4.1 Triangle Inequality Theorem

Learning Target: Understand and apply the Triangle Inequality Theorem.

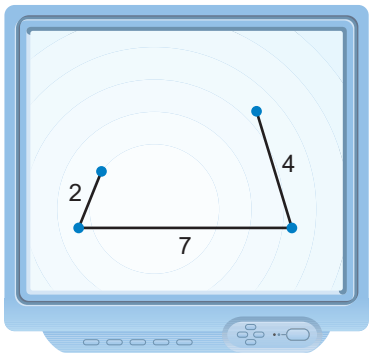
Success Criteria:
- I can determine whether three side lengths form a triangle.
- I can draw triangles given three side lengths.
- I can use two side lengths of a triangle to determine the possible lengths of the third side.

In mathematics, a rule is called a **theorem**. In this lesson, you will learn about a theorem that describes relationships among side lengths of triangles.

Exploration 1 Using Tools to Draw Triangles

Work with a partner.

a. Use geometry software to draw each triangle with the given side lengths, if possible. Complete the table.



Side Lengths	Possible to draw a triangle?
i. 4 cm, 6 cm, 7 cm	
ii. 1 in., 3 in., 5 in.	
iii. 2 cm, 4 cm, 7 cm	
iv. 2 cm, 6 cm, 7 cm	
v. 2 in., 3 in., 5 in.	
vi. 2 cm, 4 cm, 6 cm	
vii. 1 in., 2 in., 2 in.	

3 MTR REFLECT ON YOUR METHOD

How does geometry software help you learn about characteristics of triangles?

b. Choose one set of possible side lengths in part (a). Use a compass and a straightedge to draw the triangle. Explain your method.

c. Without drawing or using geometry software, how can you tell whether it is possible to draw a triangle given three side lengths? Explain your reasoning.

Geometric Reasoning

MA.8.GR.1.3 Use the Triangle Inequality Theorem to determine if a triangle can be formed from a given set of sides. Use the converse of the Pythagorean Theorem to determine if a right triangle can be formed from a given set of sides.

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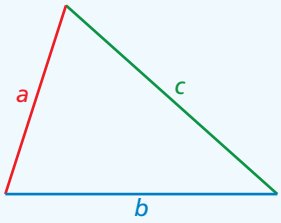
4.1 Lesson

Key Vocabulary
theorem, *p. 187*

Key Idea

Triangle Inequality Theorem

The sum of the lengths of any two sides of a triangle is greater than the length of the third side.



$a + b > c$ $\qquad$ $b + c > a$ $\qquad$ $a + c > b$

Example 1 Using Side Lengths to Identify Triangles

Determine whether the side lengths form a triangle.

a. 3 yd, 12 yd, 13 yd

Find the sum of the lengths of each pair of sides. Compare each sum to the length of the third side.

$3 \text{ yd} + 12 \text{ yd} = 15 \text{ yd}$ $\qquad$ $15 \text{ yd} > 13 \text{ yd}$ ✓

$12 \text{ yd} + 13 \text{ yd} = 25 \text{ yd}$ $\qquad$ $25 \text{ yd} > 3 \text{ yd}$ ✓

$3 \text{ yd} + 13 \text{ yd} = 16 \text{ yd}$ $\qquad$ $16 \text{ yd} > 12 \text{ yd}$ ✓

▶ Because each sum of two side lengths is greater than the length of the third side, the side lengths form a triangle.

b. 4 ft, 5 ft, 9 ft

Find the sum of the lengths of each pair of two sides. Compare each sum to the length of the third side.

$4 \text{ ft} + 5 \text{ ft} = 9 \text{ ft}$ $\qquad$ $9 \text{ ft} \not> 9 \text{ ft}$ ✗

$5 \text{ ft} + 9 \text{ ft} = 14 \text{ ft}$ $\qquad$ $14 \text{ ft} > 4 \text{ ft}$ ✓

$4 \text{ ft} + 9 \text{ ft} = 13 \text{ ft}$ $\qquad$ $13 \text{ ft} > 5 \text{ ft}$ ✓

▶ Because $4 \text{ ft} + 5 \text{ ft} \not> 9 \text{ ft}$, the side lengths do not form a triangle.

5 MTR USE STRUCTURE
How can you change one of the side lengths in part (b) so that they form a triangle? Compare answers with a classmate.

Try It **Determine whether the side lengths form a triangle.**

1. 6 ft, 8 ft, 5 ft **2.** 2 mm, 5 mm, 8 mm **3.** 12 in., 5 in., 7 in.

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Example 2 Constructing Triangles Using Side Lengths

Draw a triangle with the given side lengths, if possible.

a. 4 cm, 2 cm, 3 cm

The sum of the lengths of any two sides is greater than the length of the third side.

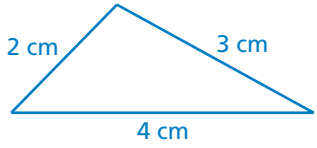
$4 \text{ cm} + 2 \text{ cm} > 3 \text{ cm}$ $4 \text{ cm} + 3 \text{ cm} > 2 \text{ cm}$ $2 \text{ cm} + 3 \text{ cm} > 4 \text{ cm}$

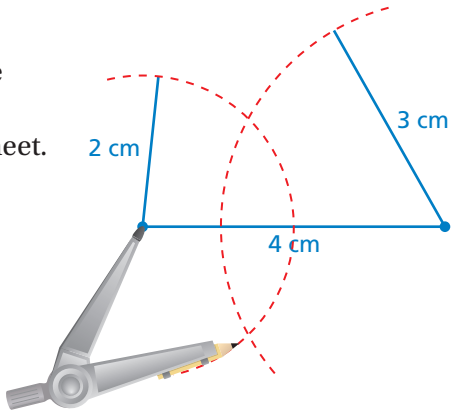
So, you can draw a triangle with the given side lengths.

Step 1: Draw a 4-centimeter side.

Step 2: Use a compass to determine where the 2-centimeter side and the 3-centimeter side meet.

Step 3: The third vertex can be at either intersection point. Draw the triangle.





6 MTR ASSESS REASONABLENESS

Try to draw the triangle in part (b). Explain how the result shows that it is not possible to draw the triangle.

b. 2.5 in., 1 in., 1 in.

Because 1 in. + 1 in. < 2.5 in., it is not possible to draw the triangle.

Try It **Draw a triangle with the given side lengths, if possible.**

4. 2 cm, 2 cm, 5 cm **5.** 4 cm, 3 cm, 3 cm **6.** 1 cm, 4 cm, 5 cm

In-Class Practice

1 I don't understand yet. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

IDENTIFYING TRIANGLES **Determine whether the side lengths form a triangle.**

7. 13 ft, 7 ft, 6 ft **8.** 36 mm, 18 mm, 40 mm

CONSTRUCTING TRIANGLES **Draw a triangle with the given side lengths, if possible.**

9. 25 mm, 36 mm, 38 mm **10.** 6 cm, 2 cm, 7 cm

4 MTR **11. WHICH ONE DOESN'T BELONG?** Which group of side lengths does *not* belong with the other three? Explain your reasoning.

5 cm, 2 cm, 6 cm

8 cm, 4 cm, 4 cm

3 cm, 7 cm, 6 cm

3 cm, 3 cm, 5 cm

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Example 3 Modeling Real Life

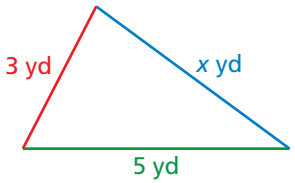
You want to enclose a flower bed using three landscaping boards. You have two boards with lengths of 3 yards and 5 yards. Describe the possible lengths that you can use for the third board.

The third board could be one of the shorter sides or the longest side. Use the Triangle Inequality Theorem to write and solve an inequality for each case. Let x represent the length of the third board.

1 MTR ANALYZE A PROBLEM
In Case 1, does it matter whether x represents the shortest side or the second-shortest side? Explain.

Case 1: x represents one of the shorter sides.

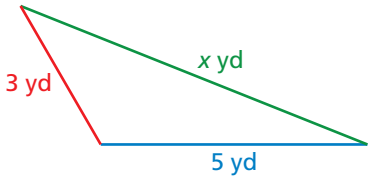
$$3 + x > 5$$
$$x > 2$$



Case 2: x represents the longest side.

5 MTR RELATE CONCEPTS
Choose a value for x between 2 and 8. Then show that it is possible to draw a triangle using the three side lengths. What scale factor did you use?

$$3 + 5 > x$$
$$8 > x$$



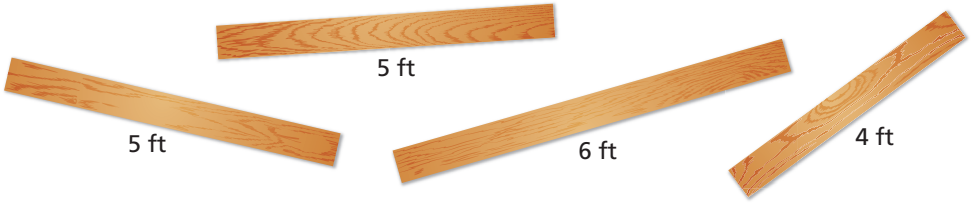
▶ You can use a board that is greater than 2 yards long and less than 8 yards long for the third side.

In-Class Practice

1 I don't understand yet. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

12. A triangular pen has two fence lengths of 6 feet and 8 feet. Describe the possible lengths of the third fence. Then choose a possible length and create a scale drawing of the pen.

13. How can you use all the boards shown to create a triangle? Justify your answer. (*Hint*: Join boards together at their ends to create a longer board.)



14. **Dig Deeper** Two rooftops have triangular patios. One patio has side lengths of 9 meters, 10 meters, and 11 meters. The other has side lengths of 6 meters, 10 meters, and 15 meters. Which patio has a greater area? Explain.

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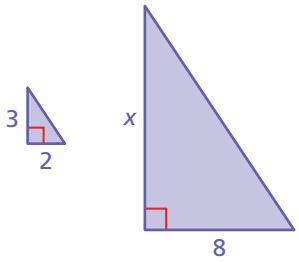
4.1 Practice WITH CalcChat® AND CalcView®

Review & Refresh

The figures are similar. Find x.

1.



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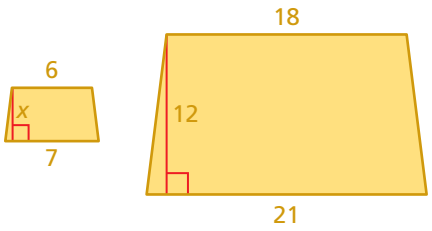

Find the sample space and the total number of possible outcomes of the indicated event.

3. choosing a toothbrush

Toothbrush	
Type	Electric, Traditional
Strength	Extra soft, Soft, Medium

4. choosing a toy hoop

Toy Hoop	
Size	Small, Medium, Large
Color	Blue, Green, Orange, Pink, Purple, Yellow

Concepts, Skills, & Problem Solving

USING TOOLS TO DRAW TRIANGLES **Use geometry software to draw the triangle with the given side lengths, if possible.** (See Exploration 1.)

5. 2 ft, 7 ft, 9 ft

6. 3 cm, 4 cm, 5 cm

USING SIDE LENGTHS TO IDENTIFY TRIANGLES **Determine whether the side lengths form a triangle.** (See Example 1.)

7. 16 m, 21 m, 27 m

8. 10 in., 9 in., 22 in.

9. 6 mm, 1 mm, 5 mm

10. 7 mi, 9 mi, 7 mi

4 MTR

11. **YOU BE THE TEACHER** Your friend wants to determine whether the side lengths 11 cm, 43 cm, and 55 cm form a triangle. Is your friend correct? Explain your reasoning.

$11 \text{ cm} + 43 \text{ cm} < 55 \text{ cm}$

Because the sum of the lengths of two sides is not greater than the length of the third side, you cannot form a triangle.

CONSTRUCTING TRIANGLES USING SIDE LENGTHS **Draw a triangle with the given side lengths, if possible.** (See Example 2.)

12. 4 in., 5 in., 10 in.

13. 10 mm, 30 mm, 50 mm

14. 5 cm, 5 cm, 8 cm

15. 8 mm, 12 mm, 13 mm

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16. **MODELING REAL LIFE** Can you construct a warning triangle using three pieces of plastic that are each 6 inches long? Explain.

17. **MODELING REAL LIFE** You watch a rocket launch at the Kennedy Space Center at a distance of 5 miles from the launchpad. In the first 66 seconds after launch, the rocket travels 11.5 miles from the launchpad. (See Example 3.)

a. Describe the possible distances between you and the rocket after 66 seconds.

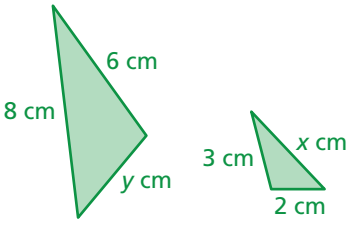
b. Are all distances in part (a) reasonable? Explain.

DISCUSS MATHEMATICAL THINKING **Determine whether you can construct *one*, *many*, or *no* triangle(s) with the given description. Explain your reasoning.**

18. a scalene triangle with side lengths of 3 centimeters and 7 centimeters

19. an isosceles triangle with side lengths of 4 feet and 1 foot

20. a triangle with side lengths of 5 meters and 2 meters



21. **MAKE A CONNECTION** The triangles are similar. Find possible side lengths for the missing sides. Explain your reasoning.

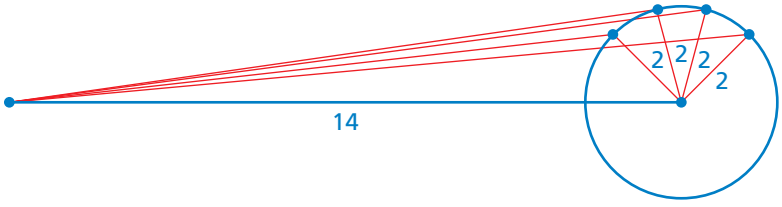
22. **MODELING REAL LIFE** A chemical spill expert sets up a triangular caution zone using cones. Cones A and B are 14 meters apart. Cones B and C are 22 meters apart.

a. Describe possible distances between Cone C and Cone A.

b. The caution zone triples in size as the spill spreads. The new caution zone is similar to the original caution zone. Describe the possible distances between the cones for the new caution zone. Explain.

23. **REASONING** A triangle has fixed side lengths of 2 and 14.

a. How many triangles can you construct? Use the figure below to explain your reasoning. (*Hint:* Each point on a circle is the same distance from a point called the center.)



b. Is the unknown side length of the triangle also fixed? Explain.

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