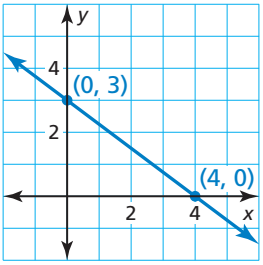
$y = 3$ Solve for y.

Step 2 Plot the points and draw the line.

The x-intercept is 4, so plot the point (4, 0).

The y-intercept is 3, so plot the point (0, 3).

Draw a line through the points.



STUDY TIP

You can check your answer by finding other solutions of the equation and verifying that the corresponding points are on the graph.

SELF-ASSESSMENT

1 I don't understand yet. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

4 MTR **7. DISCUSS MATHEMATICAL THINKING** What are some advantages of using the standard form of a linear equation?

Use intercepts to graph the linear equation. Label the points corresponding to the intercepts.

8. $2x - y = 4$ **9.** $x + 3y = -9$ **10.** $\frac{3}{4}x + 2y = 6$

2 MTR **11. MAKE A CONNECTION** Describe the graph of a linear equation written in the form $Ax + By = C$ when $C = 0$.

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Solving Real-Life Problems

EXAMPLE 3 Modeling Real Life

WATCH

5 MTR DECOMPOSE A PROBLEM
What do the terms $6x$ and $10y$ represent in this situation?

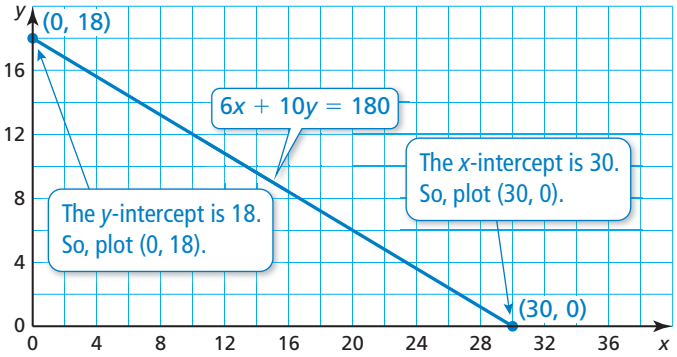
You are planning an awards banquet and need to rent tables to seat 180 people. There are two table sizes available. Small tables seat 6 people, and large tables seat 10 people. The equation $6x + 10y = 180$ models this situation, where x is the number of small tables and y is the number of large tables.

a. Graph the equation. Interpret the intercepts.

b. Find three possible solutions in the context of the problem.

SOLUTION

a. Use intercepts to graph the equation. Neither x nor y can be negative, so graph the equation only in the first quadrant.



STUDY TIP
Although x and y represent discrete data, it is convenient to draw a line segment that includes points whose coordinates are not whole numbers.

Use the graph to interpret the intercepts.

▶ The x-intercept shows that you can rent 30 small tables when you do not rent any large tables. The y-intercept shows that you can rent 18 large tables when you do not rent any small tables.

b. Only whole-number values of x and y make sense in the context of the problem. Besides the intercepts, it appears that the line passes through the point (10, 12). To verify that this point is a solution, check it in the equation.

$$6x + 10y = 180$$
$$6(10) + 10(12) \stackrel{?}{=} 180$$
$$180 = 180 \checkmark$$

▶ So, three possible combinations of tables that will seat 180 people are 0 small and 18 large, 10 small and 12 large, and 30 small and 0 large.

SELF-ASSESSMENT

1 I don't understand yet. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

12. WHAT IF? You decide to rent tables from a different company. The situation can be modeled by the equation $4x + 6y = 180$, where x is the number of small tables and y is the number of large tables.

a. Interpret the terms and coefficients in the equation.

b. Graph the equation. Interpret the intercepts.

c. Find three possible solutions in the context of the problem.

13. The number of people attending the banquet in Example 3 increases by 25%. Your friend claims that not all of the tables will be completely filled. Is your friend correct? Explain.

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3.5 Practice WITH CalcChat® AND CalcView®

In Exercises 1–4, graph the linear equation. *(See Example 1.)*

1. $x = 4$

2. $y = -3$

3. $y = \frac{1}{2}$

4. $x = -1.5$

In Exercises 5–8, find the x- and y-intercepts of the graph of the linear equation.

5. $2x + 3y = 12$

6. $-6x + 9y = -18$

7. $3x = 6y + 2$

8. $\frac{3}{4} + x = \frac{1}{2}y$

In Exercises 9–18, use intercepts to graph the linear equation. Label the points corresponding to the intercepts. *(See Example 2.)*

9. $5x + 3y = 30$

10. $4x + 6y = 12$

11. $-12x + 3y = 24$

12. $-2x + 6y = 18$

13. $-4x + 3y = -30$

14. $-2x + 7y = -21$

15. $2y - x = 7$

16. $3x + 5 = y$

17. $\frac{4}{3} + \frac{2}{3}x = \frac{1}{6}y$

18. $y = \frac{1}{4} - \frac{5}{2}x$

2 MTR MULTIPLE REPRESENTATIONS In Exercises 19–22, match the equation with its graph.

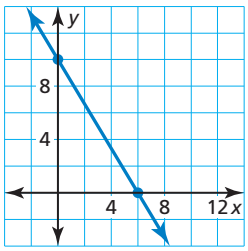
19. $5x + 3y = 30$

20. $5x + 3y = -30$

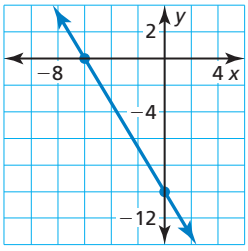
21. $5x - 3y = 30$

22. $5x - 3y = -30$

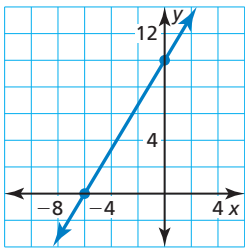
A.



C.



B.



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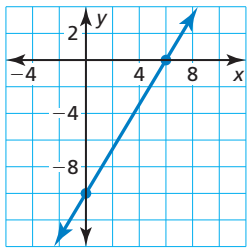


7 MTR 23. MODELING REAL LIFE You have a budget of \$300 to order shirts for a math club. The equation $10x + 12y = 300$ models the total cost, where x is the number of short-sleeved shirts and y is the number of long-sleeved shirts. *(See Example 3.)*

a. Interpret the terms and coefficients in the equation.

b. Graph the equation. Interpret the intercepts.

c. Find three possible solutions in the context of the problem.

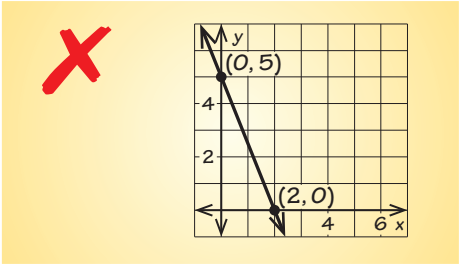
7 MTR 24. MODELING REAL LIFE Your goal is to bike and jog a total of 150 miles this month. You want to bike no more than 120 miles this month. The equation $12.5x + 6y = 150$ models this situation, where x is the number of hours you bike and y is the number of hours you jog.

a. Interpret the terms and coefficients in the equation.

b. Graph the equation. Interpret the intercept(s).

c. You bike for 9 hours this month. How many hours must you jog to reach your goal? How many miles do you bike? jog?

4 MTR 25. ERROR ANALYSIS Describe and correct the error in using intercepts to graph the linear equation $4x + 10y = 20$.



4 MTR 26. MAKING AN ARGUMENT To find the x-intercept of the graph of a linear equation, can you substitute 0 for x and solve the equation? Explain.

5 MTR CONNECTING CONCEPTS In Exercises 27–30, write a set of linear equations that intersect to form the enclosed shape.

27. rectangle

28. square

29. right triangle

30. trapezoid

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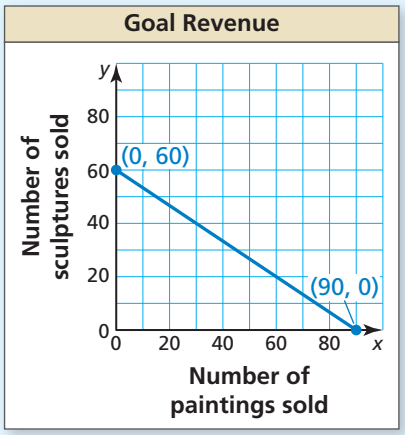
31. REASONING Are the equations of horizontal and vertical lines written in standard form? Explain.

32. HOW DO YOU SEE IT?
An artist wants to earn a revenue of \$2700 by selling paintings for \$30 each and sculptures for \$45 each.



a. Interpret the intercepts of the graph.

b. Describe the domain and range in the context of the problem.

33. B.E.S.T. TEST PREP Which of the following is *not* true about the graph of $-\frac{2}{5}x + \frac{1}{10}y = -\frac{4}{5}$?

Ⓐ The x-intercept is 2 and the y-intercept is -8.

Ⓑ The function is decreasing when $x < 2$ and increasing when $x > 2$.

Ⓒ The graph passes through $(1, -4)$ and $(5, 12)$.

Ⓓ $y \to -\infty$ as $x \to -\infty$ and $y \to +\infty$ as $x \to +\infty$.

34. THOUGHT PROVOKING
The x- and y-intercepts of the graph of $ax + by = k$ are integers. Describe the possible values of k. Explain your reasoning.

35. DIG DEEPER You have \$99 to buy stamps and envelopes. A sheet of 20 stamps costs \$11. A box of 50 envelopes costs \$7.50.

a. Write an equation in standard form that models this situation. Do the intercepts of the graph make sense in this context? Explain.

b. Can you use all of the money to buy the same numbers of stamps and envelopes? Explain.

REVIEW & REFRESH

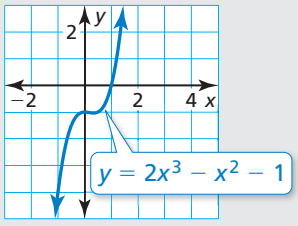
WATCH

7 MTR **36. MODELING REAL LIFE** The function $D(t) = 75 - 0.3t$ represents the number of gigabytes left after downloading a video game for t minutes.

a. How many gigabytes are left to download after 90 minutes?

b. How long will it take to download the entire video game?

37. Estimate the intercepts of the graph of the function.



38. WRITING Explain how you can determine whether a graph represents a *linear* or a *nonlinear* function.

39. Determine whether the equation $y = x(2 - x)$ represents a *linear* or *nonlinear* function. Explain.

In Exercises 40 and 41, solve the inequality. Graph the solution.

40. $b + 5 \leq -12$

41. $-\frac{c}{3} > -15$

42. REASONING Complete the equation $\square x + \square y = 30$ so that the x-intercept of the graph is -10 and the y-intercept of the graph is 5.

In Exercises 43–46, solve the equation. Check your solutions.

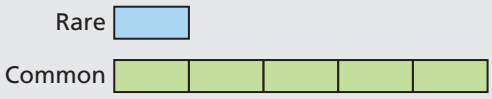
43. $6.8 + g = 14.1$

44. $-11 = 7 - 3(h + 2)$

45. $3(4 - 8k) = -4(6k - 3)$

46. $5|6n - 9| + 4 = 29$

47. The tape diagram represents the ratio of rare cards to common cards in a collection. There are 9 rare cards. How many common cards are in the collection?



48. For $f(x) = -\frac{2}{3}x + 1$, find the value of x for which $f(x) = 9$.

49. Find the x- and y-intercepts of the graph of $-4x + 8y = -16$.

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