2.2 Solving Inequalities Using Addition or Subtraction

Learning Target: Write and solve inequalities using addition or subtraction.

Success Criteria:
- I can apply the Addition and Subtraction Properties of Inequality to produce equivalent inequalities.
- I can solve inequalities using addition or subtraction.
- I can use inequalities to model real-life problems.

EXPLORE IT! Quarterback Passing Efficiency

Work with a partner. The National Collegiate Athletic Association (NCAA) uses the following formula to rank the passing efficiencies P of quarterbacks.

$$P = \frac{8.4Y + 100C + 330T - 200N}{A}$$

Y = total length of all completed passes (in Yards)

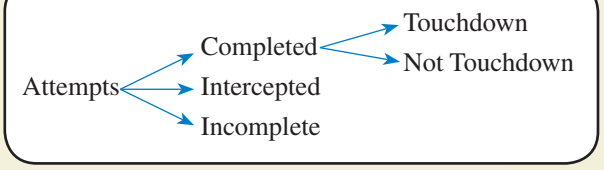
C = Completed passes

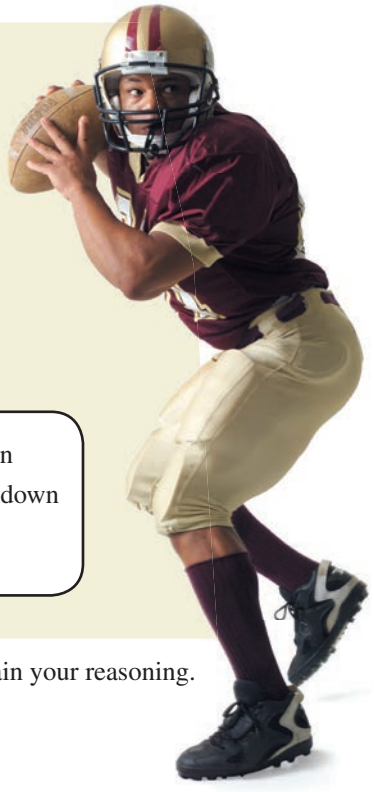
T = passes resulting in a Touchdown

N = iNtercepted passes

A = Attempted passes

M = incoMplete passes





1 MTR ANALYZE A PROBLEM

Which has a greater effect on the value of P, a touchdown or an interception? How do you know?

a. Determine whether each inequality must be true. Explain your reasoning.

i. $T < C$ **ii.** $C + N \leq A$

iii. $N < A$ **iv.** $A - C \geq M$

b. You have used properties of equality to solve equations involving addition or subtraction. Can you use similar properties to solve inequalities involving addition or subtraction? Explain your reasoning.

Attempts	
Completions	
Yards	
Touchdowns	
Interceptions	

c. Use $C + N \leq A$ and $A - C \geq M$ from part (a) to support your answer in part (b).

d. For each inequality below, complete a table with passing statistics that satisfy the inequality. Then describe the values of P that make each inequality true.

i. $P < 0$ **ii.** $P + 100 \geq 250$ **iii.** $P - 250 > -80$

Algebraic Reasoning
MA.912.AR.2.6 Given a mathematical or real-world context, write and solve one-variable linear inequalities, including compound inequalities. Represent solutions algebraically or graphically.

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Solving Inequalities Using Addition

Vocabulary

equivalent inequalities, *p. 60*

Just as you used the properties of equality to produce equivalent equations, you can use the properties of inequality to produce equivalent inequalities. **Equivalent inequalities** are inequalities that have the same solutions.

KEY IDEA

Addition Property of Inequality

Words Adding the same number to each side of an inequality produces an equivalent inequality.

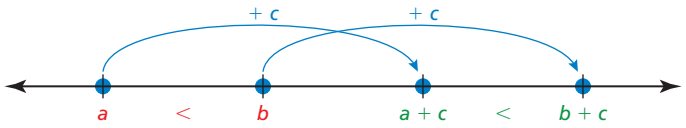
Numbers

$$\begin{array}{rrrr} -3 & < & 2 & \\ +4 & & +4 & \\ \hline 1 & < & 6 & \end{array} \qquad \begin{array}{rrrr} -3 & \geq & -10 & \\ +3 & & +3 & \\ \hline 0 & \geq & -7 & \end{array}$$

Algebra If $a > b$, then $a + c > b + c$. If $a \geq b$, then $a + c \geq b + c$.

If $a < b$, then $a + c < b + c$. If $a \leq b$, then $a + c \leq b + c$.

The diagram shows one way to visualize the Addition Property of Inequality when $c > 0$.



EXAMPLE 1 Solving an Inequality Using Addition

WATCH

Solve $x - 6 \geq -10$. Graph the solution.

SOLUTION

$$\begin{array}{rrrr} x & - & 6 & \geq -10 \\ +6 & & +6 & \\ \hline x & \geq & -4 & \end{array}$$

Write the inequality.

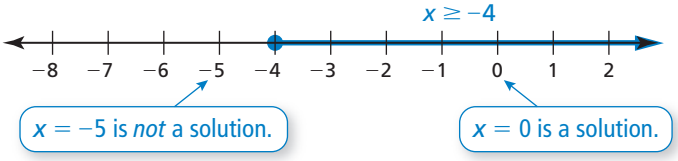
Undo the subtraction. → Addition Property of Inequality

Simplify.

▶ The solution is $x \geq -4$.

REMEMBER

To check this solution, substitute numbers to the left and right of -4 into the original inequality.



SELF-ASSESSMENT

1 I don't understand yet. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

Solve the inequality. Graph the solution.

1. $b - 2 > -9$ **2.** $m - 3 \leq 5$ **3.** $\frac{1}{4} > y - \frac{1}{4}$ **4.** $0.07 \leq -4.05 + z$

5. VOCABULARY Is the inequality $x \leq 6$ equivalent to the inequality $x - 5 \leq 6 - 5$? Explain your reasoning.

6. REASONING The possible values of x are given by $x - 4 \geq -1$. What is the greatest possible value of $-5x$? Explain your reasoning.

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Solving Inequalities Using Subtraction

KEY IDEA

Subtraction Property of Inequality

Words Subtracting the same number from each side of an inequality produces an equivalent inequality.

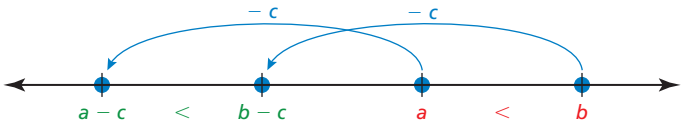
Numbers

$$\begin{array}{rcl} -3 & \leq & 1 \\ \underline{-5} & & \underline{-5} \\ -8 & \leq & -4 \end{array} \qquad \begin{array}{rcl} 7 & > & -20 \\ \underline{-7} & & \underline{-7} \\ 0 & > & -27 \end{array}$$

Algebra If $a > b$, then $a - c > b - c$. If $a \geq b$, then $a - c \geq b - c$.

If $a < b$, then $a - c < b - c$. If $a \leq b$, then $a - c \leq b - c$.

The diagram shows one way to visualize the Subtraction Property of Inequality when $c > 0$.



EXAMPLE 2 Solving Inequalities Using Subtraction

WATCH

Solve each inequality. Graph each solution.

a. $y + 8 \leq 5$ **b.** $-8 < 1.4 + m$

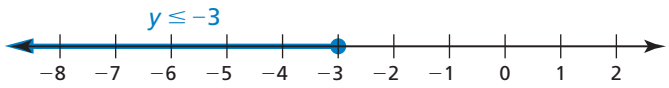
SOLUTION

a. $y + 8 \leq 5$ Write the inequality.

Undo the addition. → $\underline{-8} \quad \underline{-8}$ Subtraction Property of Inequality

$y \leq -3$ Simplify.

▶ The solution is $y \leq -3$.

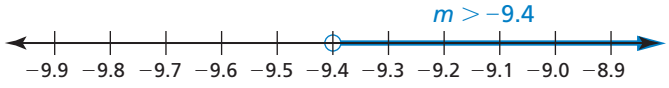


b. $-8 < 1.4 + m$ Write the inequality.

$\underline{-1.4} \quad \underline{-1.4}$ Subtraction Property of Inequality

$-9.4 < m$ Simplify.

▶ The solution is $m > -9.4$.



SELF-ASSESSMENT

1 I don't understand yet. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

Solve the inequality. Graph the solution.

7. $k + 5 \leq -3$ **8.** $75 < 58 + w$ **9.** $\frac{5}{6} \leq z + \frac{1}{6}$ **10.** $p + 0.7 > -2.3$

11. OPEN-ENDED Write two inequalities that have a solution of $y \geq -3$. One must require using subtraction to solve the inequality, and the other must require using addition.

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Solving Real-Life Problems

EXAMPLE 3 Modeling Real Life

A circuit overloads at 1800 watts of electricity. You plug an amplifier that uses 900 watts of electricity into the circuit. In addition to the amplifier, which of the following equipment can you plug into the circuit at the same time without overloading the circuit?

Equipment	Watts
Lighting	600
Fog machine 1	450
Fog machine 2	450
Television	200

SOLUTION

1. **Understand the Problem** You know that a circuit overloads at 1800 watts. You also know the numbers of watts used by several pieces of equipment. You are asked to determine what you can plug in without overloading the circuit.

2. **Make a Plan** Use a verbal model to write an inequality that represents the numbers of watts you can add without overloading the circuit. Then solve the inequality and identify other equipment that you can plug in without overloading the circuit.

3. **Solve and Check**

Verbal Model Watts used by amplifier + Additional watts < Overload wattage

Variable Let w be the additional watts you can add to the circuit.

Inequality

$$900 + w < 1800$$ Write the inequality.

$$-900 \quad -900$$ Subtraction Property of Inequality

$$w < 900$$ Simplify.

▶ You can add up to 900 watts to the circuit. So, you can also plug in the lighting and the television, one of the fog machines and the television, or any individual piece of equipment.

Check

You can check that your answer is correct by adding the numbers of watts used by each grouping of equipment.

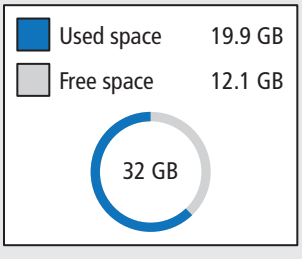
$$900 + 600 + 200 = 1700 \qquad 900 + 450 + 200 = 1550$$

The circuit will not overload because the total wattage is less than 1800 watts. ✓

SELF-ASSESSMENT

1 I don't understand yet. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

12. The capacity of a flash drive is shown in the diagram. You copy two files to the flash drive, one of which is 1.92 gigabytes (GB) and the other 3.4 gigabytes. Which of the following folders can you also copy to the flash drive?



13. **HELP A CLASSMATE** In Example 3, explain to a classmate why you cannot plug in both fog machines along with the amplifier.

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2.2 Practice WITH CalcChat® AND CalcView®

In Exercises 1–14, solve the inequality. Graph the solution. *(See Examples 1 and 2.)*

1. $x - 4 < -5$

2. $1 \leq s - 8$

3. $6 \geq m - 1$

4. $c - 12 > -4$

5. $r + 4 < 5$

6. $-8 \leq 8 + y$

7. $9 + w > 7$

8. $15 \geq q + 3$

9. $h - (-2) \geq 10$

10. $-6 > t - (-13)$

11. $j + 1.7 < -2.1$

12. $-5.2 + y \geq -7.4$

13. $-\frac{4}{5} \geq p - \frac{2}{5}$

14. $\frac{1}{6} + z > 2\frac{2}{3}$

In Exercises 15–18, write the sentence as an inequality. Then solve the inequality.

15. A number plus 8 is greater than 11.

16. A number minus 3 is at least -5.

17. The difference of a number and 9 is fewer than 4.

18. Six is less than or equal to the sum of a number and $3\frac{1}{4}$.

7 MTR **19. MODELING REAL LIFE** A Tampa Bay Lightning player has 59 goals so far in a season. What are the possible numbers of additional goals the player can score to break the NHL record of 92 goals in a season?

7 MTR **20. MODELING REAL LIFE** You want your daily sodium intake to be less than 2300 milligrams. For breakfast, you eat a cereal bar with the nutrition label shown. What are the possible amounts of sodium you can eat during the rest of the day?

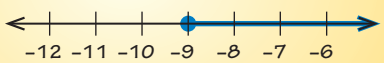
Nutrition Facts

Serving Size	1 Bar (37g)
Servings Per Container	8
Amount Per Serving	
Calories 120	Calories from Fat 30
	% Daily Value*
Total Fat 3g	**5%**
Saturated Fat 0.5g	**3%**
Trans Fat 0g	
Cholesterol 0mg	**0%**
Sodium 125mg	**5%**

4 MTR **ERROR ANALYSIS** **In Exercises 21 and 22, describe and correct the error in solving the inequality and graphing the solution.**

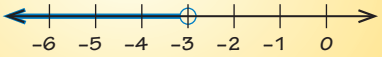
21.

$$-10 + x \geq -9$$
$$-10 + 10 + x \geq -9$$
$$x \geq -9$$



22.

$$-17 < x - 14$$
$$-17 + 14 < x - 14 + 14$$
$$-3 < x$$



7 MTR **23. MODELING REAL LIFE** An airline charges an extra fee for a checked bag that weighs more than 50 pounds. Your bag weighs 44.9 pounds. You have a 2.5-pound hair dryer, a 1.3-pound souvenir, and a 3.6-pound pair of boots. Which items can you add to the bag without paying the extra fee? *(See Example 3.)*

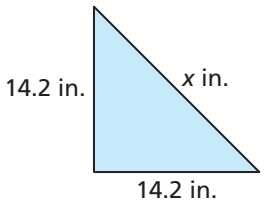
7 MTR **24. MODELING REAL LIFE** A website offers free shipping on orders of \$75 or more. You have items totaling \$34.95 in your shopping cart. You want to add one sweater and one shirt to your cart. With which combination can you get free shipping and also spend the least amount of money?

Sweater	Price
A	\$29.95
B	\$24.50

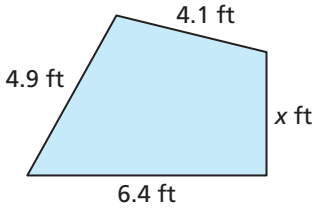
Shirt	Price
A	\$15.75
B	\$12.95

5 MTR **CONNECTING CONCEPTS** **In Exercises 25 and 26, write and solve an inequality to find the possible values of x.**

25. Perimeter < 51.3 in.



26. Perimeter ≤ 18.7 ft



4 MTR **27. MAKING AN ARGUMENT** In an aerial ski competition, you perform two acrobatic ski jumps. The scores on the two jumps are then added together.

Ski jump	Competitor's score	Your score
1	117.1	119.5
2	119.8	

a. Describe the possible scores that you can earn on your second jump to beat your competitor.

b. Your coach says that you will beat your competitor if you score 118.4 points. A teammate says that you need only 117.5 points. Who is correct? Explain.

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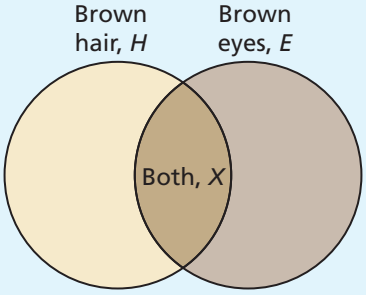
28. HOW DO YOU SEE IT?
The diagram represents the numbers of students in a school with brown eyes, brown hair, or both.



Determine whether each inequality must be true. Explain your reasoning.

a. $H \geq E$
b. $H + 10 \geq E$
c. $H \geq X$
d. $H + 10 \geq X$
e. $H > X$
f. $H + 10 > X$

29. REASONING Write and graph an inequality that represents the numbers that are *not* solutions of each inequality.

a. $x + 8 < 14$
b. $x - 12 \geq 5.7$

30. REASONING What is the greatest value of $2n + 7$ when $\frac{7}{12} \geq \frac{5}{6} + n$?

31. B.E.S.T. TEST PREP Which of the following inequalities are equivalent to the inequality $x - b < 3$, where b is a constant? Select all that apply.

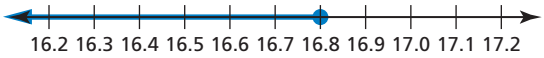
Ⓐ $x - b - 3 < 0$
Ⓑ $0 > b - x + 3$
Ⓒ $x < 3 - b$
Ⓓ $-3 < b - x$

32. THOUGHT PROVOKING
Use the inequalities

$$c - 3 \geq d,\ b + 4 < a + 1, \text{ and } a - 2 \leq d - 7$$

to order a, b, c, and d from least to greatest.

33. DIG DEEPER Write an inequality that requires using addition or subtraction to solve and has the solution shown in the graph. Then describe a real-life situation that can be modeled by the inequality.



REVIEW & REFRESH

WATCH

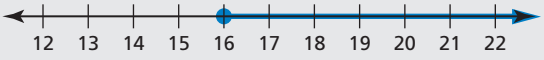
In Exercises 34–37, find the product or quotient.

34. $7 \cdot (-9)$
35. $-11 \cdot (-12)$
36. $-27 \div (-3)$
37. $20 \div (-5)$

In Exercises 38 and 39, solve the equation. Check your solution.

38. $-3y = -18$
39. $\frac{n}{4} = -7.3$

7 MTR **40. MODELING REAL LIFE** The graph shows the age requirement x (in years) for obtaining a driver's license in a state. Write and interpret an inequality that represents the age requirement.



41. Solve the equation $2|x + 5| = |3x - 2|$. Check your solutions.

42. REASONING Describe the values of a for which the equation $|x + 12| - 10 = a$ has no solution.

43. You randomly choose one of the letters shown. What is the theoretical probability of choosing a vowel?

In Exercises 44 and 45, solve the inequality. Graph the solution.

44. $6 \geq w + 11$
45. $x - 4.5 > -1.8$

7 MTR **46. MODELING REAL LIFE** You average 55 miles per hour while driving to a relative's house. On the return trip, you average 50 miles per hour because of bad weather. The total driving time is 5 hours and 15 minutes. How long does each trip take?

In Exercises 47 and 48, find the square root.

47. $\sqrt{121}$
48. $-\sqrt{\frac{4}{81}}$

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