4.7 Using Graphs of Proportional Relationships

Learning Target: Represent proportional relationships using graphs and equations.

Success Criteria:
- I can graph an equation in two variables.
- I can determine whether quantities are proportional using a graph.
- I can find the unit rate of a proportional relationship using a graph.
- I can create equations to represent proportional relationships.

Exploration 1 Representing Relationships Graphically

Work with a partner. The tables represent two different ways that red and blue food coloring are mixed.

Mixture 1

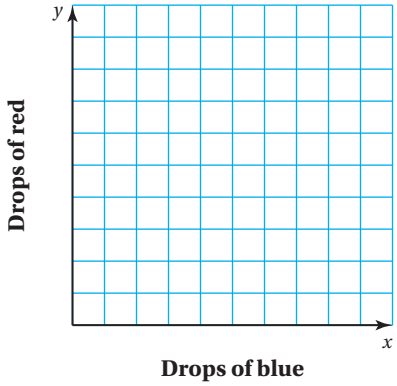
Drops of Blue, x	Drops of Red, y
1	2
2	4
3	6
4	8

Mixture 2

Drops of Blue, x	Drops of Red, y
0	2
2	4
4	6
6	8

2 MTR MAKE A CONNECTION
How is the graph of the proportional relationship different from the other graph?

a. Represent each table in the same coordinate plane. Which graph represents a proportional relationship? How do you know?



b. Find the unit rate of the proportional relationship. How is the unit rate shown on the graph?

c. What is the multiplicative relationship between x and y for the proportional relationship? How can you use this value to write an equation that relates y and x?

Algebraic Reasoning
MA.7.AR.4.1 Determine whether two quantities have a proportional relationship by examining a table, graph or written description.
MA.7.AR.4.2 Determine the constant of proportionality within a mathematical or real-world context given a table, graph or written description of a proportional relationship.
Also MA.7.AR.4.3, MA.7.AR.4.4, MA.7.AR.4.5

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4.7 Lesson

Key Vocabulary

equation in two variables, *p. 210*
solution of an equation in two variables, *p. 210*
independent variable, *p. 210*
dependent variable, *p. 210*
constant of proportionality, *p. 211*

An **equation in two variables** represents two quantities that change in relationship to one another. A **solution of an equation in two variables** is an ordered pair that makes the equation true. The variable representing the quantity that can change freely is the **independent variable**. The other variable is called the **dependent variable** because its value *depends* on the independent variable.

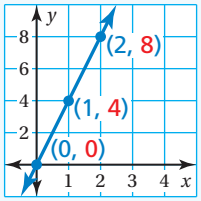
Key Idea

Tables, Graphs, and Equations

You can use tables and graphs to represent equations in two variables. The independent variable is graphed on the horizontal axis, and the dependent variable is graphed on the vertical axis. The table and graph below represent the equation $y = 4x$.

When you draw a line through the points, you graph *all* the solutions of the equation.

Independent Variable, x	Dependent Variable, y	Ordered Pair, (x, y)
0	0	(0, 0)
1	4	(1, 4)
2	8	(2, 8)



Example 1 Graphing an Equation in Two Variables

Graph $y = 10x$.

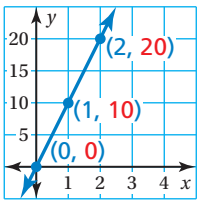
To graph the equation, first make a table.

Independent Variable, x	$y = 10x$	Dependent Variable, y	Ordered Pair, (x, y)
0	$y = 10(0)$	0	(0, 0)
1	$y = 10(1)$	10	(1, 10)
2	$y = 10(2)$	20	(2, 20)

Remember

In a coordinate plane, the horizontal axis is often called the *x-axis*. The vertical axis is often called the *y-axis*. In real-life problems, other variables can be used.

Then plot the ordered pairs and draw a line through the points.



Try It

Graph the equation.

1. $y = 12x$
2. $y = \frac{1}{2}x$
3. $y = x + 1$

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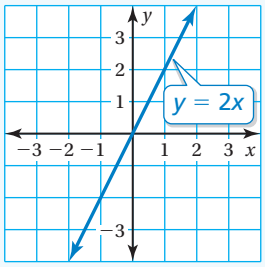
Key Idea

Graphs of Proportional Relationships

Words Two quantities x and y are proportional when $y = kx$, where k is a number and $k \neq 0$. The number k represents the multiplicative relationship between the quantities and is called the **constant of proportionality**.

Graph The graph of $y = kx$ is a line that passes through the origin.



The equation $y = kx$ can also be written as $\frac{y}{x} = k$. So, k is equal to the value of the ratio $y : x$.

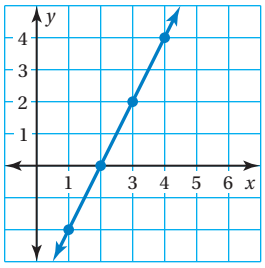
Example 2 Determining Whether Two Quantities Are Proportional

Tell whether x and y are proportional. If so, find the constant of proportionality. Explain your reasoning.

a.

x	1	2	3	4
y	−2	0	2	4

Plot the points. Draw a line through the points.

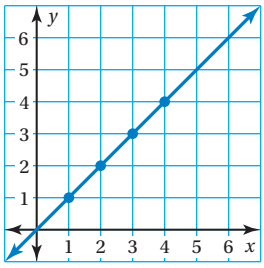


▶ The line does *not* pass through the origin. So, x and y are not proportional.

b.

x	1	2	3	4
y	1	2	3	4

Plot the points. Draw a line through the points.



▶ The line passes through the origin. So, x and y are proportional. The constant of proportionality is $k = 1$ because the value of the ratio $\frac{y}{x}$ is equal to 1.

Try It

Tell whether x and y are proportional. If so, find the constant of proportionality. Explain your reasoning.

4.

x	1	2	3	4
y	1	4	7	10

5.

x	1	2	3	4
y	4	8	12	16

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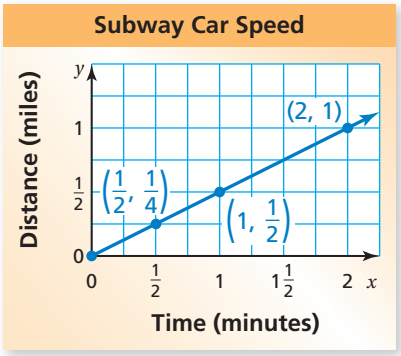
Example 3 Finding a Unit Rate from a Graph



The graph shows the speed of a subway car. Find the speed in miles per minute.

The graph is a line through the origin, so time and distance are proportional. To find the speed in miles per minute, use a point on the graph to find the unit rate.

One Way: Use the point (2, 1) to find the speed.

The point (2, 1) indicates that the subway car travels 1 mile every 2 minutes. So, the unit rate is

$\frac{1}{2}$ mile per minute.

▶ The speed of the subway car is $\frac{1}{2}$ mile per minute.

Another Way: Use the point $\left(1, \frac{1}{2}\right)$ to find the speed.

The point $\left(1, \frac{1}{2}\right)$ indicates that the subway car travels $\frac{1}{2}$ mile every 1 minute. This is the unit rate.

▶ The speed of the subway car is $\frac{1}{2}$ mile per minute.

On the graph of a proportional relationship, the point $(1, k)$ indicates the unit rate, $k : 1$, and the constant of proportionality, k. This value is a measure of the steepness, or slope, of the line.

Try It

6. **WHAT IF?** Does your answer change when you use the point $\left(\frac{1}{2}, \frac{1}{4}\right)$ to find the speed of the subway car? Explain your reasoning.

In-Class Practice

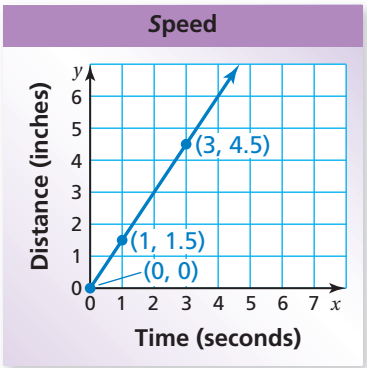
1 I don't understand yet. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

GRAPHING EQUATIONS Graph the equation.

7. $y = 7x$
8. $y = \frac{1}{4}x$
9. $y = 2x + 3$

10. **IDENTIFYING A PROPORTIONAL RELATIONSHIP** Use the graph shown to tell whether x and y are proportional. Explain your reasoning.

11. **FINDING A UNIT RATE** Interpret each plotted point in the graph. Then identify the unit rate, if possible.



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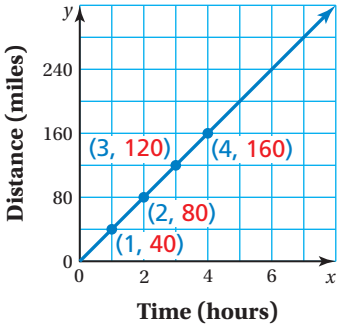
Example 4 Modeling Real Life

A train averages 40 miles per hour between two cities. Write and graph an equation that represents the relationship between the time and the distance traveled. How long does it take the train to travel 220 miles?

The rate is 40 miles per hour. Because all the rates you can write using this relationship are equivalent, the distance traveled is proportional to the time spent traveling. The constant of proportionality is 40, so an equation for the distance traveled y (in miles) after x hours is $y = 40x$.



Make a table and graph the equation.

Time (hours), x	1	2	3	4
Distance (miles), y	40	80	120	160

Use the equation to find the value of x when $y = 220$.

$y = 40x$ Write the equation.

$220 = 40x$ Substitute 220 for y.

$5.5 = x$ Divide each side by 40.

▶ The train travels 220 miles in 5.5 hours.

You can also tell from the graph that the distance is about 220 miles after 5.5 hours.

Another Method

Use a ratio table.

	÷ 2	× 11	
Time (hours), x	1	0.5	5.5 ✓
Distance (miles), y	40	20	220
	÷ 2	× 11	

In-Class Practice

1 I don't understand yet. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

12. A sky lantern rises at an average speed of 8 feet per second. Write and graph an equation that represents the relationship between the time and the distance risen. How long does it take the lantern to rise 100 feet?

13. You and a friend start biking in opposite directions from the same point. You travel 108 feet every 8 seconds. Your friend travels 63 feet every 6 seconds. How far apart are you and your friend after 15 minutes?

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4.7 Practice WITH CalcChat® AND CalcView®

Review & Refresh

Find the value of x so that the ratios are equivalent.

1. $2 : 7$ and $8 : x$ **2.** 3 to 2 and x to 18 **3.** $9 : x$ and $54 : 8$

Find the quotient, if possible.

4. $36 \div 4$ **5.** $42 \div (-6)$ **6.** $-39 \div 3$ **7.** $-44 \div (-4)$

Solve the inequality. Graph the solution.

8. $-\frac{x}{3} < 2$ **9.** $\frac{1}{3}p \geq 4$ **10.** $-8 < \frac{2}{3}n$ **11.** $-2w \leq 10$

Concepts, Skills, & Problem Solving

REPRESENTING RELATIONSHIPS GRAPHICALLY **Represent the table graphically. Does the graph represent a proportional relationship? How do you know?** (See Exploration 1.)

12.

Hours, x	Miles, y
0	50
1	100
2	150

13.

Cucumbers, x	Tomatoes, y
2	4
3	6
4	8

GRAPHING EQUATIONS **Graph the equation.** (See Example 1.)

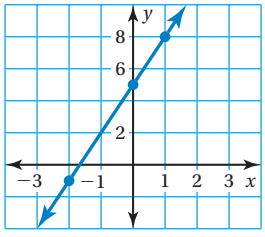
14. $y = 2x$ **15.** $y = 5x$ **16.** $y = 6x$

17. $y = x + 2$ **18.** $y = x + 0.5$ **19.** $y = x + 4$

20. $y = x + 10$ **21.** $y = 3x + 2$ **22.** $y = 2x + 4$

23. $y = \frac{2}{3}x + 8$ **24.** $y = \frac{1}{4}x + 6$ **25.** $y = 2.5x + 12$

26. **B.E.S.T. Test Prep** Which equation is shown in the graph?



Ⓐ $y = 3x - 5$ Ⓒ $y = 4x - 2$

Ⓑ $y = 3x + 5$ Ⓓ $y = 4x + 2$

7 MTR **27.** **MODELING REAL LIFE** The number of people y entering the Dalí Museum in St. Petersburg each hour x can be represented by $y = 20x$. Graph the equation. After how many hours do 120 people enter the Dalí Museum?

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IDENTIFYING A PROPORTIONAL RELATIONSHIP **Tell whether x and y are proportional. If so, find the constant of proportionality. Explain your reasoning.** (See Example 2.)

28.

x	1	2	3	4
y	2	4	6	8

29.

x	-2	-1	0	1
y	0	2	4	6

30.

x	-1	0	1	2
y	-2	-1	0	1

31.

x	3	6	9	12
y	2	4	6	8

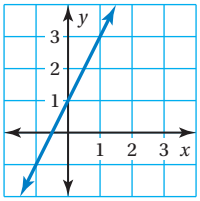
32.

x	1	2	3	4
y	3	4	5	6

33.

x	1	3	5	7
y	0.5	1.5	2.5	3.5

34. **YOU BE THE TEACHER** Your friend uses the graph to determine whether x and y are proportional. Is your friend correct? Explain your reasoning.



> The graph is a line, so x and y are proportional.

CONSTANT OF PROPORTIONALITY **Identify the constant of proportionality in the situation.**

35. A car travels 75 miles in 1 hour.

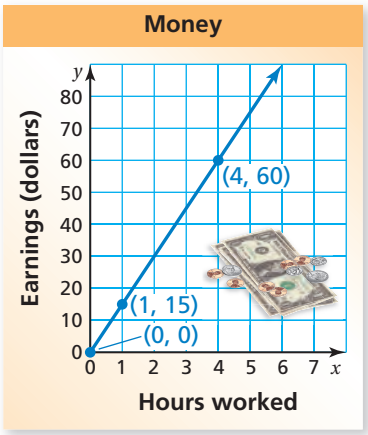
36. In 2019, there were about 80 alligators per 4 square miles in Florida.

37. A restaurant prepares 15 *pan con bistec* in 30 minutes.

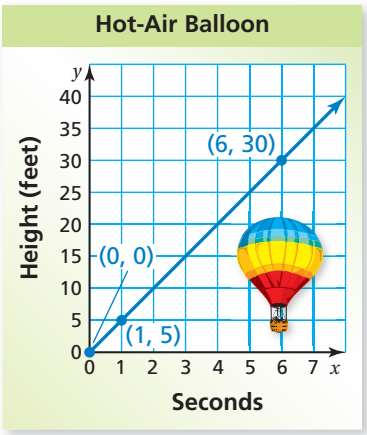
38. A veterinarian sees 48 pets in 8 hours.

FINDING A UNIT RATE **Interpret each plotted point in the graph. Then identify the unit rate.** (See Example 3.)

39.



40.



IDENTIFYING A PROPORTIONAL RELATIONSHIP **Tell whether x and y are proportional. If so, identify the constant of proportionality. Explain your reasoning.**

41. $x - y = 0$ **42.** $\frac{x}{y} = 2$ **43.** $8 = xy$ **44.** $x^2 = y$

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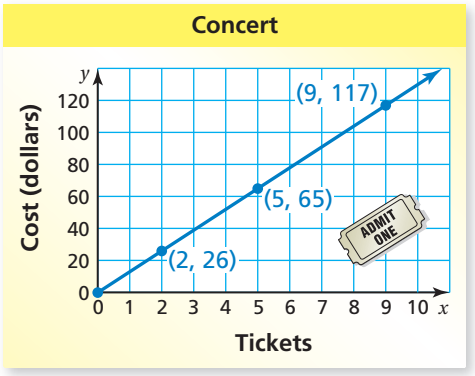
45. MODELING REAL LIFE The table shows the profit y for recycling x pounds of aluminum. Find the profit for recycling 75 pounds of aluminum.

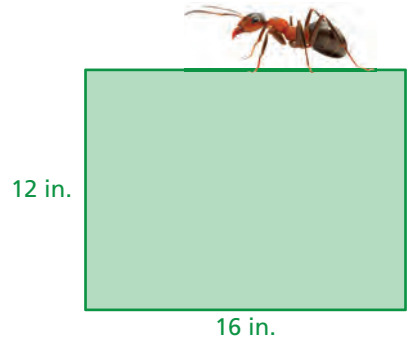
Aluminum (lb), x	10	20	30	40
Profit, y	$4.50	$9.00	$13.50	$18.00



46. MODELING REAL LIFE The graph shows the cost of buying concert tickets. Tell whether x and y are proportional. If so, find and interpret the constant of proportionality. Then find the cost of 14 tickets.

47. MODELING REAL LIFE You charge $10 to mow a lawn. Write and graph an equation that represents the amount you earn (in dollars) for mowing lawns in your neighborhood. How much do you earn when you mow 17 lawns? (See Example 4.)

48. MODELING REAL LIFE It costs $35 a month for membership at a wholesale store. Write and graph an equation that represents the monthly cost (in dollars) of a membership. What is the cost of a membership for an entire year?



49. GEOMETRY How fast should the ant walk to go around the rectangle in 4 minutes?

50. REASONING The graph of a proportional relationship passes through (12, 16) and (1, y). Find y.

51. PROBLEM SOLVING The amount of chlorine in a swimming pool is proportional to the volume of water. The pool has 2.5 milligrams of chlorine per liter of water. How much chlorine is in the pool?


8000 gallons

52. MODELING REAL LIFE To estimate how far you are from lightning (in miles), count the number of seconds between a lightning flash and the thunder that follows. Then divide the number of seconds by 5. Use two different methods to find the number of seconds between a lightning flash and the thunder that follows when a storm is 2.4 miles away.

53. Dig Deeper A vehicle travels 250 feet every 3 seconds. Find the value of the ratio, the unit rate, and the constant of proportionality. How are they related?

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