1.3 Solving Equations with Variables on Both Sides

Learning Target: Write and solve equations with variables on both sides.

Success Criteria:
- I can apply properties of equality using variable terms.
- I can solve equations with variables on both sides.
- I can recognize when an equation has zero, one, or infinitely many solutions.

EXPLORE IT! Solving a Real-Life Problem

Work with a partner. You earn \$9.75 per hour at a part-time job. Your friend earns \$9.35 per hour at a part-time job. The only other income you and your friend earn is a weekly allowance.

a. Determine whether it is possible for you and your friend to work the same number of hours and earn the same total amount in a given week for the following situations. Explain your reasoning.

i. Your allowance is \$20 per week, and your friend's allowance is \$10 per week.

ii. Your allowance is \$10 per week, and your friend's allowance is \$20 per week.

iii. Your allowance is \$10 per week, your friend's allowance is \$20 per week, and your friend receives a \$0.40 raise.

iv. Your allowance is \$20 per week, your friend's allowance is \$20 per week, and your friend receives a \$0.40 raise.

5 MTR USE STRUCTURE
What do you notice about the coefficients of the variable terms in each equation? What do they tell you about the number of solutions an equation may have?

b. The following equation represents one of the situations in part (a).

$$9.75p + 10 = 9.35p + 20$$

Interpret each term and each side of the equation. Which situation does it represent?

c. Solve the equation in part (b). Explain how you solved the equation and what the solution represents. Can you start with a different first step?

d. Write and solve an equation for each of the other three situations in part (a). Compare your solutions to your answers in part (a). What do you notice?

Algebraic Reasoning
MA.912.AR.2.1 Given a real-world context, write and solve one-variable multi-step linear equations.

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Solving Equations with Variables on Both Sides

Vocabulary

identity, *p. 21*

KEY IDEA

Solving Equations with Variables on Both Sides

To solve an equation with variables on both sides, use inverse operations to collect the variable terms on one side and the constant terms on the other side. Then isolate the variable.

EXAMPLE 1 Solving an Equation with Variables on Both Sides

Solve $10 - 4x = -9x$. Check your solution.

SOLUTION

$10 - 4x = -9x$	Write the equation.
$+ 4x \quad + 4x$	Addition Property of Equality
$10 = -5x$	Simplify.
$\frac{10}{-5} = \frac{-5x}{-5}$	Division Property of Equality
$-2 = x$	Simplify.

▶ The solution is $x = -2$.

Check

$$10 - 4x = -9x$$
$$10 - 4(-2) \stackrel{?}{=} -9(-2)$$
$$18 = 18 \checkmark$$

EXAMPLE 2 Solving an Equation with Grouping Symbols

Solve $3(3x - 4) = \frac{1}{4}(32x + 24)$.

SOLUTION

$3(3x - 4) = \frac{1}{4}(32x + 24)$	Write the equation.
$9x - 12 = 8x + 6$	Distributive Property
$+ 12 \quad + 12$	Addition Property of Equality
$9x = 8x + 18$	Simplify.
$- 8x \quad - 8x$	Subtraction Property of Equality
$x = 18$	Simplify.

▶ The solution is $x = 18$.

SELF-ASSESSMENT

1 I don't understand yet. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

Solve the equation. Check your solution.

1. $-2x = 3x + 10$
2. $0.5(6h - 4) = -5h + 1$
3. $-\frac{3}{4}(8n + 12) = 3(n - 3)$
4. **WRITING** Describe the steps in solving the linear equation $3(3x - 8) = 4x + 6$. Explain why the steps produce a valid solution.
5. **REASONING** Your friend first multiplies each side of $\frac{1}{2}(x + 9) = \frac{3}{4}(5x + 1)$ by 4 when solving the equation. Why might your friend do this?

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Identifying the Number of Solutions

Equations do not always have one solution. An equation that is true for all values of the variable is an **identity** and has *infinitely many solutions*. All real numbers are solutions of any identity. An equation that is not true for any value of the variable has *no solution*.

WORDS AND MATH

Think about the meaning of *identity* in everyday life. Your identity is who you are. It can be the name, distinguishing characteristics, or personality that identifies you. Your identity is the same no matter what changes in your day.

EXAMPLE 3 Solving Equations with Variables on Both Sides

Solve each equation.

a. $3(5x + 2) = 15x$ **b.** $\frac{n}{6} = -\frac{n}{6} + \frac{1}{2}$ **c.** $-2(4y + 1) = -8y - 2$

SOLUTION

a.
$$3(5x + 2) = 15x$$ Write the equation.
$$15x + 6 = 15x$$ Distributive Property
$$-15x \quad -15x$$ Subtraction Property of Equality
$$6 = 0$$ ✗ Simplify.

▶ The statement $6 = 0$ is never true. So, the equation has no solution.

b.
$$\frac{n}{6} = -\frac{n}{6} + \frac{1}{2}$$ Write the equation.
$$6 \cdot \frac{n}{6} = 6 \cdot \left(-\frac{n}{6} + \frac{1}{2}\right)$$ Multiplication Property of Equality
$$n = -n + 3$$ Simplify.
$$+n \quad +n$$ Addition Property of Equality
$$2n = 3$$ Simplify.
$$\frac{2n}{2} = \frac{3}{2}$$ Division Property of Equality
$$n = \frac{3}{2}$$ Simplify.

▶ The only solution is $n = \frac{3}{2}$.

c.
$$-2(4y + 1) = -8y - 2$$ Write the equation.
$$-8y - 2 = -8y - 2$$ Distributive Property
$$+8y \quad +8y$$ Addition Property of Equality
$$-2 = -2$$ Simplify.

▶ Because the statement $-2 = -2$ is always true, the original equation is an identity and has infinitely many solutions. So, the solution is all real numbers.

5 MTR USE STRUCTURE

Why is it helpful to multiply each side by 6? How else could you begin to solve this equation?

SELF-ASSESSMENT

1 I don't understand yet. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

Solve the equation.

6. $6m - m = \frac{5}{6}(-6m - 30)$ **7.** $10k + 7 = -3 + 10k$ **8.** $3(2a - 2) = 2(3a - 3)$

9. VOCABULARY Is the equation $-2(4 - x) = 2x + 8$ an identity? Explain your reasoning.

10. WRITING In Example 3, your friend says that there is enough information to determine the numbers of solutions of the equations in parts (a) and (c) once you obtain $15x + 6 = 15x$ and $-8y - 2 = -8y - 2$. Is your friend correct? Explain.

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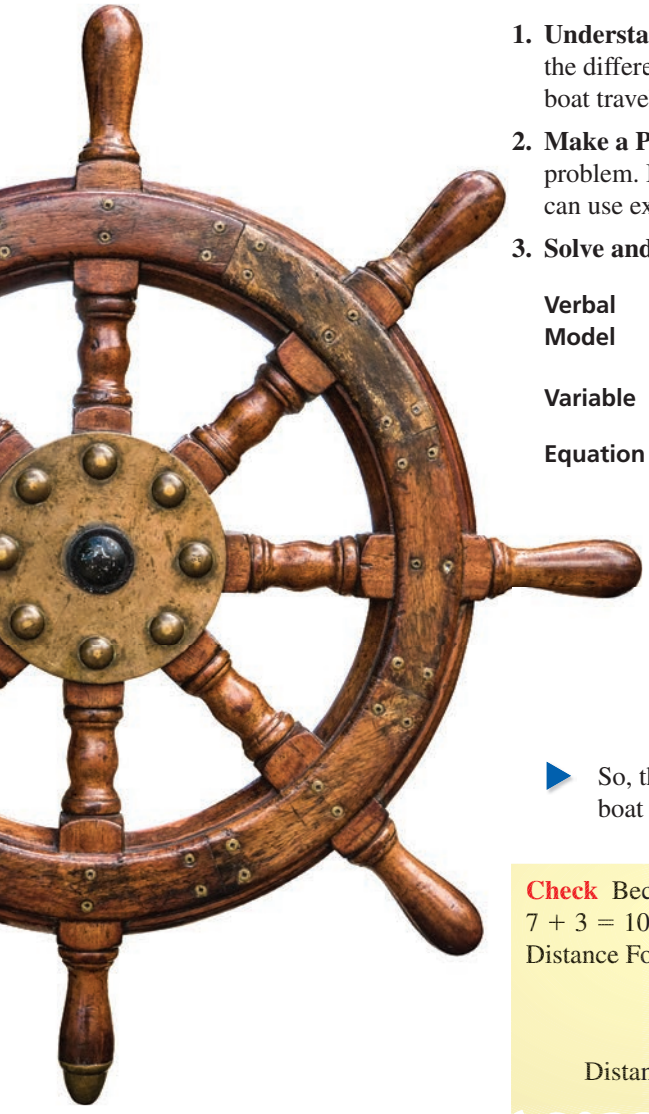
Solving Real-Life Problems

EXAMPLE 4 **Modeling Real Life** WATCH STEM

A boat leaves New Orleans and travels upstream on the Mississippi River for 4 hours. The return trip takes only 2.8 hours because the boat travels 3 miles per hour faster downstream due to the current. How far does the boat travel upstream?



SOLUTION

1. **Understand the Problem** You are given the amounts of time the boat travels and the difference in speeds for each direction. You are asked to find the distance the boat travels upstream.
2. **Make a Plan** Use the Distance Formula to write expressions that represent the problem. Because the distance the boat travels in both directions is the same, you can use expressions for the distance to write an equation.
3. **Solve and Check** The distance is equal to the product of speed and time.

Verbal Model Distance upstream = Distance downstream

Variable Let x be the speed (in miles per hour) of the boat traveling upstream.

Equation

$$x \cdot 4 = (x + 3) \cdot 2.8 \quad \text{Write the equation.}$$

$$4x = 2.8x + 8.4 \quad \text{Distributive Property}$$

$$-2.8x \quad -2.8x \quad \text{Subtraction Property of Equality}$$

$$1.2x = 8.4 \quad \text{Simplify.}$$

$$\frac{1.2x}{1.2} = \frac{8.4}{1.2} \quad \text{Division Property of Equality}$$

$$x = 7 \quad \text{Simplify.}$$

▶ So, the boat travels 7 miles per hour upstream. To determine how far the boat travels upstream, multiply 7 miles per hour by 4 hours to obtain 28 miles.

Check Because the speed upstream is 7 miles per hour, the speed downstream is $7 + 3 = 10$ miles per hour. When you substitute each speed and time into the Distance Formula, you get the same distance upstream and downstream.

Upstream	Downstream
$\text{Distance} = \frac{7 \text{ mi}}{1 \text{ h}} \cdot 4 \text{ h} = 28 \text{ mi}$	$\text{Distance} = \frac{10 \text{ mi}}{1 \text{ h}} \cdot 2.8 \text{ h} = 28 \text{ mi}$ ✓

SELF-ASSESSMENT

1 I don't understand yet. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

11. A boat travels upstream on the Missouri River for 3.5 hours. The return trip takes only 2.5 hours because the boat travels 2 miles per hour faster downstream due to the current. How far does the boat travel downstream?

12. You ask a deli clerk for x pounds of ham and x pounds of cheese. You end up getting 4 extra ounces of ham and 3 fewer ounces of cheese. The ham costs \$6.24 per pound, and the cheese costs \$4.80 per pound. You spend twice as much on ham as you do on cheese. How much do you spend in total?

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1.3 Practice WITH CalcChat® AND CalcView®

In Exercises 1–14, solve the equation. Check your solution. *(See Examples 1 and 2.)*

1. $15 - 2x = 3x$

2. $26 - 4s = 9s$

3. $5p - 9 = 2p + 12$

4. $8g + 10 = 35 + 3g$

5. $5t + 16 = 6 - 5t$

6. $-3r + 10 = 15r - 8$

7. $7 + 3x - 12x = 3x + 1$

8. $w - 2 + 2w = 6 + 5w$

9. $10(g + 5) = 2(g + 9)$

10. $-9(t - 2) = 4(t - 15)$

11. $\frac{2}{3}(3x + 9) = -2(2x + 6)$

12. $2(2t + 4) = \frac{3}{4}(24 - 8t)$

13. $1.5(3y + 2) - y = 2(8y - 6)$

14. $\frac{1}{2}(4x + 5) = 9x - 12(x - 1)$

In Exercises 15–22, solve the equation. *(See Example 3.)*

15. $3t + 4 = 12 + 3t$

16. $6d + 8 = 14 + 3d$

17. $2(h + 1) = 5h - 7$

18. $12y + 6 = 6(2y + 1)$

19. $-\frac{w}{5} = \frac{w}{5} - \frac{1}{10}$

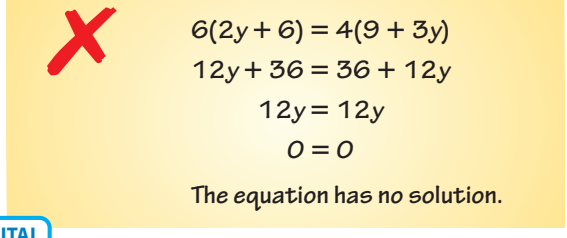
20. $\frac{x}{12} + 1 = \frac{x}{3} - \frac{x}{4}$

21. $3(4g + 6) = 2(6g + 9)$

22. $5(1 + 2m) = \frac{1}{2}(8 + 20m)$

23. MODELING REAL LIFE You and your friend drive toward each other. The equation $50h = 190 - 45h$ represents the number h of hours until you and your friend meet. After how many hours will you meet?

24. ERROR ANALYSIS Describe and correct the error in solving the equation.



25. MODELING REAL LIFE A cheetah that is running 90 feet per second is 120 feet behind an antelope that is running 60 feet per second. How long will it take the cheetah to catch up to the antelope? *(See Example 4.)*

26. MAKING AN ARGUMENT A cheetah can run at top speed for only about 20 seconds. If an antelope is too far away for a cheetah to catch it in 20 seconds, the antelope is probably safe. Your friend claims the antelope in Exercise 25 will not be safe if the cheetah starts running 650 feet behind it. Is your friend correct? Explain.

27. MODELING REAL LIFE You want to create a piece of pottery at an art studio. The total cost is the cost of the piece plus an hourly studio fee. The costs at two studios are shown.

a. After how many hours are the total costs the same at both studios? Justify your answer.

b. Studio B increases its hourly studio fee by $1.50. How does this affect your answer in part (a)? Explain.

28. PROBLEM SOLVING Taking a shower accounts for about 20% of your daily water usage. You use 60 gallons of water daily for other purposes. How many gallons of water do you use daily while taking a shower?

REASONING In Exercises 29 and 30, find the value of a for which the equation is an identity. Explain your reasoning.

29. $a(2x + 3) = 9x + 15 + x$

30. $8x - 8 + 3ax = 5ax - 2a$

31. DIG DEEPER Two times the greater of two consecutive integers is 9 less than three times the lesser integer. What are the integers?

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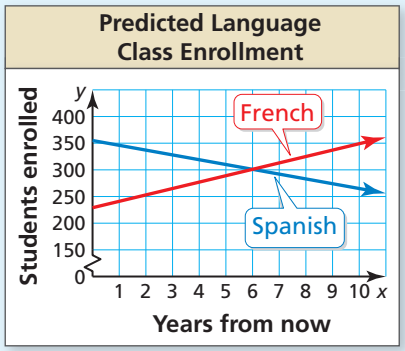
32. HOW DO YOU SEE IT?
The table and the graph show information about students at a high school.

	Students enrolled this year	Average rate of change
Spanish	355	9 fewer students each year
French	229	12 more students each year

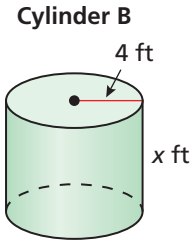


a. Use the graph to determine after how many years there will be equal enrollment in both classes.

b. How does the equation $355 - 9x = 229 + 12x$ relate to the table and the graph? How can you use this equation to determine whether your answer in part (a) is reasonable?

33. REASONING Without solving, determine whether the equation $3n + 5 = 3n - 5$ has *one solution*, *no solution*, or *infinitely many solutions*. Explain your reasoning.

34. CONNECTING CONCEPTS
Cylinder A has a radius of 6 feet and a height that is 5.5 feet less than Cylinder B. The cylinders have the same surface area. Find the height of each cylinder.

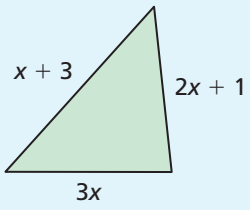


35. B.E.S.T. TEST PREP For which of the following values of c and d does the equation $cx - 2 = 7x + d$ have no solution? Select all that apply.

Ⓐ $c = -7, d = -2$ Ⓒ $c = 7, d = 0$

Ⓑ $c = 7, d = -2$ Ⓓ $c = 7, d = 2$

36. THOUGHT PROVOKING
Draw a different figure that has the same perimeter as the triangle shown. Explain why your figure has the same perimeter.



REVIEW & REFRESH

WATCH

37. The diameter of a circular watch face is 3 centimeters. Find the area of the watch face.

In Exercises 38–43, solve the equation. Check your solution.

38. $5 = 10 - v$

39. $2k - 3(2k - 3) = 45$

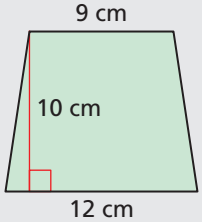
40. $x - \frac{1}{5} = -\frac{4}{5}$

41. $\frac{n}{7} = 4.5$

42. $7t - 20 = -50 - 8t$

43. $\frac{1}{2}(6c + 2) = -3(c - 1)$

44. Find the area of the trapezoid.

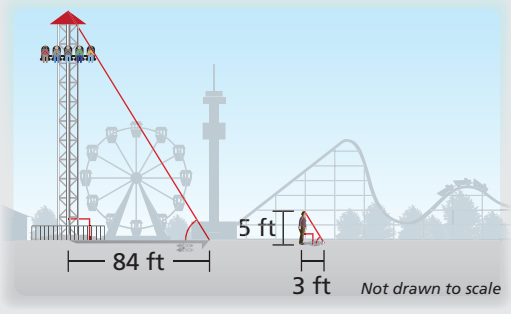


45. You type 168 words in $3\frac{1}{2}$ minutes. How many words do you type per minute?

46. Order the values from least to greatest.

$$|-32|, 22, -16, -|21|, |-10|$$

47. You want to find the height of a drop tower at an amusement park. You take the measurements shown in the diagram. The right triangles created by each object and its shadow are similar. How tall is the drop tower?



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