

5.3

Graphing Proportional Relationships



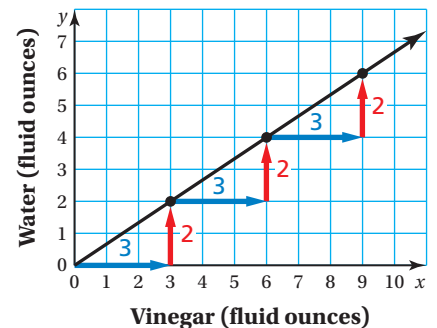
Learning Target: Graph proportional relationships.

- Success Criteria:**
- I can determine whether a linear relationship is a proportional relationship.
 - I can graph an equation that represents a proportional relationship.
 - I can write an equation that represents a proportional relationship.
 - I can use graphs to compare proportional relationships.

Exploration 1 Using a Ratio Table to Find Slope

Work with a partner. The graph shows amounts of vinegar and water that can be used to make a cleaning product.

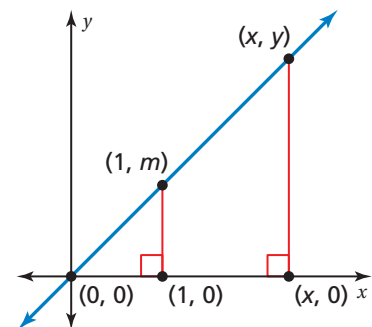
- Use the graph to make a ratio table relating the quantities. Explain how the slope of the line is represented in the table.
- Make a ratio table that represents a different ratio of vinegar to water. Use the table to describe the slope of the graph of the new relationship.



Exploration 2 Deriving an Equation

Work with a partner. Let (x, y) represent any point on the graph of a proportional relationship.

- Describe the relationship between the corresponding side lengths of the triangles shown in the graph. Explain your reasoning.
- Use the relationship in part (a) to write an equation relating y , m , and x . Then solve the equation for y .
- What does your equation in part (b) describe? What does m represent? Explain your reasoning.



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HELP A CLASSMATE

Explain to a classmate how you can find the side lengths of the triangles in the graph.

Algebraic Reasoning

MA.8.AR.3.1 Determine if a linear relationship is also a proportional relationship.

MA.8.AR.3.4 Given a mathematical or real-world context, graph a two-variable linear equation from a written description, a table or an equation in slope-intercept form.

GO DIGITAL



5.3 Lesson

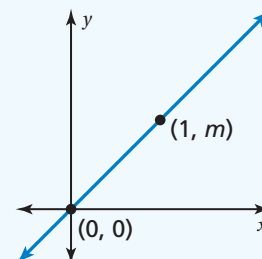
Key Idea

In the equation $y = mx$, m represents the constant of proportionality, the slope, and the unit rate.

Proportional Relationships

Words When two quantities x and y are proportional, the relationship can be represented by the equation $y = mx$, where m is the constant of proportionality.

Graph The graph of $y = mx$ is a line with a slope of m that passes through the origin.



Example 1 B.E.S.T. Test Prep: Identifying Proportional Relationships

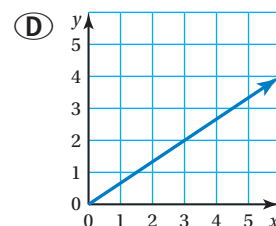
Which of the following linear relationships is also a proportional relationship? Select all that apply.

(A) $y = 7x$

(B) $y = x + 1$

(C)

x	y
0	1
1	3
2	5
3	7



4 MTR JUSTIFY A RESULT

How can you use ratios to justify that the table in Choice C does not represent a proportional relationship?

The equation in Choice A is of the form $y = mx$, where $m = 7$. The equation in Choice B is not of this form. So, the equation in Choice A represents a proportional relationship, and the equation in Choice B does not.

The points in the table in Choice C lie on a line. Because the line intersects the y -axis at $(0, 1)$, the line does not pass through the origin. So, the table does not represent a proportional relationship.

The graph in Choice D is a line that passes through the origin. So, the graph represents a proportional relationship.

► The correct answers are (A) and (D).

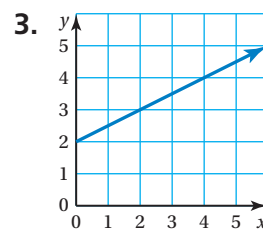
Try It

Tell whether x and y are proportional. Explain your reasoning.

1. $y = \frac{1}{3}x$

2.

x	y
0	0
1	3
2	6
3	9



Example 2 Graphing Proportional Relationships

Graph the equation for each situation and interpret the slope.

- a. The cost y (in dollars) for x ounces of frozen yogurt is represented by $y = 0.5x$.

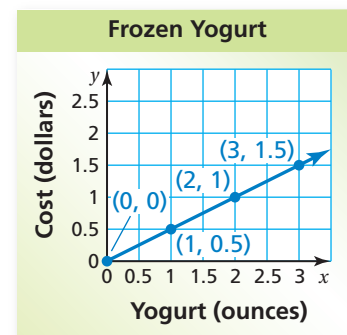
One way to graph the equation is to make a table of values.

x	$y = 0.5x$	y	(x, y)
0	$y = 0.5(0)$	0	(0, 0)
1	$y = 0.5(1)$	0.5	(1, 0.5)
2	$y = 0.5(2)$	1	(2, 1)
3	$y = 0.5(3)$	1.5	(3, 1.5)

1 MTR ANALYZE A PROBLEM

Why does it make sense to graph the equations in Example 2 in the first quadrant only?

Plot the ordered pairs and draw a line through the points.



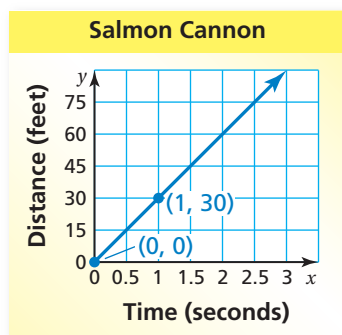
- The slope indicates that the unit cost is \$0.50 per ounce.

- b. The distance y (in feet) that a salmon travels in a salmon cannon after x seconds is represented by $y = 30x$.

One way to graph the equation is to use proportionality and the slope.

The equation represents a proportional relationship because it is of the form $y = mx$, where $m = 30$. So, the graph is a line that passes through $(0, 0)$ and $(1, 30)$.

Plot the ordered pairs and draw a line through the points.



- The slope indicates that the salmon travels 30 feet per second.

A salmon cannon uses air pressure and a tube to propel salmon across dams, allowing the fish to swim upstream to spawn.

Try It

4. **WHAT IF?** The cost of frozen yogurt is represented by $y = 0.75x$. Graph the equation and interpret the slope.



Example 3 Writing and Using an Equation

The weight y of an object on Titan, one of Saturn's moons, is proportional to the weight x of the object on Earth. An object that weighs 105 pounds on Earth would weigh 15 pounds on Titan.

- a. Write an equation that represents the situation.

Use the point (105, 15) to find the slope of the line.

$$y = mx \quad \text{Equation of a proportional relationship}$$

$$15 = m(105) \quad \text{Substitute 15 for } y \text{ and 105 for } x.$$

$$\frac{1}{7} = m \quad \text{Simplify.}$$

► So, an equation that represents the situation is $y = \frac{1}{7}x$.

- b. How much would a chunk of ice that weighs 3.5 pounds on Titan weigh on Earth?

$$3.5 = \frac{1}{7}x \quad \text{Substitute 3.5 for } y.$$

$$24.5 = x \quad \text{Multiply each side by 7.}$$

► So, the chunk of ice would weigh 24.5 pounds on Earth.

7 MTR APPLY MATHEMATICS

What does the slope tell you about weights of objects on Titan and on Earth?

Try It

5. How much would a spacecraft that weighs 3500 kilograms on Earth weigh on Titan?

In-Class Practice

- 1 I don't understand yet. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

GRAPHING A PROPORTIONAL RELATIONSHIP Graph the equation.

6. $y = 4x$

7. $y = -3x$

8. $y = 8x$

9. **WRITING AND USING AN EQUATION** The number y of objects a machine produces is proportional to the time x (in minutes) that the machine runs. The machine produces five objects in four minutes.

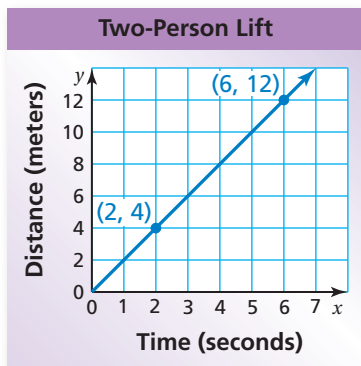
- Write an equation that represents the situation.
- Graph the equation in part (a) and interpret the slope.
- How many objects does the machine produce in one hour?

10. **OPEN-ENDED** Graph a linear equation that does not represent a proportional relationship. How can you *translate* the graph so that it does represent a proportional relationship? Explain your reasoning.



Example 4 Modeling Real Life

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The distance y (in meters) that a four-person ski lift travels in x seconds is represented by the equation $y = 2.5x$. The graph shows the distance that a two-person ski lift travels.

- a. Which ski lift is faster?

Identify the slope of the graph for each lift. Then interpret each slope as a unit rate.

Four-Person Lift

$$y = 2.5x$$

The slope is 2.5.

The four-person lift travels 2.5 meters per second.

Two-Person Lift

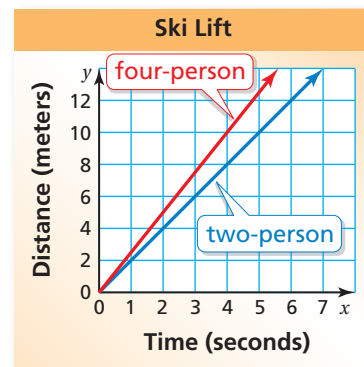
$$\begin{aligned} \text{slope} &= \frac{\text{change in } y}{\text{change in } x} \\ &= \frac{8}{4} = 2 \end{aligned}$$

The two-person lift travels 2 meters per second.

► So, the four-person lift is faster than the two-person lift.

- b. Graph the equation that represents the four-person lift in the same coordinate plane as the two-person lift. Compare and interpret the steepness of each graph.

► The graph that represents the four-person lift is steeper than the graph that represents the two-person lift. So, the four-person lift is faster.



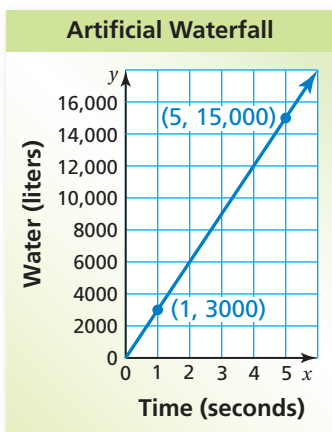
In-Class Practice

1 I don't understand yet.

2 I can do it with help.

3 I can do it on my own.

4 I can teach someone else.



11. The amount y (in liters) of water that flows over a natural waterfall in x seconds is represented by the equation $y = 500x$. The graph shows the number of liters of water that flow over an artificial waterfall. Which waterfall has a greater flow? Justify your answer.
12. The speed of sound in air is 343 meters per second. You see lightning and hear thunder 12 seconds later.
- Is there a proportional relationship between the amount of time that passes and your distance from a lightning strike? Explain.
 - Estimate your distance from the lightning strike.

GO DIGITAL

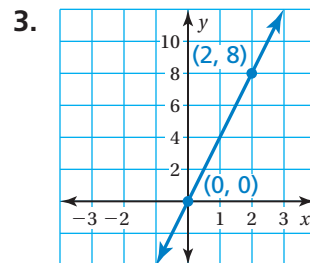
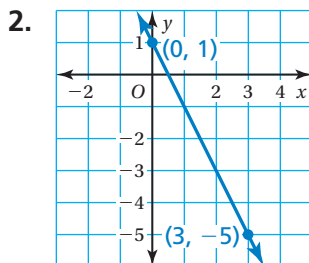
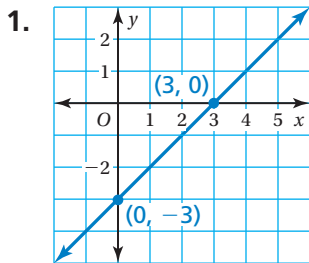


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Practice WITH CalcChat® AND CalcView®

Review & Refresh

Find the slope of the line.



Solve the equation. Check your solution.

4. $2x + 3x = 10$

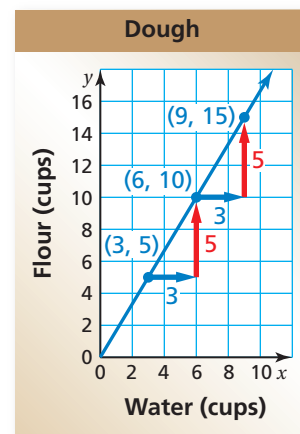
5. $x + \frac{1}{6} = 4 - 2x$

6. $2(1 - x) = 11$

Concepts, Skills, & Problem Solving

USING EQUIVALENT RATIOS The graph shows amounts of water and flour that can be used to make dough. (See Exploration 1.)

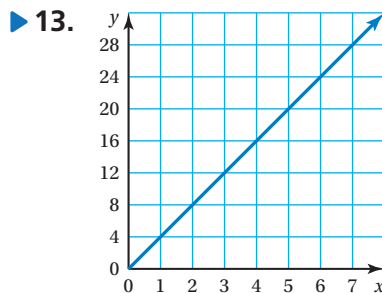
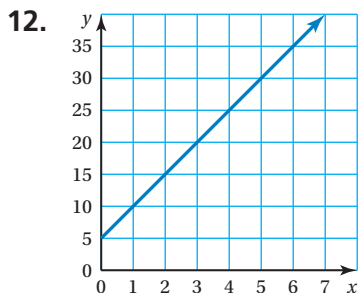
- Use the graph to make a ratio table relating the quantities. Explain how the slope of the line is represented in the table.
- Make a ratio table that represents a different ratio of flour to water. Use the table to describe the slope of the graph of the new relationship.
- GRAPHING AN EQUATION** The amount y (in dollars) that you raise by selling x fundraiser tickets is represented by the equation $y = 5x$. Graph the equation and interpret the slope.



IDENTIFYING PROPORTIONAL RELATIONSHIPS Tell whether x and y are in a proportional relationship. Explain your reasoning. (See Example 1.)

10. $y = 7.5x$

11. $y = 9x - 4$



14.

x	3	6	9	12
y	1	2	3	4

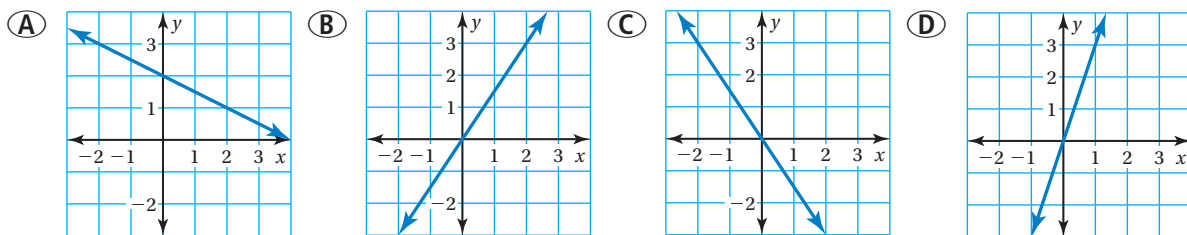
15.

x	2	5	8	10
y	4	8	13	23



GRAPHING A PROPORTIONAL RELATIONSHIP For the given situation, graph the equation and interpret the slope. (See Example 2.)

16. The total price y (in dollars) for x adults to visit Zoo Miami is represented by $y = 23x$.
- ▶ 17. During a cross country drill, the distance y (in miles) your friend runs in x minutes is represented by $y = \frac{1}{10}x$.
18. In the first 24 hours of a hurricane, the total amount of rain y (in inches) that falls in x hours is represented by $y = 0.5x$.
19. The cost y (in dollars) of x pounds of asparagus is represented by $y = 2.8x$.
20. **B.E.S.T. Test Prep** Which is the graph of the relationship represented by $y = -\frac{3}{2}x$?

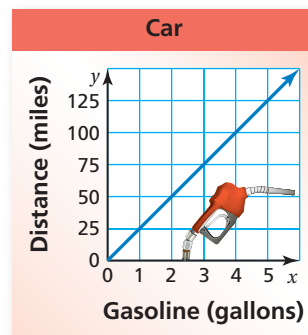


- 7 MTR** ▶ 21. **MODELING REAL LIFE** The cost y (in dollars) to rent a kayak is proportional to the number x of hours that you rent the kayak. It costs \$27 to rent the kayak for 3 hours. (See Example 3.)

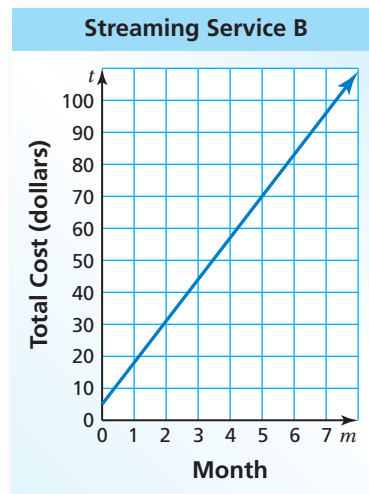
- Write an equation that represents the situation.
- Interpret the slope of the graph of the equation.
- How much does it cost to rent the kayak for 5 hours? Justify your answer.

- 7 MTR** ▶ 22. **MODELING REAL LIFE** The distance y (in miles) that a truck travels on x gallons of gasoline is represented by the equation $y = 18x$. The graph shows the distance that a car travels. (See Example 4.)

- Which vehicle gets better gas mileage? Explain how you found your answer.
- How much farther can the vehicle you chose in part (a) travel on 8 gallons of gasoline?



23. **REASONING** You are investigating streaming service prices. Service A's total cost c (in dollars) for m months can be represented by $c = 15m$. Service B's total cost t for m months is represented in the graph.



- Determine whether the price of each streaming service is proportional to the number of months. Explain your reasoning.
- Which service would you choose? Explain.

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24. **ASSESS REASONABLENESS** You time your bike rides and find that the distance d (in miles) you travel in t hours can be represented by $d = 8t$. Your friend claims that you can bike to a playground that is 18 miles away in 1.5 hours. Is your friend's claim reasonable? Justify your answer using a graph.

25. **PROBLEM SOLVING** Toenails grow about 13 millimeters per year. The table shows fingernail growth.

Weeks	1	2	3	4
Fingernail Growth (millimeters)	0.7	1.4	2.1	2.8

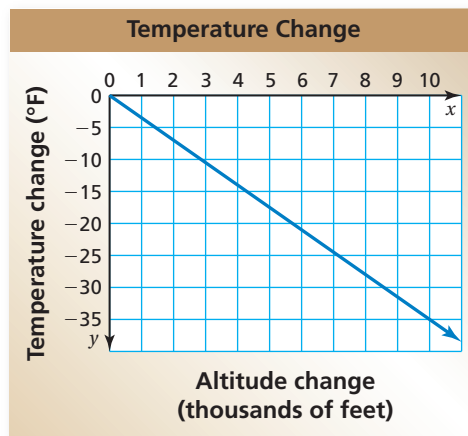
- Do fingernails or toenails grow faster? Explain.
- In the same coordinate plane, graph equations that represent the growth rates of toenails and fingernails. Compare and interpret the steepness of each graph.

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26. **CHOOSE A METHOD** The cost of blackberries is proportional to their weight. You buy blackberries that cost \$3.39 per pound. Graph the relationship and explain why you chose the method you used.

27. **REASONING** The quantities x and y are in a proportional relationship. What do you know about the ratio of y to x for any point (x, y) on the graph of x and y ?

28. **Dig Deeper** The graph relates the temperature change y (in degrees Fahrenheit) to the altitude change x (in thousands of feet).



- Is the relationship proportional? Explain.
- Write an equation of the line. Interpret the slope.
- You are at the bottom of a mountain where the temperature is 74°F . The top of the mountain is 5500 feet above you. What is the temperature at the top of the mountain? Justify your answer.

29. **REASONING** Consider the distance equation $d = rt$, where d is the distance (in feet), r is the rate (in feet per second), and t is the time (in seconds). You run for 50 seconds. Are the distance you run and the rate at which you run proportional? Use a graph to justify your answer.

