6.2 Representations of Functions

Learning Target: Represent functions in a variety of ways.

Success Criteria:
- I can determine whether a table or a graph represents a function.
- I can find the domains and ranges of relations and functions represented by tables and graphs.
- I can write a rule to represent a function.
- I can evaluate functions for given inputs.

Exploration 1 Identifying a Function

Work with a partner. Use the table.

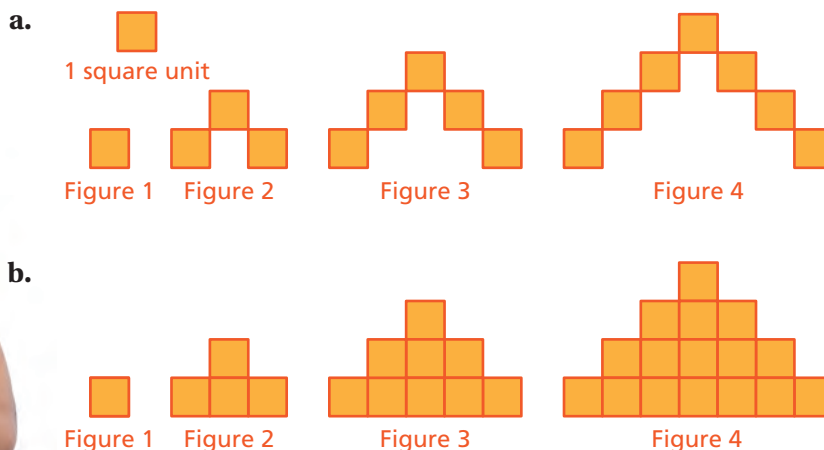| Input, x	−2	−1	0	1	2
Output, y	−4	−2	0	2	4

4 MTR CONSTRUCT AN ARGUMENT
Add the point (1, 3) to the graph in part (b). Does the graph represent a function? Explain.

a. Determine whether the table represents a function.

b. Graph the relation represented by the table. How can you use the graph to determine whether the relation is a function?

c. Draw line segments between the points in the graph as if you were making a line graph. How does this affect the domain and range of the graph? Explain.

Exploration 2 Using a Table to Describe Relationships

Work with a partner. Make a table that shows the relationship between the figure number x and the area A of each figure. Find the area of Figure 9.

a.

1 square unit

Figure 1   Figure 2   Figure 3   Figure 4

b.

Figure 1   Figure 2   Figure 3   Figure 4

Functions
MA.8.F.1.1 Given a set of ordered pairs, a table, a graph or mapping diagram, determine whether the relationship is a function. Identify the domain and range of the relation.

You have seen relations represented as ordered pairs and mapping diagrams. Relations can also be represented using tables or graphs.

Example 1 Determining Whether Tables Represent Functions

Determine whether each table represents a function. Then find the domain and range.

a.

Input, x	Output, y
-2	-8
-1	-4
0	0
1	4
2	8

b.

Input, x	Output, y
1	0
2	3
2	7
4	10
6	12

▶ Each input has exactly one output. So, the table represents a function. The domain is -2, -1, 0, 1, and 2. The range is -8, -4, 0, 4, and 8.

▶ The input 2 has two outputs, 3 and 7. So, the table does *not* represent a function. The domain is 1, 2, 4, and 6. The range is 0, 3, 7, 10, and 12.

Try It Determine whether the table represents a function. Then find the domain and range.

1.

Input, x	Output, y
-1	-2
0	-1
1	0
1	1
2	2

2.

Input, x	Output, y
0	4
1	6
2	8
3	10
4	12

3.

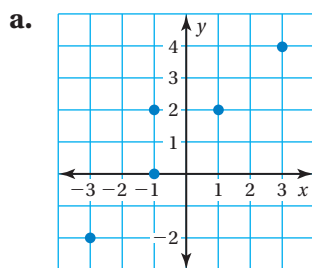| Input, x	2	4	6	8	10
Output, y	2	2	3	3	4

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Example 2 Determining Whether Graphs Represent Functions

Determine whether each graph represents a function. Then find the domain and range.
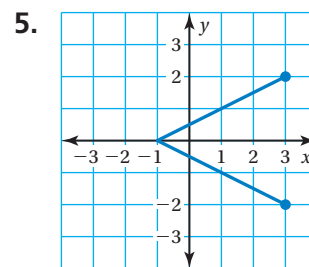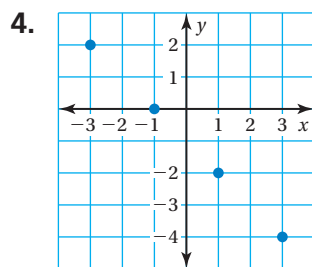
a.

b.

▶ The input -1 has two outputs, 0 and 2. So, the graph does *not* represent a function. The domain is -3, -1, 1, and 3. The range is -2, 0, 2, and 4.
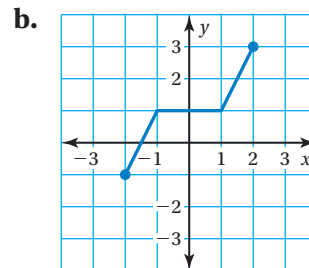
▶ Each input has exactly one output. So, the graph represents a function. The domain is $-2 \leq x$ and $x \leq 2$. The range is $-1 \leq y$ and $y \leq 3$.

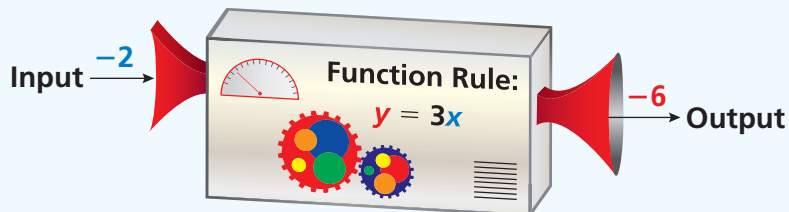**Try It** **Determine whether the graph represents a function. Then find the domain and range.**

4.

5.

Key Idea

Functions as Equations

A **function rule** is an equation that describes the relationship between inputs (independent variable) and outputs (dependent variable).

Remember

An independent variable represents a quantity that can change freely. A dependent variable depends on the independent variable.

Input $\xrightarrow{-2}$ Function Rule: $y = 3x$ $\xrightarrow{-6}$ Output

Example 3 Writing a Function Rule

Write a function rule for "The output is five less than the input."

Words	The output	is	five less than	the input.
Equation	y	$=$		$x - 5$

▶ A function rule is $y = x - 5$.

> **Try It**

6. Write a function rule for "The output is one-fourth of the input."

Example 4 Evaluating a Function

What is the value of $y = 2x + 5$ when $x = 3$?

$$y = 2x + 5 \qquad \text{Write the equation.}$$
$$= 2(3) + 5 \qquad \text{Substitute 3 for } x.$$
$$= 11 \qquad \text{Simplify.}$$

> **Try It** **Find the value of y when $x = 5$.**

7. $y = 4x - 1$ 8. $y = 10x$ 9. $y = 7 - 3x$

In-Class Practice

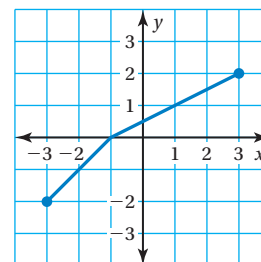| **1** I don't understand yet.	**2** I can do it with help.	**3** I can do it on my own.	**4** I can teach someone else.

10. **IDENTIFYING A FUNCTION** Determine whether the table represents a function. Then find the domain and range.

Input, x	−1	0	1	2	2
Output, y	0	1	2	3	4

11. **IDENTIFYING A FUNCTION** Determine whether the graph represents a function. Then find the domain and range.

12. **EVALUATING A FUNCTION** Find the value of $y = 6x$ when $x = -5$.

4 MTR 13. **DIFFERENT WORDS, SAME QUESTION** Which is different? Find "both" answers.

What output is 4 more than twice the input 3?

What output is twice the sum of the input 3 and 4?

What output is the sum of 2 times the input 3 and 4?

What output is 4 increased by twice the input 3?

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Example 5 Modeling Real Life **7 MTR**

A car produces 20 pounds of carbon dioxide for every gallon of gasoline burned. Write and graph a function that describes the relationship. Then find the domain and range.

Use a verbal model to write a function rule.

Verbal Model

| Carbon dioxide (pounds) | = | Pounds per gallon | · | Gasoline used (gallons) |

Variables Let p represent the number of pounds of carbon dioxide, and let g represent the number of gallons of gasoline used.

Equation $p = 20 \cdot g$

Make an input-output table that represents the function $p = 20g$.

5 MTR RELATE CONCEPTS
Which variable is the independent variable? the dependent variable? Explain.

Input, g	$20g$	Output, p	Ordered Pair, (g, p)
1	20(1)	20	(1, 20)
2	20(2)	40	(2, 40)
3	20(3)	60	(3, 60)

Plot the ordered pairs and draw a line through the points.

Because you cannot burn a negative number of gallons of gasoline, use only positive values of g. The domain is $g \geq 0$ and the range is $p \geq 0$.
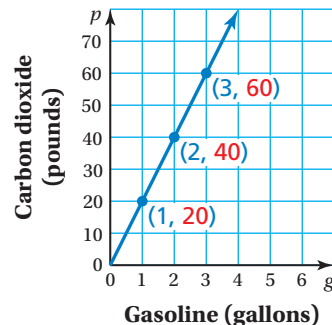
In-Class Practice

1 I don't understand yet. **2** I can do it with help. **3** I can do it on my own. **4** I can teach someone else.

14. The World Health Organization (WHO) suggests having 23 health-care workers for every 10,000 people. How many health-care workers are needed to meet the WHO suggestion for a population of 250,000 people? Justify your answer using a graph.

15. Dig Deeper A truck produces 22 pounds of carbon dioxide for every gallon of diesel fuel burned. The fuel economy of the truck is 18 miles per gallon. Write and graph a function that describes the relationship between carbon dioxide produced and distance traveled. Then find the domain and range.
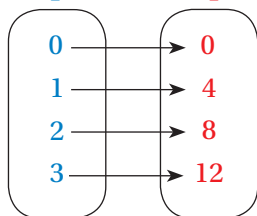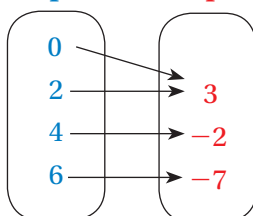
Practice WITH CalcChat® AND CalcView®

Review & Refresh

Determine whether the relation is a function. Then find the domain and range.
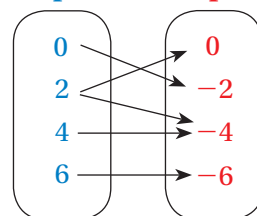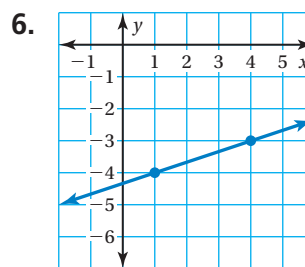
1.

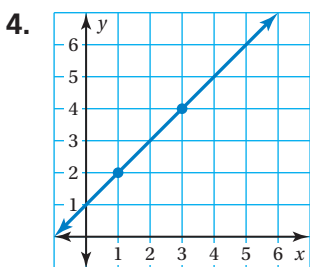| Input	Output
0 → 0
1 → 4
2 → 8
3 → 12

2.

Input	Output
0
2 → 3
4 → -2
6 → -7

3.

Input	Output
0 → 0
2 → -2
4 → -4
6 → -6

Find the slope of the line.

4.

5.

6.
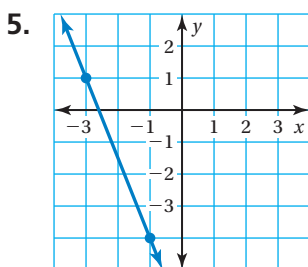
Concepts, Skills, & Problem Solving

IDENTIFYING A FUNCTION Graph the relation represented by the table. Use the graph to determine whether the relation is a function. (See Exploration 1.)

7.

Input, x	Output, y
-2	-6
-1	-3
0	0
1	3
2	6

8.

Input, x	Output, y
-4	1
-1	5
2	7
5	6
8	5

DETERMINING WHETHER TABLES REPRESENT FUNCTIONS Determine whether the table represents a function. Then find the domain and range. (See Example 1.)

▶ 9.

Input, x	Output, y
0	2
1	3
2	4
3	5
4	6

10.

| Input, x	Output, y
-3	0
-2	4
-1	8
-1	12
0	16

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11. YOU BE THE TEACHER Your friend determines whether the table represents a function. Is your friend correct? Explain your reasoning.

Mile, x	1	2	3	4	5
Change in Elevation (feet), y	8	3	0	-3	-3

The output -3 has two inputs, 4 and 5. So, the table does not represent a function.

DETERMINING WHETHER GRAPHS REPRESENT FUNCTIONS Determine whether the graph represents a function. Then find the domain and range. (See Example 2.)

12.

▶**13.**

14.

15.
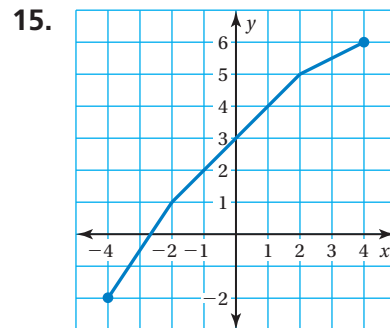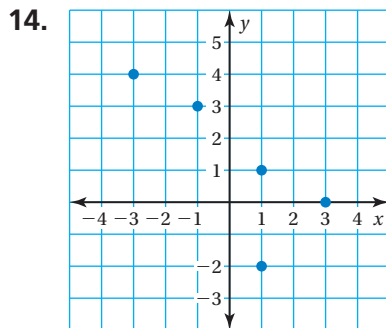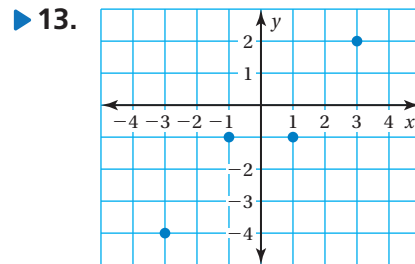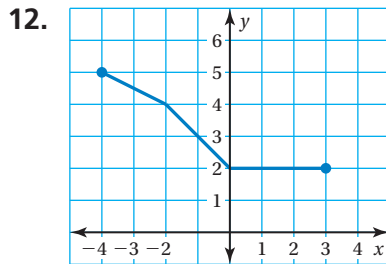
WRITING FUNCTION RULES Write a function rule for the statement. (See Example 3.)

16. The output is half of the input.

17. The output is eleven more than the input.

18. The output is three less than the input.

▶**19.** The output is six times the input.

20. The output is the cube of the input.

21. The output is one more than twice the input.

EVALUATING A FUNCTION Find the value of y for the given value of x. (See Example 4.)

22. $y = 7x;\ x = -5$

▶**23.** $y = x + 5;\ x = 3$

24. $y = 1 - 2x;\ x = 9$

25. $y = 3x + 2;\ x = 0.5$

26. $y = 2x^3;\ x = 3$

27. $y = \dfrac{x}{2} + 9;\ x = -12$

28. B.E.S.T. Test Prep What is the value of $y = \dfrac{1}{3}x + 2$ when $x = 9$?

Ⓐ 4 　　　　　 Ⓑ 5 　　　　　 Ⓒ 6 　　　　　 Ⓓ 8

7 MTR **29. MODELING REAL LIFE** A dolphin eats 30 pounds of fish per day. (See Example 5.)

    a. Write and graph a function that represents the number p of pounds of fish that a dolphin eats in d days.

    b. How many total pounds of fish does a dolphin eat in 30 days?

7 MTR **30. MODELING REAL LIFE** A dance studio teaches ballet folkórico at a rate of $12 per class.

    a. Write and graph a function that represents the cost d for taking c classes.

    b. How much does the dance studio charge for 7 classes?

7 MTR **31. MODELING REAL LIFE** You fill a fish tank with 55 gallons of water on Saturday. The water evaporates at a rate of 1.5 gallons per day. You plan to add water when the tank reaches 49 gallons. When will you add water? Justify your answer.

Ballet folkórico is a traditional Mexican dance.

32. PROBLEM SOLVING You decide to make and sell bracelets. The cost of your materials is $84.00. You charge $3.50 for each bracelet. You will *break even* when the cost of your materials equals your income. How many bracelets must you sell to break even?

33. REASONING You want to take a two-hour airboat tour. Which is a better deal? Use functions to justify your answer.

Snake Tours: $25 per hour
Gator Tours: $35 boarding fee
                  plus $5 per half hour

2 MTR **34. MULTIPLE REPRESENTATIONS** What are the different ways a function can be represented? Provide an example of each.
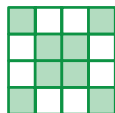
35. REASONING The graph of a function is a line that passes through the points (3, 2), (5, 8), and (8, y). What is the value of y?
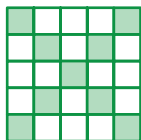
5 MTR **36. PATTERNS** The blocks that form the diagonals of each square are shaded. Each block has an area of one square unit. Find the "green area" of Square 20. Find the "green area" of Square 21. Explain your reasoning.
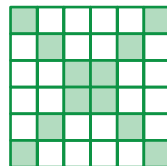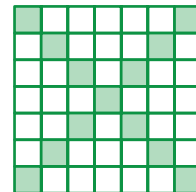
Square 1   Square 2   Square 3   Square 4   Square 5

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