4.5 Identifying Proportional Relationships

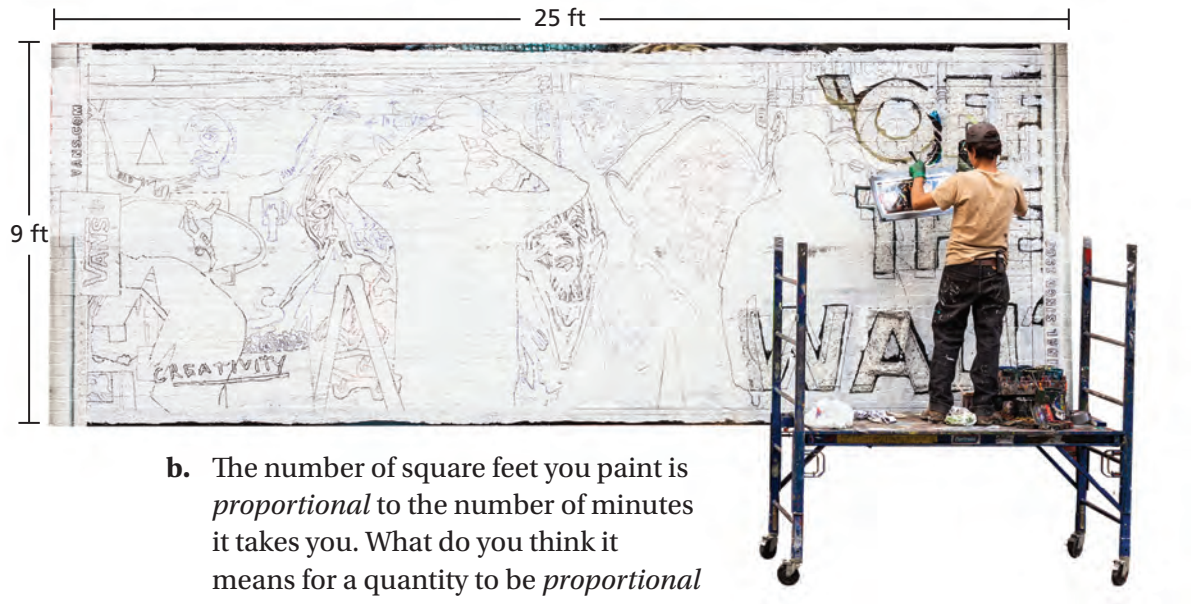
Learning Target: Determine whether two quantities are in a proportional relationship.

Success Criteria:
- I can determine whether ratios form a proportion.
- I can explain how to determine whether quantities are proportional.
- I can distinguish between proportional and nonproportional situations.

Exploration 1 Determining Proportional Relationships

Work with a partner.

a. You can paint 50 square feet of a surface every 40 minutes. How long does it take you to paint the mural shown? Explain how you found your answer.



b. The number of square feet you paint is *proportional* to the number of minutes it takes you. What do you think it means for a quantity to be *proportional* to another quantity?

c. Assume your friends paint at the same rate as you. The table shows how long it takes you and different numbers of friends to paint a fence. Is x proportional to y in the table? Explain.

Painters, x	1	2	3	4
Hours, y	4	2	$\frac{4}{3}$	1

5 MTR USE A PATTERN
How can the table in part (c) help you answer the question in part (d)?

d. How long will it take you and four friends to paint the fence? Explain how you found your answer.

Algebraic Reasoning
MA.7.AR.4.1 Determine whether two quantities have a proportional relationship by examining a table, graph or written description.
MA.7.AR.4.5 Solve real-world problems involving proportional relationships.

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4.5 Lesson

Key Vocabulary

proportion, *p. 194*

cross products, *p. 195*

proportional, *p. 196*

Key Idea

Proportions

Words A **proportion** is an equation stating that the values of two ratios are equivalent.

Numbers Equivalent ratios: $2 : 3$ and $4 : 6$

Proportion: $\frac{2}{3} = \frac{4}{6}$

Example 1 Determining Whether Ratios Form a Proportion

Tell whether the ratios form a proportion.

When you are determining whether ratios form a proportion, you are checking whether the ratios are equivalent.

a. $6 : 4$ and $8 : 12$

Compare the values of the ratios.

$$\frac{6}{4} = \frac{6 \div 2}{4 \div 2} = \frac{3}{2}$$

$$\frac{8}{12} = \frac{8 \div 4}{12 \div 4} = \frac{2}{3}$$

The values of the ratios are *not* equivalent.

▶ Because $\frac{3}{2} \neq \frac{2}{3}$, the ratios $6 : 4$ and $8 : 12$ do *not* form a proportion.

b. $10 : 40$ and $2.5 : 10$

Compare the values of the ratios.

$$\frac{10}{40} = \frac{10 \div 10}{40 \div 10} = \frac{1}{4}$$

$$\frac{2.5}{10} = \frac{2.5 \times 10}{10 \times 10} = \frac{25}{100} = \frac{25 \div 25}{100 \div 25} = \frac{1}{4}$$

The values of the ratios are equivalent.

▶ Because $\frac{1}{4} = \frac{1}{4}$, the ratios $10 : 40$ and $2.5 : 10$ form a proportion.

Try It

Tell whether the ratios form a proportion.

1. $1 : 2$ and $5 : 10$
2. $4 : 6$ and $18 : 24$
3. 4.5 to 3 and 6 to 9
4. $\frac{1}{2}$ to $\frac{1}{4}$ and 8 to 4

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Key Ideas

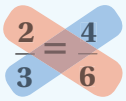
Cross Products

In the proportion $\frac{a}{b} = \frac{c}{d}$, the products $a \cdot d$ and $b \cdot c$ are called **cross products.**

Cross Products Property

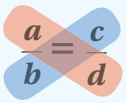
Words The cross products of a proportion are equal.

Numbers



$2 \cdot 6 = 3 \cdot 4$

Algebra



$ad = bc$, where $b \neq 0$ and $d \neq 0$

You can use the Multiplication Property of Equality to show that the cross products are equal.

$$\frac{a}{b} = \frac{c}{d}$$

$$bd \cdot \frac{a}{b} = bd \cdot \frac{c}{d}$$

$$ad = bc$$

Example 2 Using Cross Products

Tell whether the ratios form a proportion.

a. $6 : 9$ and $12 : 18$

Use the Cross Products Property to determine whether the ratios form a proportion.

$\frac{6}{9} \stackrel{?}{=} \frac{12}{18}$ Determine whether the values of the ratios are equivalent.

$6 \cdot 18 \stackrel{?}{=} 9 \cdot 12$ Find the cross products.

$108 = 108$ The cross products are equal.

▶ So, the ratios $6 : 9$ and $12 : 18$ form a proportion.

b. $2 : 3$ and $4 : 5$

Use the Cross Products Property to determine whether the ratios form a proportion.

$\frac{2}{3} \stackrel{?}{=} \frac{4}{5}$ Determine whether the values of the ratios are equivalent.

$2 \cdot 5 \stackrel{?}{=} 3 \cdot 4$ Find the cross products.

$10 \neq 12$ The cross products are *not* equal.

▶ So, the ratios $2 : 3$ and $4 : 5$ do *not* form a proportion.

Try It

Tell whether the ratios form a proportion.

5. $6 : 2$ and $12 : 1$

6. $8 : 12$ and $\frac{2}{3} : 1$

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Two quantities are **proportional** when all the ratios relating the quantities are equivalent. These quantities are said to be in a *proportional relationship*.

Example 3 Determining Whether Two Quantities Are Proportional

Tell whether x and y are proportional.

Compare the values of the ratios x to y.

$$\frac{\frac{1}{2}}{3} = \frac{1}{6} \qquad \frac{1}{6} \qquad \frac{\frac{3}{2}}{9} = \frac{1}{6} \qquad \frac{2}{12} = \frac{1}{6}$$

The values of the ratios are equivalent.

x	y
$\frac{1}{2}$	3
1	6
$\frac{3}{2}$	9
2	12

▶ So, x and y are proportional.

2 MTR USE ANOTHER METHOD
Can you use the values of the ratios y to x in Example 3? Explain.

Try It **Tell whether x and y are proportional.**

7.

x	1	2	3	4
y	2	4	6	8

8.

x	2	4	6	8	10
y	4	2	1	$\frac{1}{2}$	$\frac{1}{4}$

In-Class Practice

1 I don't understand yet. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

PROPORTIONS **Tell whether the ratios form a proportion.**

9. 4 : 14 and 12 : 40

10. 9 : 3 and 45 : 15

11. VOCABULARY Explain how to determine whether two quantities are proportional.

4 MTR 12. WHICH ONE DOESN'T BELONG? Which ratio does *not* belong with the other three? Explain your reasoning.

4 : 10 2 : 5

3 : 5 6 : 15

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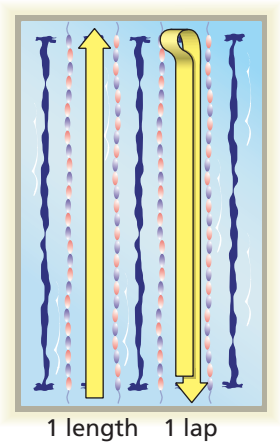
Example 4 Modeling Real Life



You swam for 16 minutes and completed 20 laps. You swam your first 4 laps in 2.4 minutes. How long does it take you to swim 10 laps?

Compare unit rates to determine whether the number of laps is proportional to your time. If it is, then you can use ratio reasoning to find the time it takes you to swim 10 laps.

2.4 minutes for every 4 laps: $\frac{2.4}{4} = 0.6$ minute per lap

16 minutes for every 20 laps: $\frac{16}{20} = 0.8$ minute per lap

The number of laps is *not* proportional to the time. So, you *cannot* use ratio reasoning to determine the time it takes you to swim 10 laps.

Because you slowed down after your first 4 laps, you can estimate that you swim 10 laps in more than

$$\frac{0.6 \text{ minute}}{1 \text{ lap}} \cdot 10 \text{ laps} = 6 \text{ minutes},$$

but less than

$$\frac{0.8 \text{ minute}}{1 \text{ lap}} \cdot 10 \text{ laps} = 8 \text{ minutes}.$$

▶ So, you can estimate that it takes you about 7 minutes to swim 10 laps.

In-Class Practice

1 I don't understand yet. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

13. After making 20 servings of pasta, a chef has used 30 cloves of garlic. The chef used 6 cloves to make the first 4 servings. How many cloves of garlic are used to make 10 servings? Justify your answer.

14. A runner completed a 25-mile race in 5 hours. The runner completed the first 7.5 miles in 1.5 hours.

a. Do these rates form a proportion? Justify your answer.

b. Can you determine, with certainty, the time it took the runner to complete 10 miles? Explain your reasoning.

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4.5 Practice WITH CalcChat® AND CalcView®

Review & Refresh

Copy and complete the statement. Round to the nearest hundredth if necessary.

1. 62 in. ≈ ____ m

2. 589 cu ≈ ____ L

3. 2725 km ≈ ____ mi

4. 48 cm ≈ ____ ft

Add or subtract.

5. $-28 + 15$

6. $-6 + (-11)$

7. $-10 - 8$

8. $-17 - (-14)$

Solve the equation.

9. $\frac{x}{6} = 25$

10. $8x = 72$

11. $150 = 2x$

12. $35 = \frac{x}{4}$

Concepts, Skills, & Problem Solving

REASONING **You can paint 75 square feet of a surface every 45 minutes. Determine how long it takes you to paint a wall with the given dimensions.** (See Exploration 1.)

13. 8 ft × 5 ft

14. 7 ft × 6 ft

15. 9 ft × 9 ft

PROPORTIONS **Tell whether the ratios form a proportion.** (See Examples 1 and 2.)

16. 1 to 3 and 7 to 21

17. 1 : 5 and 6 : 30

18. 3 to 4 and 24 to 18

19. 3.5 : 2 and 14 : 8

20. 24 : 30 and $3 : \frac{7}{2}$

21. $\frac{21}{2} : 3$ and 16 : 6

22. 0.6 : 0.5 and 12 : 10

23. 2 to 4 and 11 to $\frac{11}{2}$

24. $\frac{5}{8} : \frac{2}{3}$ and $\frac{1}{4} : \frac{1}{3}$

IDENTIFYING PROPORTIONAL RELATIONSHIPS **Tell whether x and y are proportional.** (See Example 3.)

25.

x	1	2	3
y	7	8	9

26.

x	2	4	6
y	5	10	15

27.

x	0.25	0.5	0.75
y	4	8	12

28.

x	$\frac{2}{3}$	1	$\frac{4}{3}$
y	$\frac{7}{10}$	$\frac{3}{5}$	$\frac{1}{2}$

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4 MTR **YOU BE THE TEACHER** **Your friend determines whether x and y are proportional. Is your friend correct? Explain your reasoning.**

29.

x	8	9
y	3	4

$\frac{8+1}{3+1} = \frac{9}{4}$

The values of the ratios x to y are equal. So, x and y are proportional.

30.

x	2	4	8
y	6	12	18

$\frac{2}{6} = \frac{1}{3}$ $\frac{4}{12} = \frac{1}{3}$

The values of the ratios x to y are equal. So, x and y are proportional.

PROPORTIONS **Tell whether the rates form a proportion.**

31. 7 inches in 9 hours; 42 inches in 54 hours

32. 12 players from 21 teams; 15 players from 24 teams

33. 385 calories in 3.5 servings; 300 calories in 3 servings

34. 4.8 laps every 8 minutes; 3.6 laps every 6 minutes

35. $\frac{3}{4}$ pound for every 5 gallons; $\frac{4}{5}$ pound for every $5\frac{1}{3}$ gallons

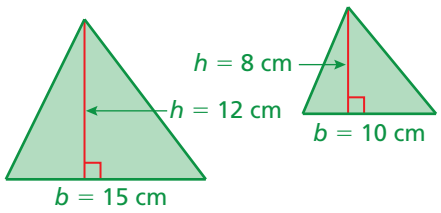
7 MTR **36.** **MODELING REAL LIFE** You do 90 sit-ups in 2 minutes. Your friend does 126 sit-ups in 2.8 minutes. Do these rates form a proportion? Explain.

7 MTR **37.** **MODELING REAL LIFE** Find the heart rates of yourself and your friend. Do these rates form a proportion? Explain.

	Heartbeats	Seconds
You	22	20
Friend	18	15

38. **PROBLEM SOLVING** You earn \$56 walking your neighbor's dog for 8 hours. Your friend earns \$36 painting your neighbor's fence for 4 hours. Are the pay rates equivalent? Explain.

39. **GEOMETRY** Are the heights and bases of the two triangles proportional? Explain.



Session Number, x	Pitches, y	Curveballs, z
1	10	4
2	20	8
3	30	12
4	40	16

40. **REASONING** A pitcher coming back from an injury limits the number of pitches thrown in bullpen sessions as shown.

a. Which quantities are proportional?

b. How many pitches that are *not* curveballs will the pitcher likely throw in Session 5?

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5 MTR

41. STRUCTURE You add the same numbers of pennies and dimes to the coins shown. Is the new ratio of pennies to dimes proportional to the original ratio of pennies to dimes? If so, illustrate your answer with an example. If not, show why with a counterexample.

a.

b.

42. REASONING You are 13 years old, and your cousin is 19 years old. As you grow older, is your age proportional to your cousin's age? Explain your reasoning.

7 MTR

43. MODELING REAL LIFE The shadow of the moon during a solar eclipse traveled 2300 miles in 1 hour. In the first 20 minutes, the shadow traveled $766\frac{2}{3}$ miles. How long did it take for the shadow to travel 1150 miles? Justify your answer. (See Example 4.)

7 MTR

44. MODELING REAL LIFE In 60 seconds, a car in a parade traveled 0.2 mile. The car traveled the last 0.05 mile in 12 seconds. How long did it take for the car to travel 0.1 mile? Justify your answer.

45. OPEN-ENDED Describe (a) a real-life situation where you expect two quantities to be proportional and (b) a real-life situation where you do *not* expect two quantities to be proportional. Explain your reasoning.

46. PROBLEM SOLVING A specific shade of red nail polish requires 7 parts red to 2 parts yellow. A mixture contains 35 quarts of red and 8 quarts of yellow. Is the mixture the correct shade? If so, justify your answer. If not, explain how you can fix the mixture to make the correct shade of red.

47. NUMBER SENSE The quantities x and y are proportional. Use each of the integers 1–5 to complete the table. Justify your answer.

x	10		6	
y				0.5

5 MTR

48. STRUCTURE Ratio A and Ratio B form a proportion. Ratio B and Ratio C also form a proportion. Do Ratio A and Ratio C form a proportion? Justify your answer.

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