

3.3 Linear Functions



Learning Target: Identify and graph linear functions.

- Success Criteria:**
- I can identify linear functions using graphs, tables, and equations.
 - I can determine whether a domain is discrete or continuous in a real-life situation.
 - I can graph linear functions with discrete and continuous domains.

EXPLORE IT! Finding a Pattern

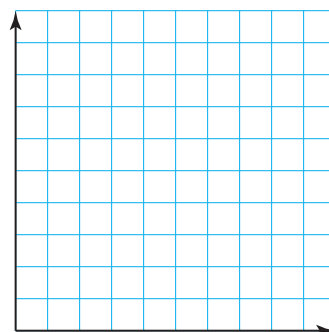
Work with a partner. Use a piece of rope that is at least 100 centimeters long. Record your data in the table.

- Measure the length of the rope. Describe your measurement.
- Make a knot in the rope, and then measure the length of the rope again. Continue to make identical knots in the rope, measuring the length of the rope after each knot is tied.



Number of knots	Length of rope
0	
1	
2	
3	
4	
5	
6	
7	
8	

- Write several observations about the data. What pattern(s) do you notice in the data? Explain.
- Make a scatter plot of the data. What pattern(s) do you notice in the scatter plot? Explain.
- How can you predict the length of the rope when it has 10 knots? Explain your reasoning.
- Does it matter where you tie the knots on the rope? Is there a maximum number of knots you can tie? Does the thickness of the rope or the type of knot you tie affect your results? Explain your reasoning.



2 MTR USE ANOTHER METHOD

Is there more than one way to predict the length in part (e)? Explain.

Algebraic Reasoning

MA.912.AR.2.4 Given a table, equation or written description of a linear function, graph that function, and determine and interpret its key features.

Functions

MA.912.F.1.1 Given an equation or graph that defines a function, determine the function type. Given an input-output table, determine a function type that could represent it.

Also **MA.912.AR.2.5**

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Vocabulary



linear equation in two variables, p. 116
 linear function, p. 116
 nonlinear function, p. 116
 solution of a linear equation in two variables, p. 119
 discrete domain, p. 119
 continuous domain, p. 119

Identifying Linear Functions

A **linear equation in two variables**, x and y , is an equation that can be written in the form

$$y = mx + b$$

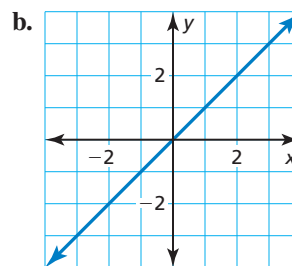
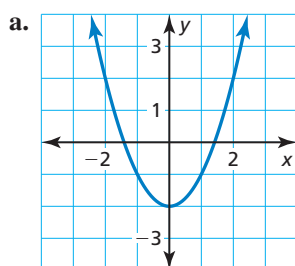
where m and b are constants. The graph of a linear equation is a line. Likewise, a **linear function** is a function whose graph is a nonvertical line. A linear function has a constant rate of change and can be represented by a linear equation in two variables. A **nonlinear function** does not have a constant rate of change. So, its graph is *not* a line.

EXAMPLE 1

Identifying Linear Functions Using Graphs



Does the graph represent a *linear* or *nonlinear* function? Explain.



SOLUTION

a. The graph is *not* a line.

▶ So, the function is nonlinear.

b. The graph is a nonvertical line.

▶ So, the function is linear.

EXAMPLE 2

Identifying Linear Functions Using Tables



Does the table represent a *linear* or *nonlinear* function? Explain.

a.

x	3	6	9	12
y	36	30	24	18

b.

x	1	3.5	6	8.5
y	2	9	20	35

SOLUTION

a.

x	3	6	9	12
y	36	30	24	18

$\xrightarrow{+3}$ $\xrightarrow{+3}$ $\xrightarrow{+3}$
 $\xleftarrow{-6}$ $\xleftarrow{-6}$ $\xleftarrow{-6}$

As x increases by 3, y decreases by 6. The rate of change is constant.

▶ So, the function is linear.

b.

x	1	3.5	6	8.5
y	2	9	20	35

$\xrightarrow{+2.5}$ $\xrightarrow{+2.5}$ $\xrightarrow{+2.5}$
 $\xleftarrow{+7}$ $\xleftarrow{+11}$ $\xleftarrow{+15}$

As x increases by 2.5, y increases by different amounts. The rate of change is *not* constant.

▶ So, the function is nonlinear.

STUDY TIP

A constant rate of change describes a quantity that changes by equal amounts over equal intervals.



EXAMPLE 3

B.E.S.T. Test Prep: Identifying Linear Functions Using Equations



Which of the following equations represent linear functions? Explain.

- Ⓐ $0 = -x - y$ Ⓒ $y = 2^x$ Ⓔ $y = -1 + x$
 Ⓑ $y = \sqrt{x}$ Ⓓ $y = |x| - 1$ Ⓕ $x^2 - y = 0$

SOLUTION

You cannot rewrite the equations $y = \sqrt{x}$, $y = 2^x$, $y = |x| - 1$, and $x^2 - y = 0$ in the form $y = mx + b$. So, they cannot represent linear functions.

You can rewrite the equation $0 = -x - y$ as $y = -x$ and the equation $y = -1 + x$ as $y = x - 1$.

► So, the correct answers are Ⓐ and Ⓔ.

CONCEPT SUMMARY

Representations of Functions

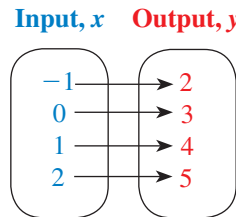
Words An output is 3 more than the input.

Equation $y = x + 3$

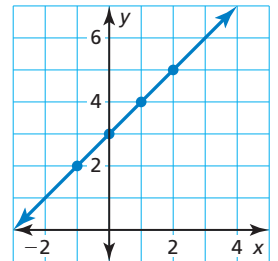
Input-Output Table

Input, x	Output, y
-1	2
0	3
1	4
2	5

Mapping Diagram



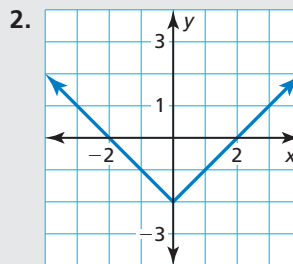
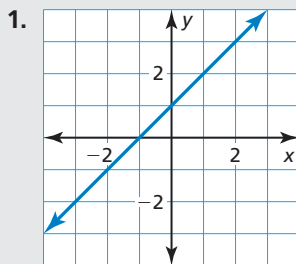
Graph



SELF-ASSESSMENT

- 1 I don't understand yet. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

Does the graph or table represent a *linear* or *nonlinear* function? Explain.



3.

x	0	1	2	3
y	3	5	7	9

4.

x	1	2	3	4
y	$\frac{1}{16}$	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$

Does the equation represent a *linear* or *nonlinear* function? Explain.

5. $y = x + 9$

6. $5 = -x + y$

7. $y = 5 - 2x^2$

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Graphing Linear Functions

EXAMPLE 4

Using Tables to Graph Linear Functions



Graph the linear function represented by each table.

a.

x	y
-2	-4
0	-3
2	-2
4	-1

b.

x	y
-1	5
2	2.5
5	0
8	-2.5

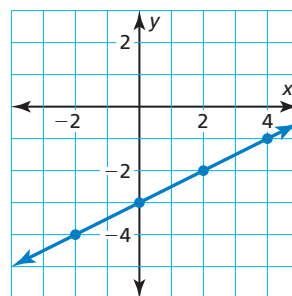
SOLUTION

STUDY TIP

When the domain of a linear function is not specified or cannot be obtained from a real-life context, it is understood to be all real numbers.

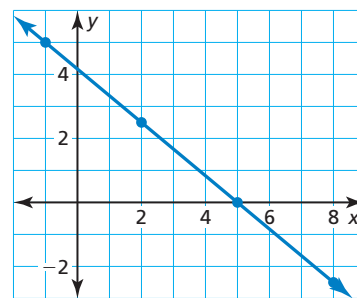
- a. Write each ordered pair (x, y) . Then plot each point in a coordinate plane and draw a line through the points.

x	y	(x, y)
-2	-4	$(-2, -4)$
0	-3	$(0, -3)$
2	-2	$(2, -2)$
4	-1	$(4, -1)$



- b. Write each ordered pair (x, y) . Then plot each point in a coordinate plane and draw a line through the points.

x	y	(x, y)
-1	5	$(-1, 5)$
2	2.5	$(2, 2.5)$
5	0	$(5, 0)$
8	-2.5	$(8, -2.5)$



4
MTR

CONSTRUCT AN ARGUMENT

How many points do you need in order to graph a linear function? Explain your reasoning.

SELF-ASSESSMENT

- 1 I don't understand yet. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

Graph the linear function represented by the table.

8.

x	0	1	2	3
y	6	4	2	0

9.

x	-2	-1	0	1
y	-1	0.5	2	3.5

5
MTR

10. **PATTERNS** The graph of a linear function passes through the point $(-3, 2.25)$. As x increases by 4, y decreases by 1.25. Graph the function.

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A **solution of a linear equation in two variables** is an ordered pair (x, y) that makes the equation true. The graph of a linear equation in two variables is the set of points (x, y) in a coordinate plane that represents all solutions of the equation. Sometimes the points are distinct, and other times the points are connected.



KEY IDEAS

Discrete and Continuous Domains

A **discrete domain** is a set of input values that consists of only certain numbers in an interval.

Example: Integers from 1 to 5



A **continuous domain** is a set of input values that consists of all numbers in an interval.

Example: All numbers from 1 to 5



STUDY TIP

The domain of a function depends on the real-life context of the function, not just the equation that represents the function.

EXAMPLE 5

Graphing Discrete Data



The linear function $y = 15.95x$ represents the cost y (in dollars) of x tickets for a museum. Each customer can buy a maximum of four tickets.

- Find the domain of the function. Is the domain discrete or continuous? Explain.
- Graph the function using its domain.

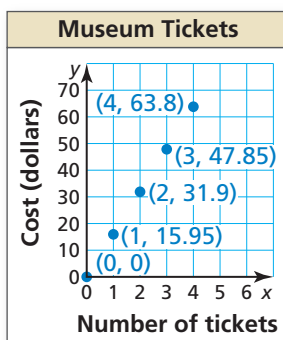
SOLUTION

- You cannot buy part of a ticket, only a certain number of tickets. Because x represents the number of tickets, it must be a whole number. The maximum number of tickets a customer can buy is four.

► So, the domain is 0, 1, 2, 3, and 4, and it is discrete.

- Step 1** Make an input-output table to find the ordered pairs.

Input, x	$15.95x$	Output, y	(x, y)
0	$15.95(0)$	0	(0, 0)
1	$15.95(1)$	15.95	(1, 15.95)
2	$15.95(2)$	31.9	(2, 31.9)
3	$15.95(3)$	47.85	(3, 47.85)
4	$15.95(4)$	63.8	(4, 63.8)



- Step 2** Plot the ordered pairs. The domain is discrete. So, the graph consists of individual points.

SELF-ASSESSMENT

- I don't understand yet.
- I can do it with help.
- I can do it on my own.
- I can teach someone else.

- The linear function $m = 50 - 9d$ represents the amount m (in dollars) of money you have left after buying d DVDs.
 - Interpret the terms and coefficient in the equation.
 - Find the domain of the function. Is the domain discrete or continuous? Explain.
 - Graph the function using its domain.

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EXAMPLE 6

Graphing Continuous Data



Each year, thousands of peregrine falcons fly over the Florida Keys as they migrate to South America. A peregrine falcon can fly 350 feet per second.

- Explain why the distance traveled is a function of the number of seconds.
- Find the domain of the function. Is the domain discrete or continuous? Explain.
- Graph the function using its domain.

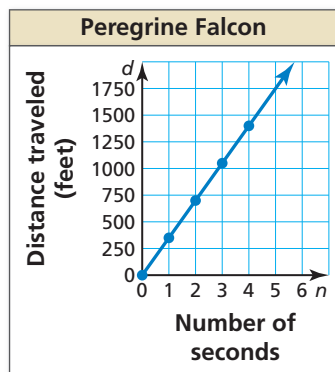
SOLUTION

- As the number n of seconds increases by 1, the distance d traveled increases by 350 feet. The rate of change is constant.
 ► So, this situation represents a linear function.
- The number n of seconds can be any value greater than or equal to 0.
 ► So, the domain is $n \geq 0$, and it is continuous.
- Step 1** Make an input-output table to find ordered pairs.

Input, n	Output, d	(n, d)
0	0	(0, 0)
1	350	(1, 350)
2	700	(2, 700)
3	1050	(3, 1050)
4	1400	(4, 1400)

Step 2 Plot the ordered pairs.

Step 3 Draw a line through the points. The line should start at $(0, 0)$ and continue to the right. Use an arrow to indicate that the line continues without end, as shown. The domain is continuous. So, the graph is a line with a domain of $n \geq 0$.



Hawk Mania is an annual bird-watching event held by the Florida Keys Audubon Society. The event teaches about the falcons, eagles, and ospreys that migrate through the Keys each fall.

SELF-ASSESSMENT

- I don't understand yet.
- I can do it with help.
- I can do it on my own.
- I can teach someone else.

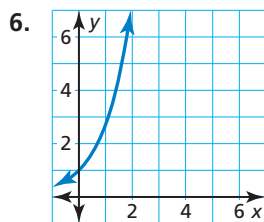
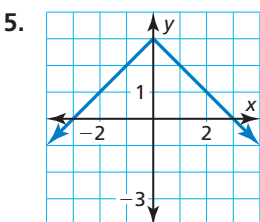
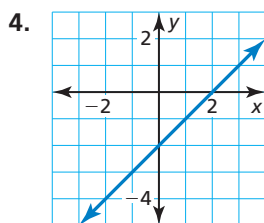
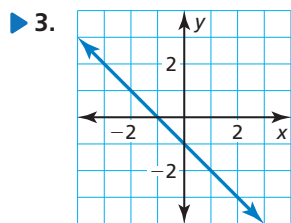
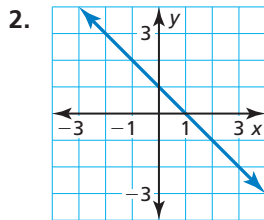
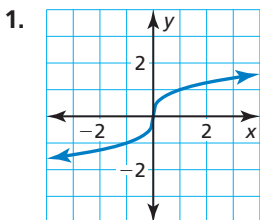
- A 20-gallon bathtub is draining at a rate of 2.5 gallons per minute.
 - Explain why the number of gallons remaining is a function of the number of minutes.
 - Find the domain of the function. Is the domain discrete or continuous? Explain.
 - Graph the function using its domain.
- When feeding, a juvenile whale shark filters about 600 cubic meters of water through its mouth each hour. About 2.8 kilograms of food are filtered out from the water each hour.
 - Graph the function that represents the amount a of water filtered by the whale shark as a function of the number m of minutes.
 - The whale shark feeds for 7.5 hours each day. Graph the function that represents the amount f of food (in pounds) filtered by the whale shark as a function of the number d of days.

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3.3 Practice WITH CalcChat® AND CalcView®

In Exercises 1–6, determine whether the graph represents a **linear** or **nonlinear** function. Explain.
(See Example 1.)



In Exercises 7–10, determine whether the table represents a **linear** or **nonlinear** function. Explain.
(See Example 2.)

7.

x	1	2	3	4
y	5	10	15	20

8.

x	5	7	9	11
y	-9	-3	-1	3

9.

x	4	8	12	16
y	16.8	12.6	7.4	1.2

10.

x	-1	0	1	2
y	35	20	5	-10

11. **REASONING** Explain why a V-shaped graph does *not* represent a linear function.

12. **REASONING** How can you tell whether a graph shows a discrete domain or a continuous domain?



4 MTR **ERROR ANALYSIS** In Exercises 13 and 14, describe and correct the error in determining whether the table or graph represents a linear function.

13.

		+2	+2	+2
x	2	4	6	8
y	4	16	64	256
		×4	×4	×4

As x increases by 2, y increases by a constant factor of 4. So, the function is linear.

14.

The graph is a line. So, the graph represents a linear function.

In Exercises 15–22, determine whether the equation represents a **linear** or **nonlinear** function. Explain.
(See Example 3.)

15. $y = x^2 + 13$

16. $y = 7 - x$

17. $y = \sqrt[3]{8} - x$

18. $y = x \cdot x \cdot x$

19. $2 + y = x + 4$

20. $7 - y = 2^x$

21. $x - y = |-54|$

22. $|x| + 12 = y$

23. **B.E.S.T. TEST PREP** Which of the following equations do *not* represent linear functions? Select all that apply.

(A) $12 = -x^2 + y$

(D) $-x = y$

(B) $y - x + 3 = 0$

(E) $y = \left(\frac{1}{2}\right)^x + 9$

(C) $x = 8$

(F) $y = \sqrt{x} + 3$

24. **WRITING** Compare discrete domains and continuous domains.

In Exercises 25–28, graph the linear function represented by the table. (See Example 4.)

▶ 25.

x	-2	-1	0	1	2
y	7	4	1	-2	-5

26.

x	-6	-3	0	3	6
y	-4	-3	-2	-1	0

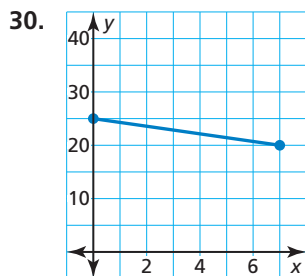
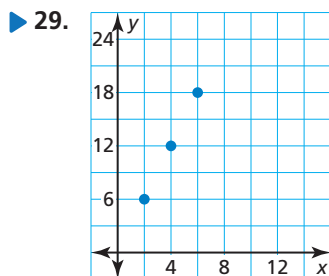
27.

x	-5	-3	3	5	8
y	-15	-11	1	5	11

28.

x	-6.3	-3	2.1	3	7
y	9.3	6	0.9	0	-4

In Exercises 29 and 30, find the domain of the function represented by the graph. Determine whether the domain is *discrete* or *continuous*. Explain.



In Exercises 31 and 32, determine whether the domain is *discrete* or *continuous*. Explain.

31.

Input Time (hours), x	3	6	9
Output Distance (miles), y	150	300	450

32.

Input Relay teams, x	0	1	2
Output Athletes, y	0	4	8

7 MTR 33. **MODELING REAL LIFE** The linear function $m = 10 - 1.44p$ represents the amount m (in dollars) of money that you have after printing p photographs. (See Example 5.)

- Interpret the terms and coefficient in the equation.
- Find the domain of the function. Is the domain discrete or continuous? Explain.
- Graph the function using its domain.

7 MTR 34. **MODELING REAL LIFE** The linear function $y = 145 + 30x$ represents the cost y (in dollars) of an airline ticket after adding x checked bags. At most 5 bags can be checked.

- Interpret the terms and coefficient in the equation.
- Find the domain of the function. Is the domain discrete or continuous? Explain.
- Graph the function using its domain.

7 MTR 35. **MODELING REAL LIFE** You fill a swimming pool with water at a rate of 17 gallons per minute. (See Example 6.)

- Is the amount of water in the pool a function of the number of minutes? Explain.
- Find the domain of the function. Is the domain discrete or continuous? Explain.
- Graph the function using its domain.

7 MTR 36. **MODELING REAL LIFE** The amount of air in a scuba diving tank with a capacity of 2400 liters is decreasing at a rate of 48 liters per minute.

- Is the amount of air in the tank a function of the number of minutes? Explain.
- Find the domain of the function. Is the domain discrete or continuous? Explain.
- Graph the function using its domain.



5 MTR 37. **STRUCTURE** Complete the table so it represents a linear function.

x	5	10	15	20	25
y	-1				11

2 MTR 38. **MULTIPLE REPRESENTATIONS** You are researching the speed of sound waves in dry air at 86°F. The linear function $d = 0.217t$ represents the distances d (in miles) sound waves travel in t seconds.

- Represent the situation using a table and a graph.
- Which of the three representations would you use to find how long it takes sound waves to travel 0.1 mile in dry air at 86°F? Explain.



39. **OPEN-ENDED** Identify a real-life situation where the domain is continuous. Justify your answer.

- 6 MTR** 40. **ASSESS REASONABLENESS** You and a friend observe the Mug Race along the St. Johns River. The table shows how many sailboats you see. Graph the linear function. Your friend claims you will see 110 sailboats by hour 6. Is your friend's claim reasonable? Explain.

Time (hours), x	0	1	2	3	4
Number of boats, y	0	15	30	45	60

41. **REASONING** Is the function represented by the ordered pairs linear or nonlinear? Explain your reasoning.

(0, 2), (3, 14), (5, 22), (9, 38), (11, 46)

42. **WRITING** Describe the end behavior of an increasing linear function and a decreasing linear function.

- 5 MTR** 43. **STRUCTURE** The table shows your earnings y (in dollars) for working x hours. You work no more than 18 hours each week.

Time (hours), x	Earnings (dollars), y
4	40.80
5	51.00
6	61.20
7	71.40

- a. What is your hourly pay rate?
b. Find the domain and range of the function.

- 4 MTR** 44. **MAKING AN ARGUMENT** The linear function $d = 50t$ represents the distance d (in miles) Car A is from a car rental store after t hours. The table shows the distances Car B is from the rental store.

Time (hours), t	Distance (miles), d
1	60
3	180
5	310
7	450
9	540

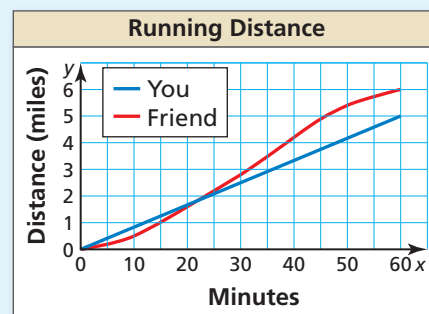
- a. Does the table represent a linear or nonlinear function? Explain.
b. Which car is moving at a faster rate? Explain.
45. **REASONING** A water company fills two different-sized jugs. The first jug can hold x gallons of water. The second jug can hold y gallons of water. The company fills A jugs of the first size and B jugs of the second size. What does each expression represent? Does each expression represent a set of discrete or continuous values?
- a. $x + y$ b. $A + B$
c. Ax d. $Ax + By$

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46. **HOW DO YOU SEE IT?**

You and your friend go running. The graph shows the distances you and your friend run.



- a. Describe your run and your friend's run. Who runs at a constant rate? How do you know? Why might a person not run at a constant rate?
b. Find the domain of each function. Describe the domains using the context of the problem.

- 7 MTR** 47. **PERFORMANCE TASK** You are ordering T-shirts for a school fundraiser and receive bids from three options. You are not sure exactly how many T-shirts you need, but you know it will be no more than 200 shirts. Create a proposal for the school principal. Include the number of T-shirts, which option you chose, and the cost per T-shirt.

Option 1	Option 2												
Charge includes an initial fee plus a cost per T-shirt. Sample pricing is shown in the table.	Cost for x T-shirts: $c = 2x + 80$												
<table border="1"> <tr> <th>Number of shirts</th><th>Price (dollars)</th></tr> <tr> <td>40</td><td>200</td></tr> <tr> <td>80</td><td>340</td></tr> <tr> <td>120</td><td>480</td></tr> <tr> <td>160</td><td>620</td></tr> <tr> <td>200</td><td>760</td></tr> </table>	Number of shirts	Price (dollars)	40	200	80	340	120	480	160	620	200	760	Option 3
Number of shirts	Price (dollars)												
40	200												
80	340												
120	480												
160	620												
200	760												
	Cost is \$5 per T-shirt.												

48. **THOUGHT PROVOKING**

A movie complex is open from 10:00 A.M. to 2:00 A.M. daily. It contains 8 theaters, each of which has 225 seats. The number of viewers in the theaters is a function of the number of hours after 10:00 A.M. each day. Describe a reasonable domain and range of the function. Then determine whether the function must be linear. Explain.

49. **REASONING** Is a linear function always increasing or always decreasing? Explain.

- 4 MTR** 50. **DISCUSS MATHEMATICAL THINKING** Explain the relationship between an equation and its graph. Why does the graph of a linear equation form a line?

- 2 MTR** 51. **MAKE A CONNECTION** What can you determine about the range of a linear function with a discrete domain? a continuous domain? Explain.

DIG DEEPER In Exercises 52 and 53, describe a real-life situation for the constraints.

52. The function has at least one negative number in the domain. The domain is continuous.
53. The function gives at least one negative number as an output. The domain is discrete.



REVIEW & REFRESH

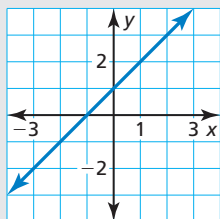
In Exercises 54–57, solve the equation. Check your solution.

54. $h + 6 = -7$ 55. $15 = 24 - 3y$
56. $9g - 12 = 6g$ 57. $-18 = -2|w + 3|$

In Exercises 58–61, multiply or divide.

58. $\frac{3}{2} \cdot \frac{4}{7}$ 59. $2\frac{5}{8} \cdot 3\frac{1}{3}$
60. $\frac{7}{8} \div \frac{1}{16}$ 61. $\frac{8}{9} \div 4$

62. Tell whether x and y are proportional. Explain your reasoning.



63. Write the sentence as an inequality. Then graph the inequality.

The sum of a number and 12 is at least 35.

In Exercises 64 and 65, determine whether the relation is a function.

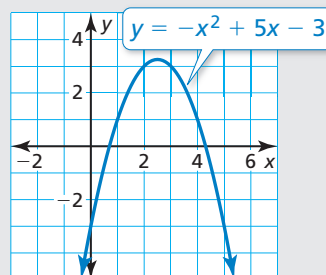
64. $(-5, 6), (0, 3), (2, 10), (4, -3), (-5, -2)$
65. $(-3, 4), (-1, -1), (1, 6), (2, 2), (4, -1)$
66. Is the domain discrete or continuous? Explain.

Input Number of stories, x	1	2	3
Output Height of building (feet), y	12	24	36

67. **OPEN-ENDED** Write an inequality that can be solved using the Division Property of Inequality where the inequality symbol needs to be reversed.

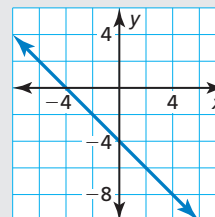
- 7 MTR** 68. **MODELING REAL LIFE** An event center charges \$59.95 for each concert ticket.

- a. Does this situation represent a function? If so, identify the independent and dependent variables.
- b. There are 9350 seats in the arena. Find the domain and range. Is the domain discrete or continuous? Explain.
69. Approximate when the function is positive, negative, increasing, or decreasing. Then describe the end behavior of the function.



In Exercises 70–73, solve the inequality. Graph the solution.

70. $-4 < -2x + 9 \leq 32$
71. $-1 \leq q - 6$ or $\frac{1}{8}q < -\frac{3}{4}$
72. $|8y + 3| > -1$ 73. $4|3 - 2z| - 9 \leq 3$
74. Does the graph represent a linear or nonlinear function? Explain.



In Exercises 75 and 76, write the number in scientific notation.

75. 107,000,000 76. 0.000002

