Exponents and Rational Numbers

Chapter Learning Target:
Understand exponents and rational numbers.

Chapter Success Criteria:

- ◆ I can evaluate a power.
- ◆ I can evaluate expressions involving whole number bases.
- ■ I can evaluate expressions involving rational number bases.
- ■ I can solve real-life problems involving powers.

◆ Surface ■ Deep

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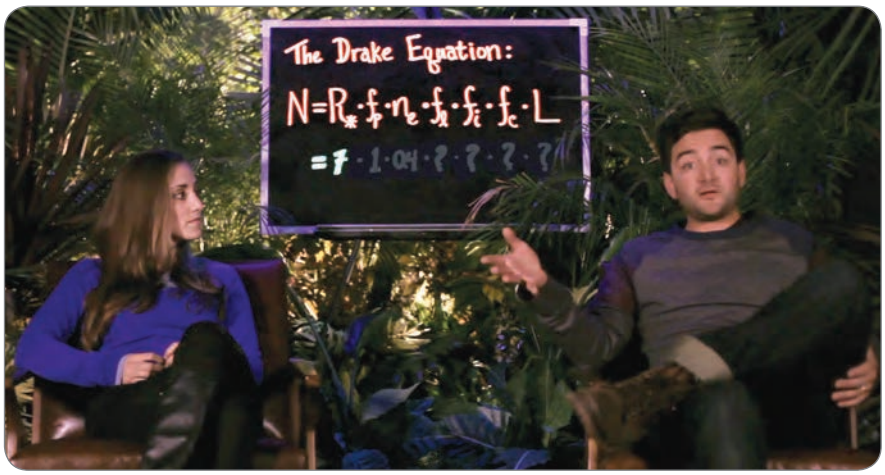
7 MTR *Mathematicians who apply mathematics to real-world contexts connect mathematical concepts to everyday experiences.*

STEAM Video The Drake Equation



Enid and Tony discuss the number of galaxies in the universe.

1. Tony says that there are approximately 10^{24} stars in our universe and 10^{11} stars in our galaxy. How can this information help you estimate how many other galaxies exist, when you assume other galaxies have about the same number of stars?

1 MTR 2. **ANALYZE A PROBLEM** Scientists have estimated the number of stars and planets in our galaxy.

   a. Describe how estimating the number of stars in our galaxy helps us estimate the number of planets in other galaxies.

   b. How accurate of an estimation do you think we can get? Explain.

STEAM Performance Task The Number of Stars in Galaxies

After completing this chapter, you will be able to use the concepts you learned to answer the questions in the *STEAM Performance Task*. You will be given information about the estimated *number of stars* in different galaxies.

Number of Stars

Milky Way galaxy: 10^{11}

Andromeda galaxy: 10^{12}

You will be asked to group the number of stars and compare the number of stars in each galaxy.

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Getting Ready for Chapter 1 WITH CalcChat®

Chapter Exploration

1 MTR **Actively Participate in Effortful Learning Collectively**
Work with a partner to prepare for concepts in this chapter.

5 MTR

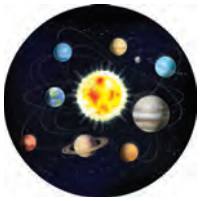
1. **STRUCTURE** Write each distance as a whole number. Which numbers do you know how to write in words? For instance, in words, 10^2 is equal to *one hundred*.

a. 10^3 meters: diameter of a meteor crater

b. 10^4 meters: diameter of Halley's Comet

c. 10^7 meters: diameter of Earth

d. 10^{16} meters: diameter of the solar system



e. 10^{21} meters: diameter of the Milky Way galaxy

f. 10^{27} meters: diameter of the observable universe

2. Write the numbers of wives, sacks, cats, and kits as powers.

As I was going to St. Ives
I met a man with seven wives
Each wife had seven sacks
Each sack had seven cats
Each cat had seven kits
Kits, cats, sacks, wives
How many were going to St. Ives?

Nursery Rhyme, 1730

Vocabulary

The following terms are defined in this chapter. Think about what each might mean and record your thoughts.

power
base of a power
exponent of a power
rational number

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1.1 Exponents

Learning Target: Use exponents to write and evaluate expressions.

Success Criteria:
- I can explain what an exponent is.
- I can evaluate expressions involving powers.
- I can use exponents to solve real-life problems.

The expression 3^5 is called a *power*. The *base* is 3. The *exponent* is 5.

base → 3^5 ← exponent

Exploration 1 Using Exponent Notation

Work with a partner.

a. Copy and complete the table.

Power	Repeated Multiplication Form	Value
2^1	2	2
2^2	$2 \cdot 2$	4
2^3		
2^4		
2^5		
2^6		
2^7		

b. Describe what is meant by the expression 2^n. How can you find the value of 2^n?

Exploration 2 Using Exponent Notation

Work with a partner. On a game show, each small cube is worth \$3. The small cubes are arranged to form a large cube. Show how you can use a power to find the total value of the large cube. Then write an explanation to convince a friend that your answer is correct.

5 MTR MAKE A PLAN

What formula can you use to find the number of cubes that form the large cube?

Number Sense and Operations

MA.7.NSO.1.1 Know and apply the Laws of Exponents to evaluate numerical expressions and generate equivalent numerical expressions, limited to whole-number exponents and rational number bases.

MA.7.NSO.2.1 Solve mathematical problems using multi-step order of operations with rational numbers including grouping symbols, whole-number exponents and absolute value.

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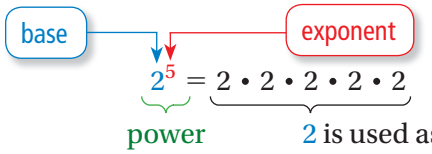
1.1 Lesson

Key Vocabulary
power, *p. 4*
base, *p. 4*
exponent, *p. 4*

A **power** is a product of repeated factors. The **base** of a power is the repeated factor. The **exponent** of a power indicates the number of times the base is used as a factor.



Example 1 Writing Expressions Using Exponents

Write each product using exponents.

a. $7 \cdot 7 \cdot 7$

Because 7 is used as a factor 3 times, its exponent is 3.

▶ So, $7 \cdot 7 \cdot 7 = 7^3$.

b. $3 \cdot 3 \cdot 5 \cdot 5 \cdot 5 \cdot 5$

Because 3 is used as a factor 2 times, its exponent is 2. Because 5 is used as a factor 4 times, its exponent is 4.

▶ So, $3 \cdot 3 \cdot 5 \cdot 5 \cdot 5 \cdot 5 = 3^2 \cdot 5^4$.

Try It **Write the product using exponents.**

1. $4 \cdot 4 \cdot 4 \cdot 4 \cdot 4$ **2.** $2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3$

Example 2 Evaluating Expressions

Evaluate each expression.

a. 5^3

$5^3 = 5 \cdot 5 \cdot 5$ Write as repeated multiplication.

The base is 5.

$= 125$ Simplify.

b. -2^4

$-2^4 = -(2 \cdot 2 \cdot 2 \cdot 2)$ Write as repeated multiplication.

The base is 2.

$= -16$ Simplify.

Try It **Evaluate the expression.**

3. 12^2 **4.** 2^6 **5.** -5^4 **6.** -4^3

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Example 3 Using Order of Operations

Evaluate each expression.

a. $3 + 2 \cdot 3^4 = 3 + 2 \cdot 81$ Evaluate the power.

$= 3 + 162$ Multiply.

$= 165$ Add.

b. $3^3 - 8^2 \div 2 = 27 - 64 \div 2$ Evaluate the powers.

$= 27 - 32$ Divide.

$= -5$ Subtract.

c. $-3 \cdot (-10^2 + 70) = -3 \cdot (-100 + 70)$ Evaluate the power.

$= -3 \cdot (-30)$ Perform operation in parentheses.

$= 90$ Multiply.

3 MTR REFLECT ON YOUR METHOD
Can you use the Distributive Property to evaluate the expression in part (c)? Explain.

Try It **Evaluate the expression.**

7. $9 - 2^5 \cdot 0.5$ **8.** $3^3 \div 9 + 18$ **9.** $(7 \cdot 4 - 4^3) \div 6$

In-Class Practice

1 I don't understand yet. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

WRITING EXPRESSIONS USING EXPONENTS **Write the product using exponents.**

10. $9 \cdot 9 \cdot 9$ **11.** $8 \cdot 8 \cdot 8 \cdot 8 \cdot 8$

12. $2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 \cdot 3$ **13.** $5 \cdot 5 \cdot 7 \cdot 7 \cdot 7 \cdot 7 \cdot 7$

EVALUATING EXPRESSIONS **Evaluate the expression.**

14. 11^2 **15.** 8^3

16. -6^3 **17.** -2^5

USING ORDER OF OPERATIONS **Evaluate the expression.**

18. $-24 \div 2^2$ **19.** $(3^3 - 6 \cdot 8) \div 7$

4 MTR 20. WHICH ONE DOESN'T BELONG? Which expression does *not* belong with the other three? Explain your reasoning.

4^3 3^4

8^2 2^6

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Example 4 Modeling Real Life

The annual profit P (in thousands of dollars) earned by a technology company x years after opening is represented by the equation $P = 0.1x^3 + 3$. How much more profit is earned in year 5 than in year 4?

Use the equation to find the profits earned in year 4 and year 5. Then subtract the profit in year 4 from the profit in year 5 to determine how much more profit is earned in year 5.

Year 4		*Year 5*
$P = 0.1x^3 + 3$	Write the equation.	$P = 0.1x^3 + 3$
$= 0.1(4)^3 + 3$	Substitute.	$= 0.1(5)^3 + 3$
$= 0.1(64) + 3$	Evaluate the power.	$= 0.1(125) + 3$
$= 9.4$	Simplify.	$= 15.5$

▶ So, the company earns $15.5 - 9.4 = 6.1$, or \$6100 more profit in year 5 than in year 4.

In-Class Practice

1 I don't understand yet. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

21. The annual profit P (in thousands of dollars) earned by a commercial fishing company x years after opening is represented by the equation $P = 1.6x^2 + 2$. How much more profit is earned in year 4 than in year 3?

22. Dig Deeper Consider the three units of time below.

Century: 10^2 years **Millennium:** 10^3 years **Gigayear:** 10^9 years

a. Write each unit of time as a whole number.

b. A megayear is $\frac{1}{1000}$ the length of a gigayear. Write the length of a megayear as a power.

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1.1 Practice with CalcChat® and CalcView®

Review & Refresh

Find the missing value(s) in the ratio table. Then write the equivalent ratios.

1.

Oranges	5		15
Apples	4	8	

2.

Cars	3	9	
Trucks	2		36

Evaluate the expression.

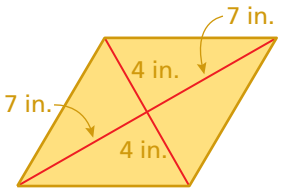
3. $3(15 - 8) + 4$

4. $2 \times 20 - 13$

5. $-7 + (6 - 2) \div 2$

Find the area of each figure.

6.



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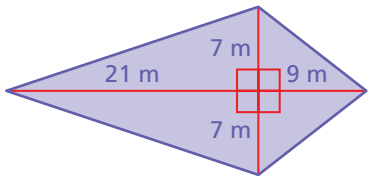

Concepts, Skills, & Problem Solving

USING EXPONENT NOTATION **Write the power in repeated multiplication form. Then find the value of the power.** (See Exploration 1.)

8. 4^4

9. 8^2

10. 5^3

WRITING EXPRESSIONS USING EXPONENTS **Write the product using exponents.** (See Example 1.)

11. $3 \cdot 3 \cdot 3 \cdot 3$

12. $6 \cdot 6$

13. $9 \cdot 9 \cdot 9 \cdot 9$

14. $4 \cdot 4 \cdot 4 \cdot 4 \cdot 4$

15. $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$

16. $5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5$

17. $7 \cdot 7 \cdot 7 \cdot 2 \cdot 2$

18. $8 \cdot 8 \cdot 8 \cdot 8 \cdot 6 \cdot 6 \cdot 6$

19. $3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 9 \cdot 9$

EVALUATING EXPRESSIONS **Evaluate the expression.** (See Example 2.)

20. 5^2

21. -11^3

22. 1^6

23. 3^4

24. 13^3

25. -9^4

26. **YOU BE THE TEACHER** Your friend evaluates the power -6^2. Is your friend correct? Explain your reasoning.

$-6^2 = (-6) \cdot (-6) = 36$

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5 MTR **STRUCTURE** **Write the prime factorization of the number using exponents.**

27. 675 **28.** 280 **29.** 363

5 MTR **30. PATTERNS** The smallest doll is 2 inches tall. The height of each of the other dolls is twice the height of the next smaller doll. Write an expression involving a power that represents the height of the largest doll. What is the height of the largest doll?

USING ORDER OF OPERATIONS **Evaluate the expression.** (See Example 3.)

31. $5 + 3 \cdot 2^3$ **32.** $2 + 7 \cdot 3^2$ **33.** $(13^2 - 12^2) \div 5$

34. $\frac{1}{2}(4^3 - 6 \cdot 3^2)$ **35.** $\frac{1}{2}(7 + 5^3)$ **36.** $-10 \times (24 - 4^2)$

37. $(9^2 - 15 \cdot 2) \div 17$ **38.** $-6 \cdot (-5^2 + 20)$ **39.** $(-4 + 12 - 6^2) \div 7$

7 MTR **40. MODELING REAL LIFE** Scientists study two bacteria colonies to learn more about how these bacteria populations grow. The number P of bacteria present in colony A after h hours is represented by the equation $P = 50(3)^h$. The number B of bacteria present in colony B after h hours can be represented by $B = 75(2)^h$. (See Example 4.)

a. Find the amount of bacteria in each colony after 4 hours.

b. Which colony has a greater number of bacteria after 9 hours? Explain.

5 MTR **41. STRUCTURE** Copy and complete the table. Compare the values of $2^h - 1$ with the values of 2^{h-1}. Are any of the values the same? If so, which values? If not, explain why not.

h	2	3	4	5	6
$2^h - 1$					
2^{h-1}					

42. Dig Deeper You create a social media page for an event you are planning at your school. The number N of people who see the post after s shares is represented by the equation $N = 2(4)^s$.

a. How many people see your event page if 3 of your classmates share your page?

b. How many classmates must share your event page for at least 8100 people to see your page? Explain.

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1.2 Product of Powers Property

Learning Target: Generate equivalent expressions involving products of powers.

Success Criteria:
- I can find products of powers that have the same base.
- I can find powers of powers.
- I can find powers of products.

Exploration 1 Finding Products of Powers

Work with a partner.

a. Copy and complete the table. Use your results to write a *general rule* for finding $a^m \cdot a^n$, a product of two powers with the same base.

Product	Repeated Multiplication Form	Power
$2^2 \cdot 2^4$	$2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$	
$7^3 \cdot 7^2$		
$5^1 \cdot 5^6$		
$10^3 \cdot 10^5$		
$6^2 \cdot 6^2$		

b. Show how to use your rule in part (a) to write each expression below as a single power. Then write a *general rule* for finding $(a^m)^n$, a power of a power.

$(7^3)^2$ $(6^2)^2$ $(3^2)^3$ $(2^2)^4$ $(5^2)^5$

5 MTR USE STRUCTURE
How are the expressions in part (b) similar to the expressions in part (a)?

Exploration 2 Finding Powers of Products

Work with a partner. Copy and complete the table. Use your results to write a *general rule* for finding $(ab)^m$, a power of a product.

Power	Repeated Multiplication Form	Product of Powers
$(2 \cdot 3)^3$		
$(2 \cdot 5)^2$		
$(5 \cdot 4)^3$		

Number Sense and Operations
MA.7.NSO.1.1 Know and apply the Laws of Exponents to evaluate numerical expressions and generate equivalent numerical expressions, limited to whole-number exponents and rational number bases.

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1.2 Lesson

Common Error

When multiplying powers, do not multiply the bases. $4^2 \cdot 4^3 = 4^5$, not 16^5.

Key Ideas

Product of Powers Property

Words To multiply powers with the same base, add their exponents.

Numbers $4^2 \cdot 4^3 = 4^{2+3} = 4^5$

Algebra $a^m \cdot a^n = a^{m+n}$

Power of a Power Property

Words To find a power of a power, multiply the exponents.

Numbers $(4^6)^3 = 4^{6 \cdot 3} = 4^{18}$

Algebra $(a^m)^n = a^{mn}$

Power of a Product Property

Words To find a power of a product, find the power of each factor and multiply.

Numbers $(3 \cdot 2)^5 = 3^5 \cdot 2^5$

Algebra $(ab)^m = a^m b^m$

Example 1 Multiplying Powers with the Same Base

a. $2^4 \cdot 2^5 = 2^{4+5}$ Product of Powers Property

$= 2^9$ Simplify.

b. $4^3 \cdot 4^7 = 4^{3+7}$ Product of Powers Property

$= 4^{10}$ Simplify.

A number written without an exponent has an exponent of 1.

c. $5 \cdot 5^6 = 5^1 \cdot 5^6$ Rewrite 5 as 5^1.

$= 5^{1+6}$ Product of Powers Property

$= 5^7$ Simplify.

Try It

Simplify the expression. Write your answer as a power.

1. $6^2 \cdot 6^4$ **2.** $2^3 \cdot 2^6$ **3.** $3 \cdot 3^{12}$

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Example 2 Finding a Power of a Power

a. $(3^4)^3 = 3^{4 \cdot 3}$ Power of a Power Property

$= 3^{12}$ Simplify.

b. $(8^5)^4 = 8^{5 \cdot 4}$ Power of a Power Property

$= 8^{20}$ Simplify.

Try It **Simplify the expression. Write your answer as a power.**

4. $(4^3)^5$ **5.** $(7^2)^4$ **6.** $(5^3)^2$

Example 3 Finding a Power of a Product

a. $(2 \cdot 5)^3 = 2^3 \cdot 5^3$ Power of a Product Property

b. $(3 \cdot 4)^2 = 3^2 \cdot 4^2$ Power of a Product Property

Try It **Simplify the expression. Write your answer as a product of powers.**

7. $(5 \cdot 7)^4$ **8.** $(2 \cdot 9)^5$ **9.** $(11 \cdot 12)^2$

In-Class Practice

1 I don't understand yet. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

FINDING POWERS **Simplify the expression. Write your answer as a power.**

10. $4^7 \cdot 4^4$ **11.** $(8^5)^3$ **12.** $3^5 \cdot 3^7$

FINDING A POWER OF A PRODUCT **Simplify the expression. Write your answer as a product of powers.**

13. $(2 \cdot 7)^4$ **14.** $(3 \cdot 5)^6$ **15.** $(8 \cdot 9)^3$

16. **REASONING** Can you use the Product of Powers Property to simplify $5^2 \cdot 6^4$? Explain.

17. **OPEN-ENDED** Write an expression that simplifies to 2^{12} using the Product of Powers Property.

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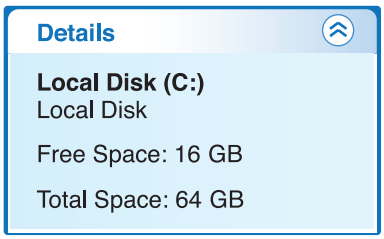
Example 4 B.E.S.T. Test Prep: Modeling Real Life

One gigabyte (GB) of computer storage space is 2^{30} bytes. The storage details of a computer are shown. How many bytes of total storage space does the computer have?



Ⓐ 2^{34} Ⓒ 2^{180}

Ⓑ 2^{36} Ⓓ 128^{30}

The computer has 64 gigabytes of total storage space. Notice that you can write 64 as a power, 2^6.

Use a verbal model to solve the problem.

$$\text{Total number of bytes} = \text{Number of bytes in a gigabyte} \cdot \text{Number of gigabytes}$$

$= 2^{30} \cdot 2^6$ Substitute.

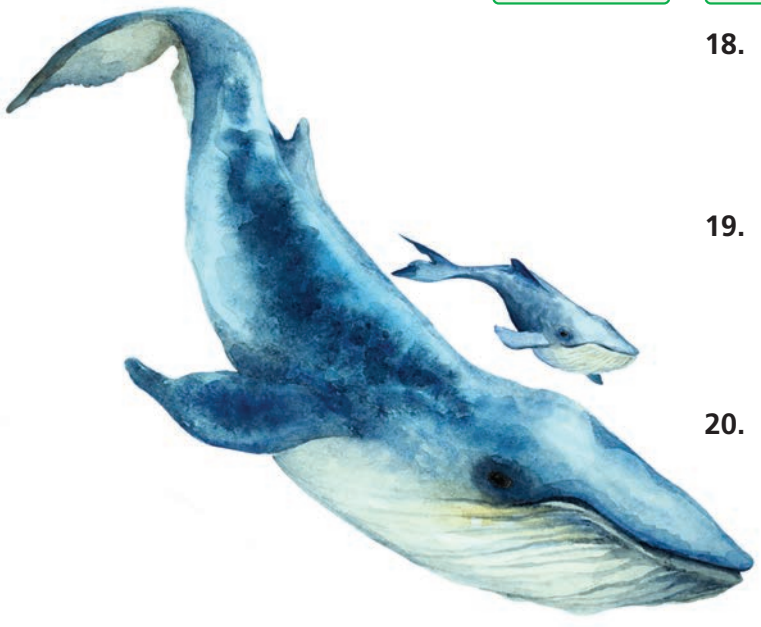
$= 2^{30 + 6}$ Product of Powers Property

$= 2^{36}$ Simplify.

▶ The computer has 2^{36} bytes of total storage space. So, the correct answer is Ⓑ.

In-Class Practice

1 I don't understand yet. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.



18. A newborn blue whale weighs 3^7 kilograms. An adult blue whale weighs 81 times the weight of the newborn. How many kilograms does the adult blue whale weigh?

19. One megabyte of cell phone storage space is 2^{20} bytes. An app uses 4^4 megabytes of storage space. How many bytes of storage space does the app use?

20. **Dig Deeper** The sides of a large square rug are 3 times longer than the sides of a square rug with sides that are 2 feet long. Write an expression for the area of the large rug as a product of powers. Justify your answer.

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1.2 Practice WITH CalcChat® AND CalcView®

Review & Refresh

Write the product using exponents.

1. $11 \cdot 11 \cdot 11 \cdot 11 \cdot 11$

2. $6 \cdot 6 \cdot 6 \cdot 3 \cdot 3$

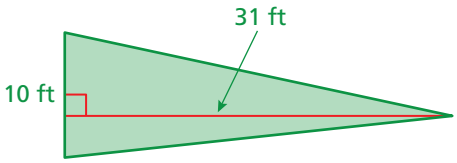
Evaluate the expression when $x = 2$ and $z = -3$.

3. $-4x$

4. xz

5. $7z + 6x$

6. Find the area of the triangle.



Concepts, Skills, & Problem Solving

FINDING PRODUCTS OF POWERS **Write the expression in repeated multiplication form. Then write the expression as a power.** (See Exploration 1.)

7. $5^6 \cdot 5^3$

8. $(6^4)^2$

9. $8^3 \cdot 8^4$

FINDING POWERS **Simplify the expression. Write your answer as a power.** (See Examples 1 and 2.)

10. $3^2 \cdot 3^2$

11. $8^{10} \cdot 8^4$

12. $(5^4)^3$

13. $(3^2)^4$

14. $4^5 \cdot 4^7$

15. $7^6 \cdot 7$

16. $(1^{12})^3$

17. $(5^2)^3$

18. $6^2 \cdot 6^4$

1 MTR

19. **HELP A CLASSMATE** Your friend wants to simplify the expression $5^2 \cdot 5^9$. Explain how your friend can complete their work.

$5^2 \cdot 5^9 = (5)$
$=$

FINDING A POWER OF A PRODUCT **Simplify the expression. Write your answer as a product of powers.** (See Example 3.)

20. $(6 \cdot 4)^3$

21. $(3 \cdot 7)^5$

22. $(2 \cdot 9)^4$

23. $(8 \cdot 7)^4$

24. $(1 \cdot 5)^{12}$

25. $(10 \cdot 3)^2$

5 MTR

26. **STRUCTURE** Is $3^2 + 3^3$ equal to 3^5? Explain.

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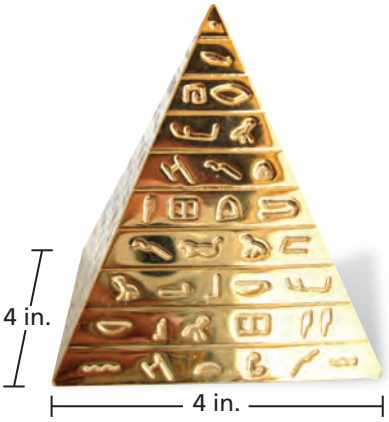


27. PROBLEM SOLVING A display case for the artifact shown is in the shape of a cube. Each side of the display case is three times longer than the base length of the artifact.

a. Write a power of a product that represents the volume of the case.

b. What is the volume of the case?

28. NUMBER SENSE Show that $(3 \cdot 8 \cdot 5)^7 = 6^7 \cdot 4^7 \cdot 5^7$.

7 MTR

29. MODELING REAL LIFE The lowest altitude of an altocumulus cloud is about 3^8 feet. The highest altitude of an altocumulus cloud is about 3 times the lowest altitude. What is the highest altitude of an altocumulus cloud? Write your answer as a power. (See Example 4.)

7 MTR

30. INVESTIGATE Each galaxy in the universe has a different number of stars. Astronomers estimate that the Milky Way galaxy has about 10^{11} stars. The Andromeda galaxy has about 10^6 as many stars as the Milky Way.

a. How many total stars are in the Andromeda galaxy?

b. Research the number of stars in another galaxy. Write a power that relates the number of stars in this galaxy to the number of stars in the Milky Way.

7 MTR

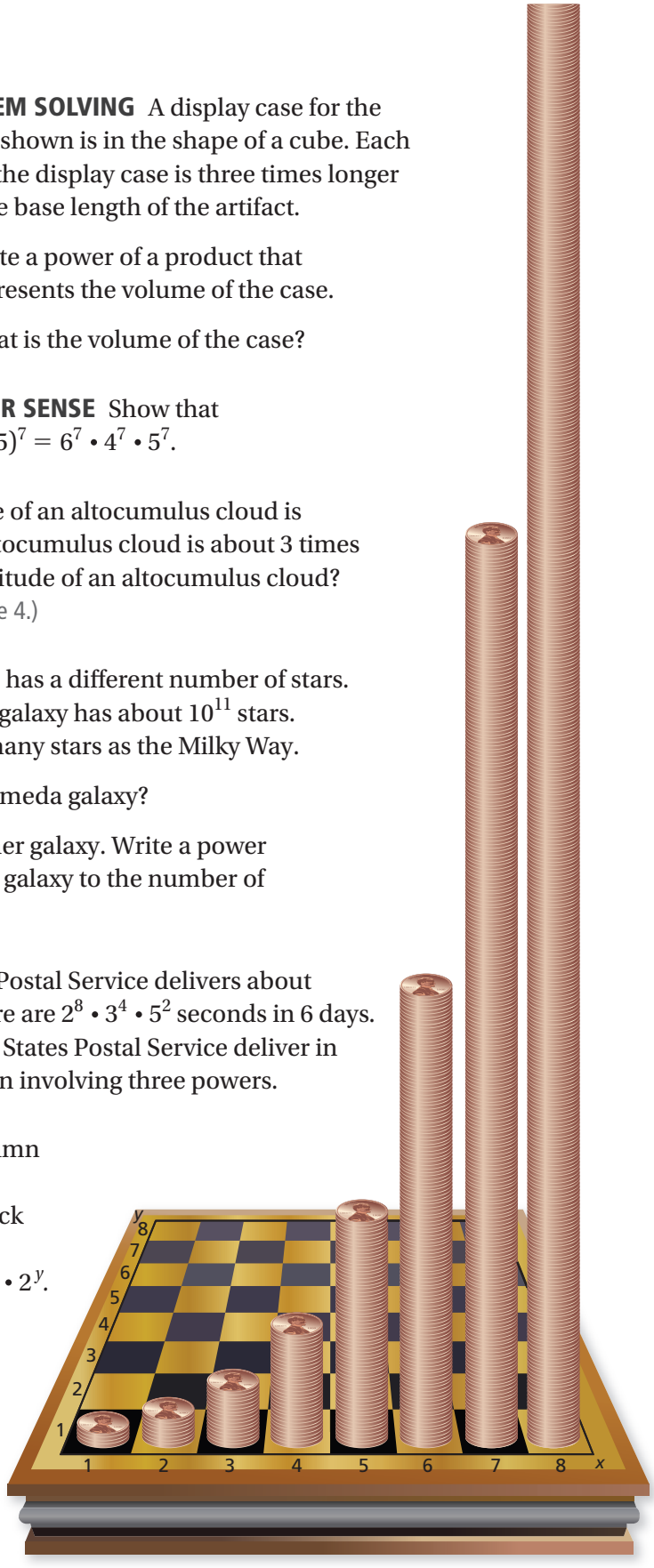
31. MODELING REAL LIFE The United States Postal Service delivers about $2^4 \cdot 3 \cdot 5^3$ pieces of mail each second. There are $2^8 \cdot 3^4 \cdot 5^2$ seconds in 6 days. How many pieces of mail does the United States Postal Service deliver in 6 days? Write your answer as an expression involving three powers.

32. REASONING The row numbers y and column numbers x of a chessboard are shown. Each position on the chessboard has a stack of pennies. (Only the first row is shown.) The number of pennies in each stack is $2^x \cdot 2^y$.

a. Which locations have 32 pennies in their stacks?

b. How much money (in dollars) is in the location with the tallest stack?

c. A penny is about 0.06 inch thick. About how tall is the tallest stack?



33. NUMBER SENSE Find the value of x in the equation without evaluating the power.

$$2^5 \cdot 2^x = 256$$

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1.3 Quotient of Powers Property

Learning Target: Generate equivalent expressions involving quotients of powers.

Success Criteria:
- I can find quotients of powers that have the same base.
- I can simplify expressions using the Quotient of Powers Property.
- I can solve real-life problems involving quotients of powers.

Exploration 1 Finding Quotients of Powers

Work with a partner.

a. Copy and complete the table. Use your results to write a *general rule* for finding $\frac{a^m}{a^n}$, a quotient of two powers with the same base.

5 MTR USE STRUCTURE
How does writing the expanded form of each expression help you find a general rule?

Quotient	Repeated Multiplication Form	Power
$\frac{2^4}{2^2}$	$\frac{2 \cdot 2 \cdot 2 \cdot 2}{2 \cdot 2}$	
$\frac{7^7}{7^3}$		
$\frac{10^8}{10^5}$		

b. Use your rule in part (a) to simplify the quotients in the first column of the table above. Does your rule give the results in the third column?

c. Copy and complete the table. Use your rule in part (a) to complete the third column. Use your results to define a^0, where $a \neq 0$.

Quotient	Value	Power
$\frac{4^3}{4^3}$	1	
$\frac{3^5}{3^5}$		
$\frac{2^6}{2^6}$		

Number Sense and Operations
MA.7.NSO.1.1 Know and apply the Laws of Exponents to evaluate numerical expressions and generate equivalent numerical expressions, limited to whole-number exponents and rational number bases.

GO DIGITAL

1.3 Lesson

Key Ideas

Quotient of Powers Property

Words To divide powers with the same base, subtract their exponents.

Numbers $\frac{4^5}{4^2} = 4^{5-2} = 4^3$ **Algebra** $\frac{a^m}{a^n} = a^{m-n}$, where $a \neq 0$

Zero Exponents

Words For any nonzero number a, $a^0 = 1$. The power 0^0 is *undefined*.

Numbers $4^0 = 1$ **Algebra** $a^0 = 1$, where $a \neq 0$

Example 1 Dividing Powers with the Same Base

Common Error

When dividing powers, do not divide the bases.

$\frac{2^6}{2^4} = 2^2$, not 1^2.

Simplify $\frac{2^6}{2^4}$. Write your answer as a power.

$\frac{2^6}{2^4} = 2^{6-4}$ Quotient of Powers Property

$= 2^2$ Simplify.

Try It **Simplify the expression. Write your answer as a power.**

1. $\frac{9^7}{9^4}$ 2. $\frac{4^6}{4^5}$ 3. $\frac{8^8}{8^4}$ 4. $\frac{5^3}{5^3}$

Example 2 Simplifying an Expression

Simplify $\frac{3^4 \cdot 3^2}{3^3}$. Write your answer as a power.

The numerator is a product of powers. Add the exponents in the numerator.

$\frac{3^4 \cdot 3^2}{3^3} = \frac{3^{4+2}}{3^3}$ Product of Powers Property

$= \frac{3^6}{3^3}$ Simplify.

$= 3^{6-3}$ Quotient of Powers Property

$= 3^3$ Simplify.

Try It **Simplify the expression. Write your answer as a power.**

5. $\frac{6^7 \cdot 6^3}{6^5}$ 6. $\frac{2^{15}}{2^3 \cdot 2^5}$ 7. $\frac{4^8 \cdot 4^6}{4^5}$

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Example 3 Simplifying Expressions

a. $\frac{4^9}{4^5} \cdot \frac{4^8}{4^2} = 4^{9-5} \cdot 4^{8-2}$ Quotient of Powers Property

$= 4^4 \cdot 4^6$ Simplify.

$= 4^{4+6}$ Product of Powers Property

$= 4^{10}$ Simplify.

b. $\frac{9^{10}}{9^6} \cdot \frac{9^7}{9^4} = 9^{10-6} \cdot 9^{7-4}$ Quotient of Powers Property

$= 9^4 \cdot 9^3$ Simplify.

$= 9^{4+3}$ Product of Powers Property

$= 9^7$ Simplify.

c. $5^3 \cdot 3^0 = 5^3 \cdot 1$ Definition of a zero exponent

$= 5^3$ Multiplicative Identity Property of One

2 MTR USE ANOTHER METHOD

Show how you can simplify the expressions in parts (a) and (b) by first multiplying the numerators and then multiplying the denominators.

Try It **Simplify the expression. Write your answer as a power.**

8. $\frac{5^7 \cdot 5^6}{5^5 \cdot 5^2}$ **9.** $\frac{7^5}{7} \cdot \frac{7^9}{7^8}$ **10.** $2^5 \cdot 8^0$

In-Class Practice

1 I don't understand yet. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

SIMPLIFYING EXPRESSIONS **Simplify the expression. Write your answer as a power.**

11. $\frac{3^9}{3^2}$ **12.** $\frac{8^6 \cdot 8^2}{8^5}$ **13.** $\frac{7^{11}}{7^4 \cdot 7^6}$

14. $\frac{5^6}{5} \cdot \frac{5^3}{5^2}$ **15.** $\frac{2^9 \cdot 2^4}{2^4 \cdot 2^4}$ **16.** $8^4 \cdot 6^0$

4 MTR **17. WHICH ONE DOESN'T BELONG?** Which quotient does *not* belong with the other three? Explain your reasoning.

$\frac{10^7}{10^2}$ $\frac{6^3}{6^2}$ $\frac{4^8}{3^4}$ $\frac{5^6}{5^3}$

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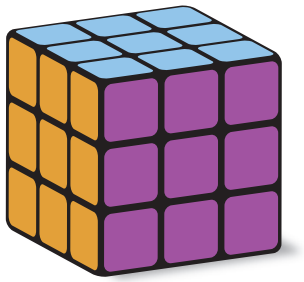
Example 4 Modeling Real Life



A warehouse is shipping boxes of cube-shaped puzzles to a store. The volume of a puzzle is 2^3 cubic inches. The puzzles can fit in a box with no space left over. The volume of the box is $8 \cdot 2^6$ cubic inches. How many puzzles can fit in the box?

You can find the number of puzzles that can fit in the box by dividing the volume of the box by the volume of a puzzle.

$$\text{Number of puzzles} = \frac{\text{Volume of the box}}{\text{Volume of a puzzle}}$$

$$= \frac{8 \cdot 2^6}{2^3} \quad \text{Substitute.}$$

$$= 8 \cdot \frac{2^6}{2^3} \quad \text{Rewrite.}$$

$$= 8 \cdot 2^3 \quad \text{Quotient of Powers Property}$$

$$= 8 \cdot 8 \quad \text{Evaluate the power.}$$

$$= 64 \quad \text{Simplify.}$$

▶ So, 64 cube-shaped puzzles can fit in the box.

In-Class Practice

1 I don't understand yet. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

18. A warehouse is shipping crates of cube-shaped boxes. The volume of a box is 2^6 cubic inches. The boxes can fit in a shipping crate with no space left over. The volume of the crate is $27 \cdot 2^9$ cubic inches. How many boxes can fit in the crate?

19. You want to purchase a cat tracker. Tracker A detects your cat within a distance of $4 \cdot 10^2$ feet of your home. Tracker B detects your cat within a distance of 10^4 feet of your home. Which tracker covers a greater distance from your home? How many times greater?

20. **Dig Deeper** An earthquake of magnitude 3.0 is 10^2 times stronger than an earthquake of magnitude 1.0. An earthquake of magnitude 8.0 is 10^7 times stronger than an earthquake of magnitude 1.0. How many times stronger is an earthquake of magnitude 8.0 than an earthquake of magnitude 3.0?

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1.3 Practice WITH CalcChat® AND CalcView®

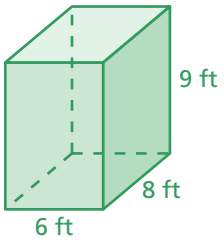
Review & Refresh

Simplify the expression. Write your answer as a power.

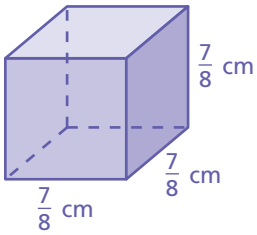
1. $4^2 \cdot 4^3$ **2.** $(8^5)^5$ **3.** $(5 \cdot 6)^7$

Find the volume of the prism.

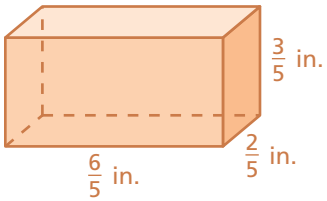
4.



5.



6.



Find the mean of the data.

7. 21, 23, 24, 24, 27, 19 **8.** 44, 37, 45, 38, 42, 40, 36, 46

Concepts, Skills, & Problem Solving

FINDING QUOTIENTS OF POWERS Write the quotient as repeated multiplication. Then write the quotient as a power. (See Exploration 1.)

9. $\frac{7^9}{7^6}$ **10.** $\frac{4^6}{4^2}$ **11.** $\frac{9^{10}}{9^5}$

DIVIDING POWERS WITH THE SAME BASE Simplify the expression. Write your answer as a power. (See Example 1.)

12. $\frac{8^9}{8^7}$ **▶ 13.** $\frac{6^{10}}{6^4}$ **14.** $\frac{3^4}{3^1}$ **15.** $\frac{4^5}{4^3}$

16. $\frac{64^4}{64^3}$ **17.** $\frac{17^5}{17^2}$ **18.** $\frac{72^8}{72^6}$ **19.** $\frac{29^{11}}{29^7}$

4 MTR

20. YOU BE THE TEACHER Your friend simplifies the quotient. Is your friend correct? Explain your reasoning.

$$\frac{6^{15}}{6^5} = 6^{15/5}$$
$$= 6^3$$

SIMPLIFYING AN EXPRESSION Simplify the expression. Write your answer as a power. (See Example 2.)

▶ 21. $\frac{6^{13}}{6^4 \cdot 6^2}$ **22.** $\frac{7^5 \cdot 7^3}{7^2}$ **23.** $\frac{8^{11}}{8^7 \cdot 8^2}$

24. $\frac{9^{30}}{9^{18} \cdot 9^4}$ **25.** $\frac{5^{22}}{5^8 \cdot 5^9}$ **26.** $\frac{11^8 \cdot 11^6}{11^8}$

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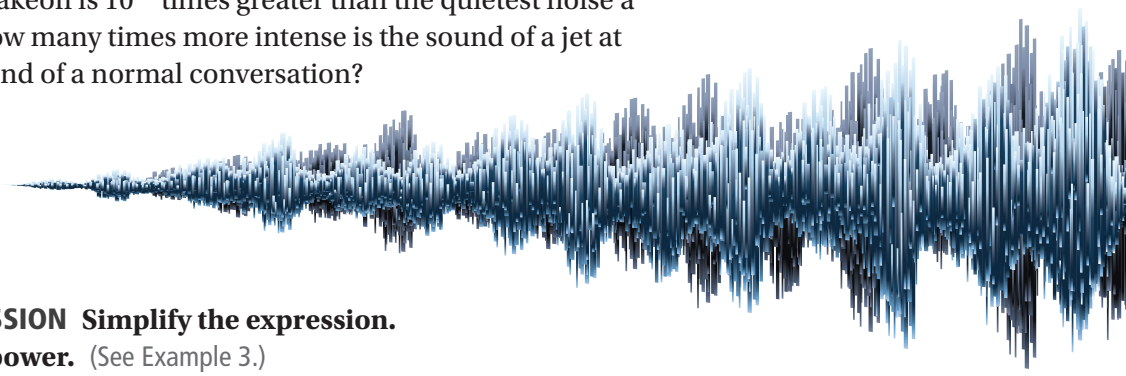
7 MTR

27. MODELING REAL LIFE The sound intensity of a normal conversation is 10^6 times greater than the quietest noise a person can hear. The sound intensity of a jet at takeoff is 10^{14} times greater than the quietest noise a person can hear. How many times more intense is the sound of a jet at takeoff than the sound of a normal conversation?



SIMPLIFYING AN EXPRESSION Simplify the expression. Write your answer as a power. (See Example 3.)

28. $\frac{4^8 \cdot 4^3}{4^4 \cdot 4^2}$

29. $\frac{3^2 \cdot 3^6}{3^2} \cdot \frac{3^5}{3}$

30. $\frac{6^2}{6} \cdot \frac{6^{12}}{6^8}$

31. $\frac{7^7 \cdot 7^6}{7 \cdot 7^2}$

32. $\frac{8^5}{8^4} \cdot \frac{8^{13}}{8^8}$

33. $\frac{9^8 \cdot 9^2}{9^7} \cdot \frac{9^4}{9} \cdot \frac{9^7}{9^2}$

Device	Storage (GB)	Price
A	2^5	\$30
B	2^6	\$50
C	2^7	\$70
D	2^8	\$90
E	2^9	\$110

5 MTR

34. PATTERNS The storage capacities and prices of five devices are shown in the table.

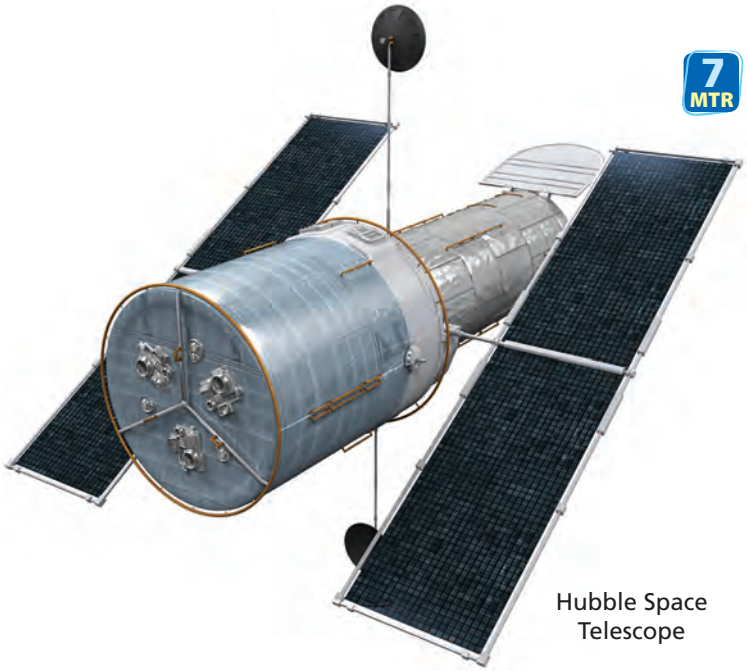
a. How many times more storage does Device D have than Device B?

b. Predict the price for a device with 2^{12} GB of storage.

35. Dig Deeper Consider the equation $\frac{9^m}{9^n} = 9^2$.

a. Find two numbers m and n that satisfy the equation.

b. Describe the number of solutions that satisfy the equation. Explain your reasoning.



Hubble Space Telescope

7 MTR

36. MODELING REAL LIFE A scientist estimates that there are about 10^{24} stars in the universe and that each galaxy has, on average, approximately the same number of stars as the Milky Way galaxy: $10 \cdot 10^{10}$ stars. About how many galaxies are in the universe? (See Example 4.)

37. NUMBER SENSE Find the value of x that makes $\frac{8^{3x}}{8^{2x+1}} = 8^9$ true. Explain how you found your answer.

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1.4 Rational Numbers

Learning Target: Understand how to compare rational numbers.

Success Criteria:
- I can graph rational numbers on a number line.
- I can find the absolute value of a rational number.
- I can use a number line to compare rational numbers.

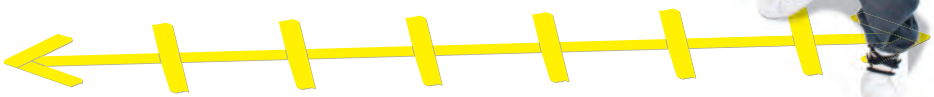
Recall that **integers** are the set of whole numbers and their opposites. A **rational number** is a number that can be written as $\frac{a}{b}$, where a and b are integers and $b \neq 0$.

Exploration 1 Using a Number Line

Work with a partner. Make a number line on the floor. Include both negative numbers and positive numbers.

a. Stand on an integer. Then have your partner stand on the opposite of the integer. How far are each of you from 0? What do you call the distance between a number and 0 on a number line?

b. Stand on a rational number that is not an integer. Then have your partner stand on any other number. Which number is greater? How do you know?



3 MTR SELECT METHODS
What are some ways to determine which of two numbers is greater?

c. Stand on any number other than 0 on the number line. Can your partner stand on a number that is
- greater than your number and farther from 0?
- greater than your number and closer to 0?
- less than your number and the same distance from 0?
- less than your number and farther from 0?

For each case in which it was not possible to stand on a number as directed, explain why it is not possible. In each of the other cases, how can you decide where your partner can stand?

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Number Sense and Operations
preparing for MA.7.NSO.2.2 Add, subtract, multiply and divide rational numbers with procedural fluency.

1.4 Lesson

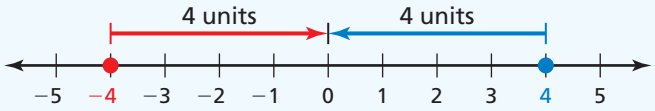
Key Vocabulary
integers, *p. 21*
rational number, *p. 21*
absolute value, *p. 22*

1 MTR HELP A CLASSMATE
Explain to a classmate why $|4| \neq -4$.

Key Idea

Absolute Value

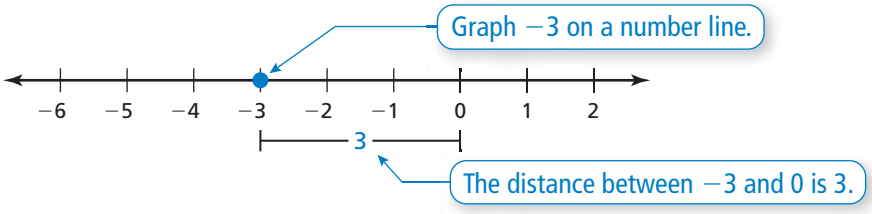
Words The **absolute value** of a number is the distance between the number and 0 on a number line. The absolute value of a number a is written as $|a|$.



Numbers $|-4| = 4$ $|4| = 4$

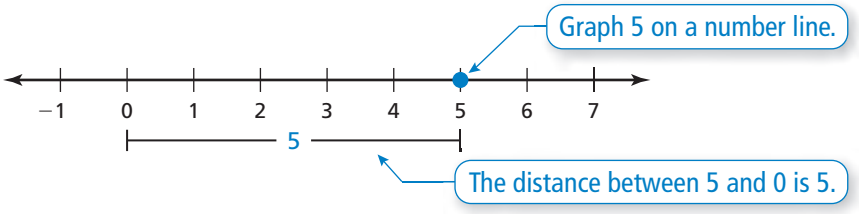
Example 1 Finding Absolute Values of Integers

a. Find the absolute value of −3.



▶ So, $|-3| = 3$.

b. Find the absolute value of 5.



▶ So, $|5| = 5$.

Try It

Find the absolute value.

1. $|-2|$ **2.** $|-6|$ **3.** $|7|$

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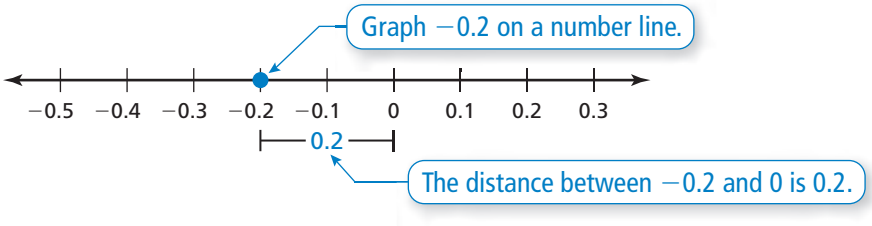
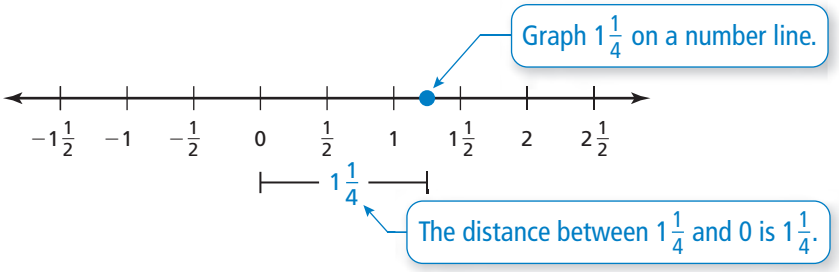
Example 2 Finding Absolute Values of Rational Numbers

a. Find the absolute value of -0.2.



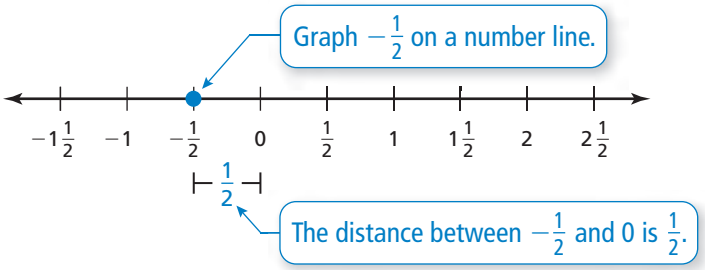
▶ So, $|-0.2| = 0.2$.

b. Find the absolute value of $1\frac{1}{4}$.



▶ So, $\left|1\frac{1}{4}\right| = 1\frac{1}{4}$.

c. Find the absolute value of $-\frac{1}{2}$.



▶ So, $\left|-\frac{1}{2}\right| = \frac{1}{2}$.

Try It **Find the absolute value.**

4. $|-2.6|$ **5.** $\left|1\frac{1}{2}\right|$ **6.** $\left|-\frac{5}{3}\right|$

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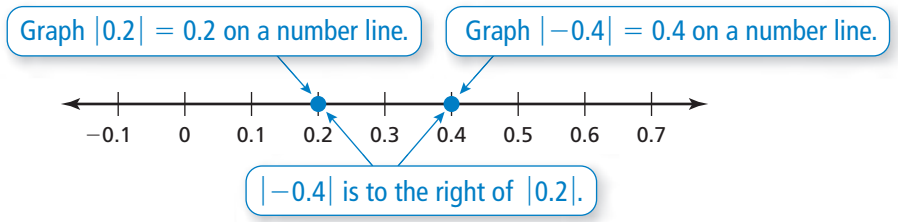
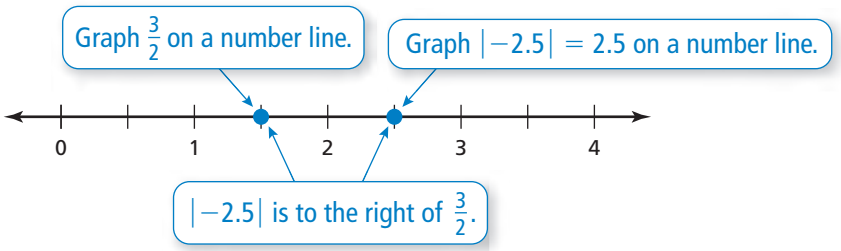
Example 3 Comparing Rational Numbers

a. Compare $|-0.4|$ and $|0.2|$.



▶ So, $|-0.4| > |0.2|$.

b. Compare $|-2.5|$ and $\frac{3}{2}$.



▶ So, $|-2.5| > \frac{3}{2}$.

Remember

Two numbers that are the same distance from 0 on a number line, but on opposite sides of 0, are called *opposites*. The opposite of a number a is $-a$.

Try It **Copy and complete the statement using <, >, or =.**

7. $|9| \square |-9|$ **8.** $-\left|\frac{1}{2}\right| \square -\frac{1}{4}$ **9.** $7 \square -|-4.5|$

In-Class Practice

1 I don't understand yet. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

10. VOCABULARY Which of the following numbers are integers?

$$9,\ 3.2,\ -1,\ \frac{1}{2},\ -0.25,\ 15$$

COMPARING RATIONAL NUMBERS **Copy and complete the statement using <, >, or =. Use a number line to justify your answer.**

11. $\frac{2}{5} \square \left|-\frac{3}{5}\right|$ **12.** $3.5 \square \left|-\frac{7}{2}\right|$ **13.** $\left|\frac{11}{4}\right| \square |-2.8|$

14. WRITING You compare two numbers, a and b. Explain how $a > b$ and $|a| < |b|$ can both be true statements.

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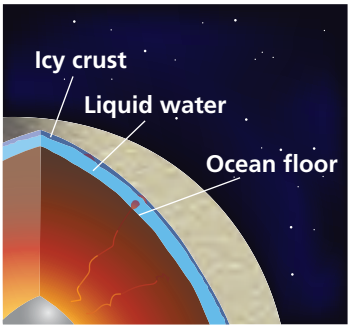
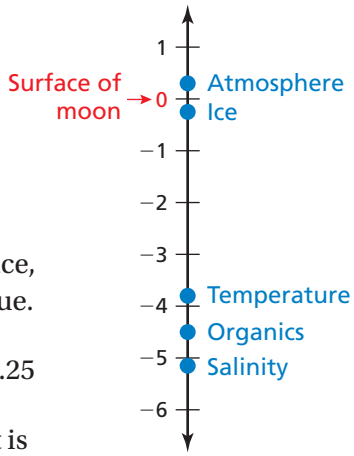
Example 4 Modeling Real Life



A moon has an ocean underneath its icy surface. Scientists run tests above and below the surface. The table shows the elevations of each test. Which test is deepest? Which test is closest to the surface?

Test	Temperature	Salinity	Atmosphere	Organics	Ice
Elevation (miles)	−3.8	−5.15	0.3	−4.5	−0.25

To determine which test is deepest, find the least elevation. Graph the elevations on a vertical number line.



The number line shows that the salinity test is deepest. The number line also shows that the atmosphere test and the ice test are closest to the surface. To determine which is closer to the surface, identify which elevation has a lesser absolute value.

Atmosphere: $|0.3| = 0.3$ **Ice:** $|-0.25| = 0.25$

▶ So, the salinity test is deepest and the ice test is closest to the surface.

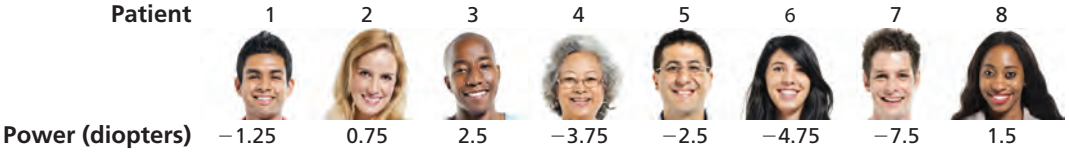
In-Class Practice

1 I don't understand yet. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

15. An airplane is at an elevation of 5.5 miles. A submarine is at an elevation of −10.9 kilometers. Which is closer to sea level? Explain.

16. A sailfish is at an elevation of $-\frac{1}{5}$ kilometer. A heron is at an elevation of 0.4 kilometer. Which is closer to sea level? Explain.

17. The image shows the corrective powers (in *diopters*) of contact lenses for eight people. The farther the number of diopters is from 0, the greater the power of the lens. Positive diopters correct *farsightedness* and negative diopters correct *nearsightedness*. Who is the most nearsighted? the most farsighted? Who has the best eyesight?



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1.4 Practice WITH CalcChat® AND CalcView®

Review & Refresh

Simplify the expression. Write your answer as a power.

1. $\frac{10^8}{10^4}$ **2.** $\frac{2^9}{2^7}$ **3.** $\frac{3^8 \cdot 3^3}{3^2}$

Find the GCF of the numbers.

4. 8, 20 **5.** 12, 30 **6.** 7, 28 **7.** 48, 72

8. What is the ratio of dogs to cats?

Concepts, Skills, & Problem Solving

NUMBER SENSE Determine which number is greater and which number is farther from 0. Explain your reasoning. (See Exploration 1.)

9. 4, −6 **10.** −3.25, $\frac{7}{2}$ **11.** $-\frac{4}{5}$, −1.3

FINDING ABSOLUTE VALUES Find the absolute value. (See Examples 1 and 2.)

12. $|8|$ **13.** $|-2|$ **14.** $|-10|$ **15.** $|10|$

16. $|0|$ **17.** $\left|\frac{1}{3}\right|$ **18.** $\left|\frac{7}{8}\right|$ **19.** $\left|-\frac{5}{9}\right|$

20. $\left|\frac{11}{8}\right|$ **21.** $|3.8|$ **22.** $|-5.3|$ **23.** $\left|-\frac{15}{4}\right|$

24. $|7.64|$ **25.** $|-18.26|$ **26.** $\left|4\frac{2}{5}\right|$ **27.** $\left|-5\frac{1}{6}\right|$

COMPARING RATIONAL NUMBERS Copy and complete the statement using <, >, or =. (See Example 3.)

28. $2 \square |-5|$ **29.** $|-1| \square |-8|$ **30.** $|5| \square |-5|$

31. $|-2| \square 0$ **32.** $0.4 \square \left|-\frac{7}{8}\right|$ **33.** $|4.9| \square |-5.3|$

34. $-|4.7| \square \frac{1}{2}$ **35.** $\left|-\frac{3}{4}\right| \square -\left|\frac{3}{4}\right|$ **36.** $-\left|1\frac{1}{4}\right| \square -\left|-1\frac{3}{8}\right|$

GO DIGITAL

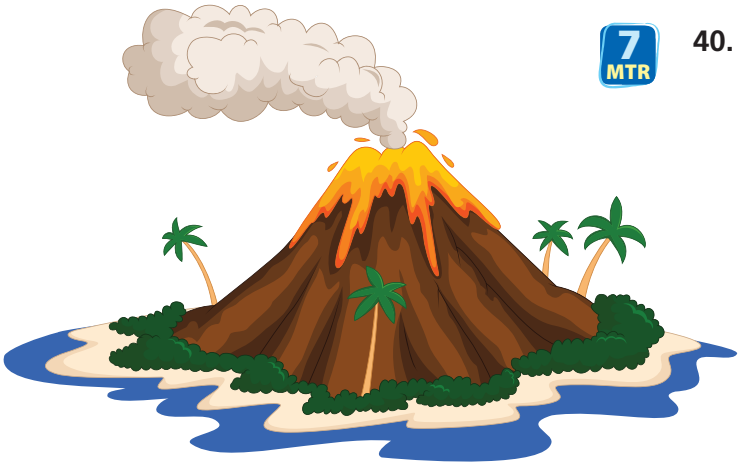
4 MTR **YOU BE THE TEACHER** **Your friend compares two rational numbers. Is your friend correct? Explain your reasoning.**

37. $|-10| = -10$

38. $\left|-\frac{4}{5}\right| > -\left|\frac{1}{2}\right|$

39. **B.E.S.T. Test Prep** Which number has an absolute value greater than 3?

Ⓐ -4 Ⓑ -2.5 Ⓒ 2 Ⓓ 3



7 MTR **40.** **MODELING REAL LIFE** The *summit elevation* of a volcano is the elevation of the top of the volcano relative to sea level. The summit elevation of Kilauea, a volcano in Hawaii, is 1277 meters. The summit elevation of Loihi, an underwater volcano in Hawaii, is -969 meters. Which summit is higher? Which summit is closer to sea level?

7 MTR **41.** **MODELING REAL LIFE** The *freezing point* of a liquid is the temperature at which the liquid becomes a solid. (See Example 4.)

a. Which liquid in the table has the lowest freezing point?

b. Is the freezing point of mercury or butter closer to the freezing point of water, 0°C?

Liquid	Freezing Point (°C)
Butter	35
Airplane fuel	-53
Honey	-3
Mercury	-39
Candle wax	53

ORDERING RATIONAL NUMBERS **Order the values from least to greatest.**

42. $8, |3|, -5, |-2|, -2$

43. $|-6.3|, -7.2, 8, |5|, -6.3$

44. $|3.5|, |-1.8|, 4.6, 3\frac{2}{5}, |2.7|$

45. $\left|-\frac{3}{4}\right|, \frac{5}{8}, \left|\frac{1}{4}\right|, -\frac{1}{2}, \left|-\frac{7}{8}\right|$

7 MTR **46.** **INVESTIGATE** The table shows the distances (in astronomical units) of several NASA spacecraft from Earth in 2020.

a. Order the spacecraft by distance from Earth from closest to farthest.

b. Research the distance these spacecrafts are from Earth now. Did the order in part (a) change?

Spacecraft	Distance (AU)
New Horizons	46.8
Voyager 1	149.2
Voyager 2	123.6
Juno	0.4

GO DIGITAL

47. PROBLEM SOLVING The table shows golf scores, relative to *par*.

Player	Score
1	+5
2	0
3	−4
4	−1
5	+2

a. The player with the lowest score wins. Which player wins?

b. Which player is closest to par?

c. Which player is farthest from par?

48. Dig Deeper You use the table below to record the temperature at the same location each hour for several hours. At what time is the temperature coldest? At what time is the temperature closest to the freezing point of water, 0°C?

Time	10:00 A.M.	11:00 A.M.	12:00 P.M.	1:00 P.M.	2:00 P.M.	3:00 P.M.
Temperature (°C)	−2.6	−2.7	−0.15	1.6	−1.25	−3.4

49. Dig Deeper A construction company demolishes a brick patio and builds a new one. Trucks deliver and remove bricks from the construction site. Trucks delivering bricks carry 150 new bricks per truckload, and trucks removing bricks carry 100 bricks per truckload.

a. Two truckloads of old bricks are removed, and three truckloads of new bricks are delivered to the site. Use the expression $2\left|-100\right| + 3(150)$ to find the total number of bricks that are moved.

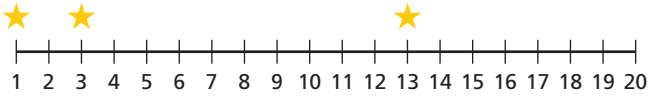
b. Describe a situation where the sum of bricks the company removes and delivers is −250 bricks.

4 MTR **DISCUSS MATHEMATICAL THINKING** **Determine whether the statement is *true* or *false*. Explain your reasoning.**

50. If $x < 0$, then $\left|x\right| = -x$.

51. The absolute value of every rational number is positive.

3 MTR **52. ADAPT A PROCEDURE** You, your friend, and your cousin visit the Calle Ocho Walk of Fame in Miami. The stars are in order from 1 to 20. You decide that everyone must stop at each star they pass and can visit a star more than once. You start at star 1, your friend starts at star 3, and your cousin starts at star 13.



a. You and your cousin walk toward each other and meet at star 5. Who saw more stars? Explain.

b. Your friend made 23 stops and ended on star 10. What is a possible order of stars your friend visited? Explain how you found your answer.

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1.5 Rational Numbers and Exponents

Learning Target: Generate equivalent expressions involving powers with rational number bases.

Success Criteria:
- I can write products and evaluate expressions involving powers.
- I can simplify expressions involving powers.
- I can solve real-life problems involving powers.

Exploration 1 Finding Powers of Quotients

Work with a partner. Copy and complete the table. Use your results to write a *general rule* for finding $\left(\frac{a}{b}\right)^m$, a power of a quotient.

Expression	Repeated Multiplication Form	Rewritten Form	Value
$\left(\frac{1}{2}\right)^2$	$\frac{1}{2} \cdot \frac{1}{2}$	$\frac{1 \cdot 1}{2 \cdot 2} = \frac{1^2}{2^2}$	$\frac{1}{4}$
$\left(\frac{3}{4}\right)^3$			
$\left(\frac{5}{2}\right)^4$			

Exploration 2 Writing Equivalent Expressions

Work with a partner. Copy and complete the table. Apply the properties of exponents to rewrite each expression as a single power. State the property used.

Expression	Simplified Expression	Property
$\left(\frac{1}{3}\right)^2 \cdot \left(\frac{1}{3}\right)^6$		
$0.4^4 \cdot 0.4^5$		
$\left[\left(\frac{1}{4}\right)^2\right]^3$		
$\frac{1.5^9}{1.5^4}$		

4 MTR JUSTIFY A RESULT
How can you verify each simplified expression?

GO DIGITAL

Number Sense and Operations
MA.7.NSO.1.1 Know and apply the Laws of Exponents to evaluate numerical expressions and generate equivalent numerical expressions, limited to whole-number exponents and rational number bases.

1.5 Lesson

Example 1 Writing Expressions Using Exponents

Write each product using exponents.

Use parentheses to write powers when the base is negative or a fraction.

a. $\left(-\frac{1}{2}\right) \cdot \left(-\frac{1}{2}\right) \cdot \left(-\frac{1}{2}\right)$

Because $-\frac{1}{2}$ is used as a factor 3 times, its exponent is 3.

▶ So, $\left(-\frac{1}{2}\right) \cdot \left(-\frac{1}{2}\right) \cdot \left(-\frac{1}{2}\right) = \left(-\frac{1}{2}\right)^3$.

b. $0.5 \cdot 0.5 \cdot 0.5 \cdot 0.5$

Because 0.5 is used as a factor 4 times, its exponent is 4.

▶ So, $0.5 \cdot 0.5 \cdot 0.5 \cdot 0.5 = 0.5^4$.

Try It **Write the product using exponents.**

1. $\frac{1}{4} \cdot \frac{1}{4} \cdot \frac{1}{4} \cdot \frac{1}{4} \cdot \frac{1}{4}$
2. $(-0.3) \cdot (-0.3) \cdot (-0.3)$

You can use repeated multiplication or the Power of a Quotient Property to evaluate expressions involving powers with rational number bases.

Key Idea

Power of a Quotient Property

Words To find a power of a quotient, divide the power of the numerator by the power of the denominator.

Numbers $\left(\frac{3}{2}\right)^2 = \frac{3^2}{2^2} = \frac{9}{4}$ **Algebra** $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$, where $b \neq 0$

Example 2 Evaluating Expressions

Evaluate each expression.

2 MTR USE ANOTHER METHOD
Evaluate the expression in part (a) by rewriting the decimal as a fraction and using the Power of a Quotient Property.

a. $-0.1^2 = -(0.1 \cdot 0.1)$ Write as repeated multiplication.

$= -0.01$ Simplify.

b. $\left(\frac{5}{4}\right)^3 = \frac{5^3}{4^3}$ Power of a Quotient Property

$= \frac{125}{64}$ Simplify.

Try It **Evaluate the expression.**

3. 0.5^2
4. -0.2^3
5. $\left(\frac{7}{6}\right)^2$
6. $-\left(\frac{2}{3}\right)^4$

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Example 3 Simplifying Expressions

The properties of exponents also apply to powers with rational number bases.

a. $\left(-\frac{1}{4}\right)^4 \cdot \left(-\frac{1}{4}\right)^3 = \left(-\frac{1}{4}\right)^{4+3}$ Product of Powers Property

$= \left(-\frac{1}{4}\right)^7$ Simplify.

b. $(0.5^2)^3 = 0.5^{2 \cdot 3}$ Power of a Power Property

$= 0.5^6$ Simplify.

c. $\frac{(-2.7)^8}{(-2.7)^5} = (-2.7)^{8-5}$ Quotient of Powers Property

$= (-2.7)^3$ Simplify.

d. $\left(\frac{1}{5}\right)^3 \cdot \left(\frac{1}{6}\right)^0 = \left(\frac{1}{5}\right)^3 \cdot 1$ Definition of a zero exponent

$= \left(\frac{1}{5}\right)^3$ Multiplicative Identity Property of One

Try It **Simplify the expression. Write your answer as a power.**

7. $0.8^6 \cdot 0.8^7$ **8.** $\frac{6.5^7}{6.5^4}$ **9.** $\left(\frac{1}{8}\right)^3 \cdot \left(\frac{1}{4}\right)^0$

In-Class Practice

1 I don't understand yet. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

WRITING EXPRESSIONS USING EXPONENTS **Write the product using exponents.**

10. $(-0.9) \cdot (-0.9) \cdot (-0.9)$ **11.** $\frac{1}{8} \cdot \frac{1}{8} \cdot \frac{1}{8} \cdot \frac{1}{8}$

EVALUATING EXPRESSIONS **Evaluate the expression.**

12. 0.4^3 **13.** $\left(\frac{3}{2}\right)^3$ **14.** $-\left(\frac{1}{2}\right)^5$

SIMPLIFYING EXPRESSIONS **Simplify the expression. Write your answer as a power.**

15. $\left(-\frac{1}{8}\right)^5 \cdot \left(-\frac{1}{8}\right)^7$ **16.** $(0.2^3)^4$ **17.** $\frac{1.8^9}{1.8^7}$

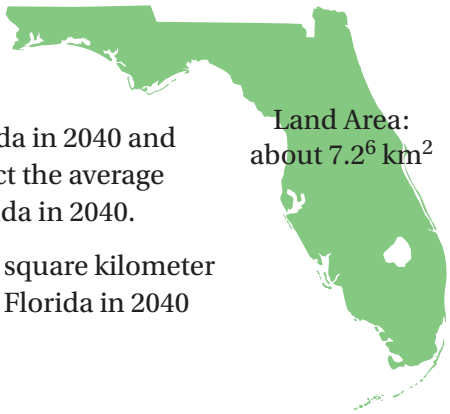
4 MTR **18. WHICH ONE DOESN'T BELONG?** Which expression does *not* belong with the other three? Explain your reasoning.

$1.2^2 \cdot 1.2^3$ $\frac{1.2^8}{1.2^3}$ $(1.2^3)^2$ $1.2^5 \cdot 1.2^0$

GO DIGITAL

Example 4 Modeling Real Life

The projected population of Florida in 2040 is about $4 \cdot 7.2^8$. Predict the average number of people per square kilometer in Florida in 2040.



Understand the problem.

You are given the projected population of Florida in 2040 and the land area of Florida. You are asked to predict the average number of people per square kilometer in Florida in 2040.

Make a plan.

You can find the average number of people per square kilometer in 2040 by dividing the projected population of Florida in 2040 by the land area.

Solve and check.

$$\text{People per square kilometer} = \frac{\text{Population in 2040}}{\text{Land area}}$$

$= \dfrac{4 \cdot 7.2^8}{7.2^6}$ Substitute.

$= 4 \cdot \dfrac{7.2^8}{7.2^6}$ Rewrite.

$= 4 \cdot 7.2^2$ Quotient of Powers Property

$= 207.36$ Evaluate.

▶ So, you can predict that there will be about 207 people per square kilometer in Florida in 2040.

Check Reasonableness

Use estimation.

$4 \cdot \dfrac{7^8}{7^6} = 4 \cdot 7^2$
$= 4 \cdot 49$
$\approx 4 \cdot 50$
$= 200$, which is close to 207. ✓

In-Class Practice

1 I don't understand yet. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

19. The edges of a cube are 1.4^5 centimeters long. Find the volume of the cube. Write your answer as a power.

20. **Dig Deeper** A scientist has 32 grams of a radioactive isotope. The amount is halved every minute due to radioactive decay.

a. Write an expression involving a power for the amount remaining after 3 minutes.

b. Write an expression for the amount remaining after t minutes. Justify your answer.

c. Use the expression in part (b) to find the amount remaining after 5 minutes.

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1.5 Practice WITH CalcChat® AND CalcView®

Review & Refresh

Copy and complete the statement using <, >, or =.

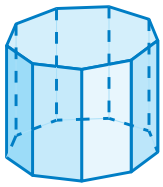
1. $5 \square |-7|$ **2.** $|-2.6| \square |-2.06|$ **3.** $\left|-\frac{3}{5}\right| \square -\left|\frac{5}{8}\right|$

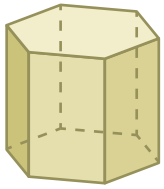
Make a box-and-whisker plot for the data.

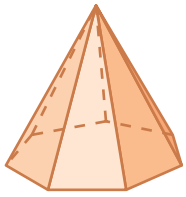
4. Time spent studying (in minutes): 35, 41, 45, 43, 33, 24, 56, 49, 37, 47, 45

5. Points scored in a basketball game: 73, 65, 64, 71, 52, 63, 74, 68, 66, 72, 59, 78

Find the numbers of faces, edges, and vertices of the solid.

6. 

7. 

8. 

Concepts, Skills, & Problem Solving

WRITING EQUIVALENT EXPRESSIONS **Apply the properties of exponents to rewrite the expression as a single power. State the property used.** (See Exploration 2.)

9. $0.6^3 \cdot 0.6^7$ **10.** $\frac{3.7^8}{3.7^5}$ **11.** $\left[\left(\frac{2}{3}\right)^3\right]^3$

WRITING EXPRESSIONS USING EXPONENTS **Write the product using exponents.** (See Example 1.)

12. $0.2 \cdot 0.2 \cdot 0.2 \cdot 0.2$

▶ 13. $\frac{1}{3} \cdot \frac{1}{3} \cdot \frac{1}{3}$

14. $\left(-\frac{1}{2}\right) \cdot \left(-\frac{1}{2}\right) \cdot \left(-\frac{1}{2}\right)$

15. $(-0.6) \cdot (-0.6) \cdot (-0.6) \cdot (-0.6)$

16. $1.5 \cdot 1.5 \cdot 1.5 \cdot 1.5 \cdot 1.5$

17. $\left(-\frac{2}{5}\right) \cdot \left(-\frac{2}{5}\right) \cdot \left(-\frac{2}{5}\right)$

18. $\left(-\frac{4}{3}\right) \cdot \left(-\frac{4}{3}\right) \cdot \left(-\frac{4}{3}\right) \cdot \left(-\frac{4}{3}\right)$

19. $-\left(\frac{1}{4} \cdot \frac{1}{4} \cdot \frac{1}{4} \cdot \frac{1}{4}\right)$

EVALUATING EXPRESSIONS **Evaluate the expression.** (See Example 2.)

20. 0.3^3

▶ 21. $\left(\frac{3}{4}\right)^4$

22. -0.5^5

23. $\left(\frac{2}{7}\right)^3$

24. $(1.2)^2$

25. $\left(\frac{1}{2}\right)^6$

26. $\left(\frac{1}{12}\right)^2$

27. $\left(-\frac{2}{9}\right)^3$

28. $-\left(\frac{3}{2}\right)^4$

GO DIGITAL

29. YOU BE THE TEACHER Your friend evaluates the expression. Is your friend correct? Explain your reasoning.

$$\left(\frac{2}{3}\right)^2 = \left(\frac{2}{3}\right) \cdot \left(\frac{2}{3}\right)$$
$$= \frac{4}{9}$$

SIMPLIFYING EXPRESSIONS **Simplify the expression. Write your answer as a power.** (See Example 3.)

30. $0.7^5 \cdot 0.7^0$

31. $\dfrac{(-6.4)^8}{(-6.4)^6}$

32. $\dfrac{4.5^5}{4.5^3}$

33. $\left(\frac{2}{3}\right)^2 \cdot \left(\frac{2}{3}\right)^6$

34. $\left[\left(-\frac{3}{4}\right)^5\right]^2$

35. $\left(-\frac{5}{7}\right)^8 \cdot \left(-\frac{5}{7}\right)^9$

36. B.E.S.T. Test Prep Which expression is equivalent to 0.47^9?

Ⓐ $0.47^3 \cdot 0.47^3$

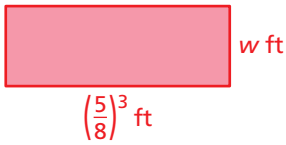
Ⓑ $((0.47)^3)^6$

Ⓒ $\dfrac{0.47^0}{0.47^9}$

Ⓓ $\dfrac{0.47^{12}}{0.47^3}$

37. MODELING REAL LIFE The Haitian Compas Festival in Miami, Florida, is one of the largest outdoor festivals in the United States. The projected number of people for next year attending the first day of the festival is about $6 \cdot \left(\frac{8}{5}\right)^{15}$ people. The projected total number of people who will attend the whole festival is $\left(\frac{8}{5}\right)^2$ times the number of people who attended the first day. Predict the total number of people who will attend the festival. (See Example 4.)

38. GEOMETRY The area of a rectangle is $\left(\frac{5}{8}\right)^{12}$ square feet. Find the width of the rectangle.



39. NUMBER SENSE Find the value of x in the equation without evaluating the power.

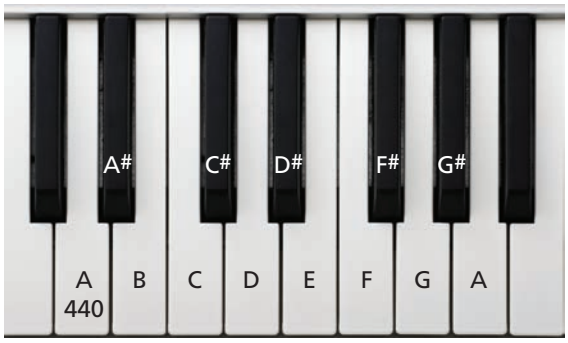
a. $\left(\frac{1}{2}\right)^5 \cdot \left(\frac{1}{2}\right)^x = \frac{1}{256}$

b. $\left(\frac{1}{3}\right)^2 \cdot \left(\frac{1}{3}\right)^x = \frac{1}{729}$

40. ASSESS REASONABLENESS The approximate frequency (in vibrations per second) of a note on a piano is represented by the equation $F = 440(1.059)^n$, where n is the number of notes above A440. Each black or white key represents one note.

a. Compare the frequency of A to the frequency of A440.

b. Estimate the frequency of D#. Is your answer reasonable? Explain.



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1 Connecting Concepts

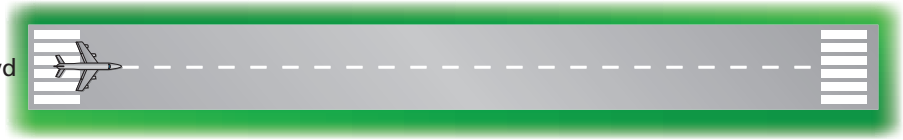
Problem-Solving Strategies

Using an appropriate strategy will help you make sense of problems as you study the mathematics in this course. You can use the following strategies to solve problems that you encounter.

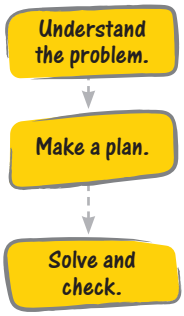
- Use a verbal model.
- Draw a diagram.
- Write an equation.
- Solve a simpler problem.
- Sketch a graph or number line.
- Make a table.
- Make a list.
- Break the problem into parts.

5 MTR *Mathematicians who use patterns and structure to help understand and connect mathematical concepts relate previously learned concepts to new concepts.*

1 MTR **1. ANALYZE A PROBLEM** The runway shown has an area of 2.14^{16} square yards. Find the perimeter (in feet) of the runway.



Using the Problem-Solving Plan



You know the area of the rectangular runway in square yards and the width of the runway in yards. You want to know the perimeter of the runway in feet.

Use the formula for the area of a rectangle to write an equation in one variable, then solve for the length of the runway (in yards) and find the perimeter of the runway. Finally, use a measurement conversion to write the perimeter in terms of feet.

Use the plan to solve the problem. Then check your solution.

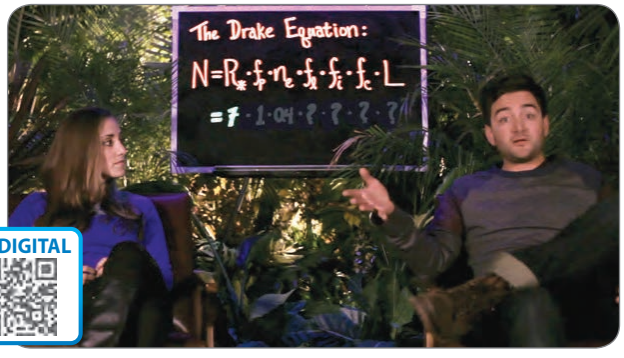
2. Two memory cards are shown. How many times more bytes can card A hold than card B? At what percentage of maximum capacity is each card?

2^8 bytes out of 2^{10} bytes used

2^5 bytes out of 2^8 bytes used

STEAM Performance Task

The Number of Stars in the Galaxies



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At the beginning of this chapter, you watched a STEAM Video called "The Drake Equation." You are now ready to complete the STEAM Performance Task related to this video, where you will use powers to group the stars in the Milky Way galaxy. Be sure to use mathematical thinking and reasoning, and the problem-solving plan as you work through the performance task.

1 Chapter Review WITH CalcChat®

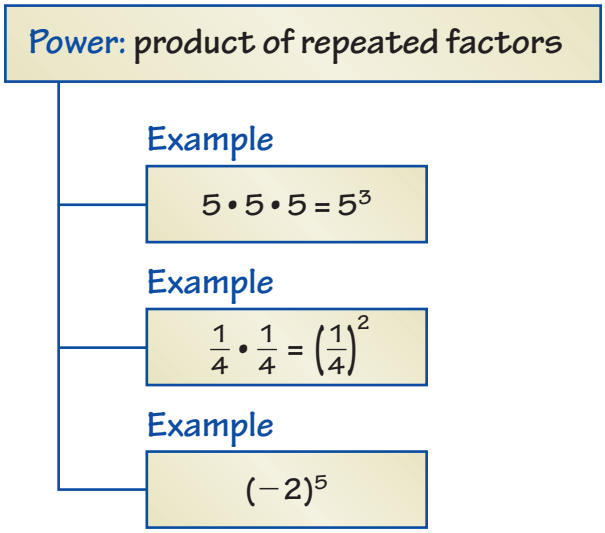
Review Vocabulary

Write the definition and give an example of each vocabulary term.

power, *p. 4*
base, *p. 4*
exponent, *p. 4*
integers, *p. 21*
rational number, *p. 21*
absolute value, *p. 22*

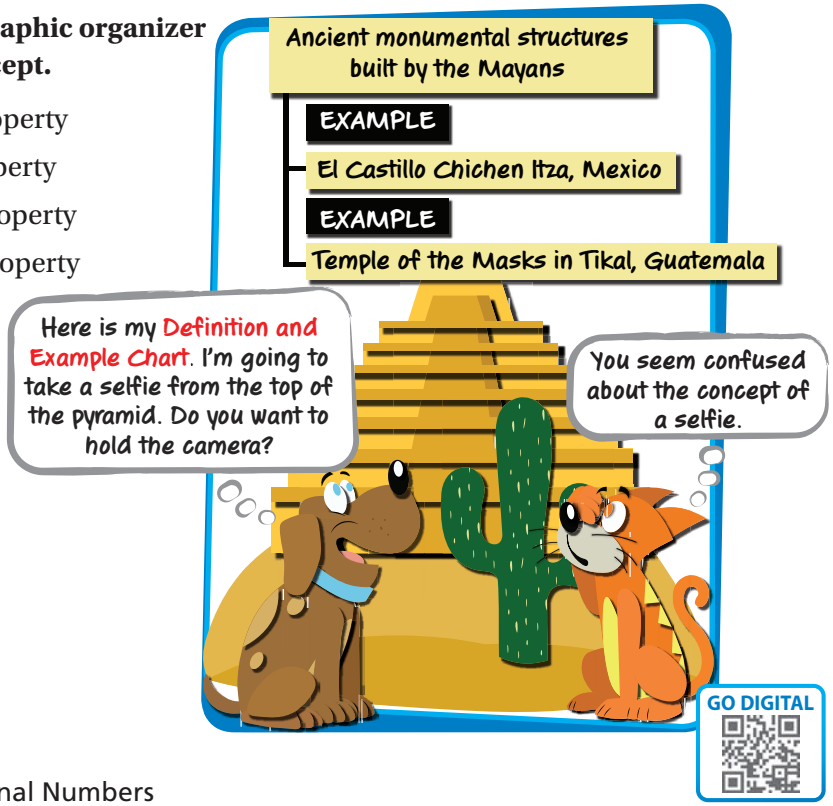
Graphic Organizers

You can use a **Definition and Example Chart** to organize information about a concept. Here is an example of a Definition and Example Chart for the vocabulary term ***power***.



Choose and complete a graphic organizer to help you study the concept.

1. Product of Powers Property
2. Power of a Power Property
3. Power of a Product Property
4. Quotient of Powers Property
5. rational number
6. absolute value



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Chapter Learning Target: Understand exponents and rational numbers.

Chapter Success Criteria:

- ◆ I can evaluate a power.
- ◆ I can evaluate expressions involving whole number bases.
- ■ I can evaluate expressions involving rational number bases.
- ■ I can solve real-life problems involving powers.

◆ Surface
■ Deep

Rate your understanding after each section.

1 I don't understand yet. | 2 I can do it with help. | 3 I can do it on my own. | 4 I can teach someone else.

1.1 Exponents *(pp. 3–8)*

Learning Target: Use exponents to write and evaluate expressions.

Write the product using exponents.

1. $9 \cdot 9 \cdot 9 \cdot 9 \cdot 9$

2. $2 \cdot 2 \cdot 2 \cdot 4 \cdot 4$

Evaluate the expression.

3. 11^3

4. -7^5

5. -12^4

6. The profit P (in dollars) earned by a local merchant selling x items is represented by the equation $P = 0.2x^3 - 10$. How much more profit does he earn selling 15 items than 5 items?

1.2 Product of Powers Property *(pp. 9–14)*

Learning Target: Generate equivalent expressions involving products of powers.

Simplify the expression. Write your answer as a power.

7. $3^5 \cdot 3^2$

8. $(8^{11})^2$

9. $(4 \cdot 6)^4$

10. Simplify $-(2 \cdot 5)^4$.

11. Write an expression that simplifies to 11^{24} using the Power of a Power Property.

12. You send an email with a file size of 4 kilobytes. One kilobyte is 2^{10} bytes. What is the file size of your email in bytes?

13. Explain how to use properties of exponents to simplify the expression $27 \cdot 3^2$.

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1.3 Quotient of Powers Property *(pp. 15–20)*

Learning Target: Generate equivalent expressions involving quotients of powers.

Simplify the expression. Write your answer as a power.

14. $\frac{8^8}{8^3}$

15. $\frac{5^2 \cdot 5^9}{5}$

16. $\frac{7^8}{7^6} \cdot \frac{7^5}{7^4}$

17. $\frac{3^9}{3^7} \cdot \frac{3^{10} \cdot 3^0}{3^5}$

18. Write an expression that simplifies to 9^3 using the Quotient of Powers Property.

19. At the end of a fiscal year, a company has made $5 \cdot 7^7$ dollars in profit. The company employs 7^4 people. How much will each person receive if the company divides the profit equally among its employees?

20. A professional video game tournament has a 4^{13} dollar cash prize. Any player who beats the game is a winner, and the prize is split evenly among the winners.

a. How many people win when each winner receives 4^9 dollars?

b. How much money will each winner receive when 4^3 people win?

1.4 Rational Numbers *(pp. 21–28)*

Learning Target: Understand how to compare rational numbers.

Find the absolute value.

21. $|3|$

22. $|-9|$

23. $\left|\frac{3}{4}\right|$

24. $|-5.2|$

25. $\left|-\frac{6}{7}\right|$

26. $|4.15|$

Copy and complete the statement using <, >, or =.

27. $|-2|$ ☐ -2

28. $\left|-\frac{1}{3}\right|$ ☐ $\left|-\frac{5}{6}\right|$

29. $-|1.7|$ ☐ -1.7

30. Order $|2.25|$, $|-1.5|$, $1\frac{1}{4}$, $\left|2\frac{1}{2}\right|$, and -2 from least to greatest.

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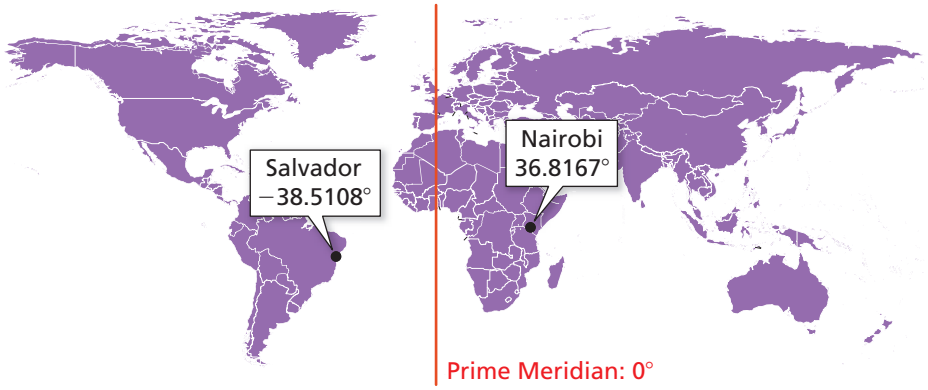
31. Your friend is in Death Valley, California, at an elevation of −282 feet. You are near the Mississippi River in Illinois at an elevation of 279 feet. Who is closer to sea level?

32. Give values for a and b so that $a < b$ and $|a| > |b|$.

33. The map shows the longitudes (in degrees) for Salvador, Brazil, and Nairobi, Kenya. Which city is closer to the prime meridian?



1.5 Rational Numbers and Exponents *(pp. 29–34)*

Learning Target: Generate equivalent expressions involving powers with rational number bases.

Evaluate the expression.

34. $(0.7)^4$

35. $-\left(\frac{1}{2}\right)^0$

36. $\left(\frac{3}{5}\right)^2$

Write the product using exponents.

37. $\frac{1}{8} \cdot \frac{1}{8} \cdot \frac{1}{8} \cdot \frac{1}{8}$

38. $\left(-\frac{3}{7}\right) \cdot \left(-\frac{3}{7}\right) \cdot \left(-\frac{3}{7}\right)$

39. $\left(-\frac{2}{5}\right)^3 \cdot \left(-\frac{2}{5}\right)^2$

40. A blue crab is 1.6^7 feet away from a trap on the ocean floor. The crab pauses every 1.6^5 feet while walking to check for predators. Write an expression using exponents that represents how many times the crab pauses before reaching the trap.

Simplify the expression. Write your answer as a power.

41. $\frac{3.6^7}{3.6^4}$

42. $\left(\frac{2}{9}\right)^3 \cdot \left(\frac{2}{9}\right)^5$

43. $\frac{1.2^5}{1.2^0}$

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1 Practice Test WITH CalcChat®

Write the product using exponents.

1. $(-15) \cdot (-15) \cdot (-15)$

2. $\frac{1}{6} \cdot \frac{1}{6} \cdot \frac{1}{6} \cdot \frac{1}{6} \cdot \frac{1}{6}$

Evaluate the expression.

3. $10 + 3^3 \div 9$

4. $3^3 - 3^4 \div 3^3$

Simplify the expression. Write your answer as a power.

5. $9^{10} \cdot 9$

6. $(6^6)^5$

7. $\frac{(-3.5)^{13} \cdot (-3.5)^2}{(-3.5)^9}$

8. Is $(5^2)^3$ the same as $(5^3)^2$? Explain.

9. One scoop of rice weighs about 3^9 milligrams.

a. Write an expression to find the weight of s scoops of rice. Use your expression to find the weight of 5 scoops of rice.

b. A grain of rice weighs about 3^3 milligrams. About how many grains of rice are in 1 scoop?

Find the absolute value.

10. $\left| -\frac{4}{5} \right|$

11. $|6.43|$

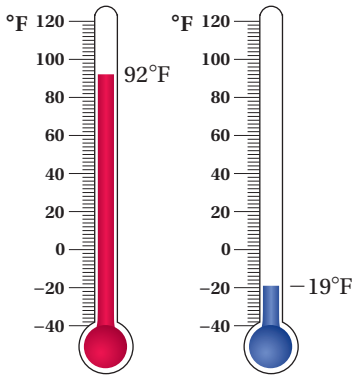
12. $|-22|$

Copy and complete the statement using <, >, or =.

13. $4 \square |-8|$

14. $|-7| \square -12$

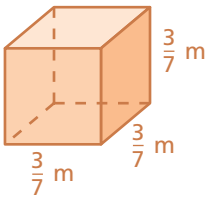
15. $-7 \square |3|$



16. The average temperature in Rapid City, South Dakota, is 45.2°F. In 2019, the lowest temperature in Rapid City was −19°F and the highest temperature was 92°F. Which of the extreme temperatures is closer to the average? Explain.

17. A snail begins to cross a road that is 1.3^{12} feet wide. The snail is moving at a pace of 1.3^3 feet per minute. How many minutes does it take the snail to cross the road? Write your answer using exponents.

18. A cube has the dimensions shown.



a. Write a power that represents the surface area of the cube.

b. Write a power that represents the volume of the cube.

GO DIGITAL

1 Review & Refresh WITH CalcChat®
while Building Fluency

Use the Distributive Property to evaluate the expression. Explain each step.

1. $9 \cdot (5 - 3)$ **2.** $(7 + 11) \cdot 3$ **3.** $-6 \cdot (-4 + 1)$

Write the product using exponents.

4. $3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3$ **5.** $(-8) \cdot (-8) \cdot (-8) \cdot (-8)$

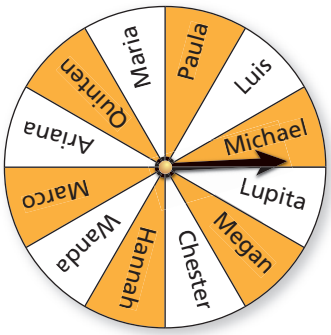
Write a positive or negative integer that represents the situation.

6. A bird flies from the top of a 25-foot-tall tree to the ground.

7. You walk up 18 stairs.

8. A charity receives a $340 donation.

9. A teacher uses a spinner to call on students in class. How many times more likely is it that the student called on has a name beginning with *M* than *L*? Explain.



10. You and a friend are biking down your street. You are traveling 8 miles per hour, and your friend is traveling 716 feet per minute. Who is traveling faster? Explain.

Simplify the expression. Write your answer as a power.

11. $4^7 \cdot 4^0$ **12.** $(-2)^8 \cdot (-2)^3$ **13.** $\dfrac{7^6 \cdot 7^7}{7^4}$

14. You are flipping a fair coin. When the coin lands on heads, you add 7 points to your score. Your score at the end of the game is 91 points.

a. Write an equation in one variable that represents the number of times the coin lands on heads.

b. Use your equation to find the number of times the coin lands on heads.

Write the ratio of the objects.

15. worms to fish

16. bottles to cans

GO DIGITAL

1 B.E.S.T. Test Prep WITH CalcChat®

Cumulative Practice

1. Which of the following expressions are equivalent to 36?

Ⓐ $1 + 3 \cdot 9$

Ⓑ $44 - 4 \cdot 2$

Ⓒ $9 \cdot 2^3 \div 2$

Ⓓ $72 \div 9 - 7$

Ⓔ $-8 + 11 \cdot 12$

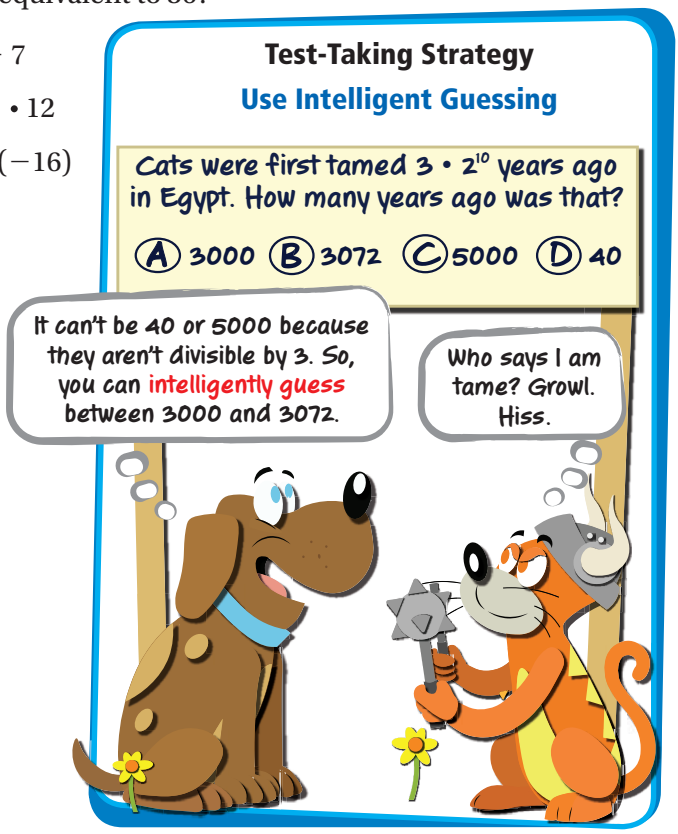
Ⓕ $4^2 \cdot 3 + (-16)$

Test-Taking Strategy

Use Intelligent Guessing

Cats were first tamed $3 \cdot 2^{10}$ years ago in Egypt. How many years ago was that?

Ⓐ 3000 Ⓑ 3072 Ⓒ 5000 Ⓓ 40



2. What is the value of $8^3 \div 8^0$?

GRIDDED RESPONSE

3. Which expression is equivalent to the expression $2^4 \cdot 2^3$?

Ⓐ 2^{12}

Ⓑ 4^7

Ⓒ 48

Ⓓ 128

4. When evaluating $40 - 4 \cdot 3^2$ the first step is to

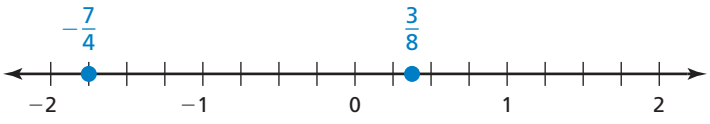
Ⓐ subtract 40 and 4
Ⓑ multiply 4 and 3
Ⓒ evaluate 3^2
Ⓓ multiply 3 and 2

and the value of the expression is

Ⓐ -104
Ⓑ 4
Ⓒ 16
Ⓓ 360

.

5. What is the distance between the two numbers on the number line?



Ⓐ $-2\frac{1}{8}$

Ⓑ $-1\frac{3}{8}$

Ⓒ $1\frac{3}{8}$

Ⓓ $2\frac{1}{8}$

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6. Which expression is *not* equivalent to 16?

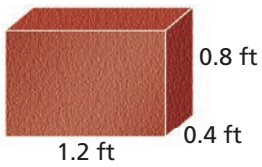
Ⓐ $2^4 \cdot 2^0$ Ⓑ $4^6 \div 4^4$ Ⓒ $\frac{2^5}{2}$ Ⓓ $16^8 \cdot 16^7$

7. Which expressions simplify to 6^5?

Ⓐ $6^7 \cdot 6^2$ Ⓒ $6^3 \cdot 6^2$ Ⓔ $\frac{6^6}{6^0}$

Ⓑ $\frac{6^8}{6^3}$ Ⓓ $6^9 \cdot 6^4$ Ⓕ $\frac{6^4 \cdot 6^6}{6^5}$

8. A privacy wall is constructed using the brick shown. How many bricks does it take to complete one row of the wall when the wall is 57.6 feet long?



Ⓐ 48 bricks Ⓓ 150 bricks

Ⓑ 72 bricks Ⓒ 144 bricks

9. What is the missing exponent?

GRIDDED RESPONSE

$\frac{1}{8}^{\square} = \frac{1}{8} \cdot \frac{1}{8} \cdot \frac{1}{8}$

10. The table shows the height (in inches) of 12 students in a class.

Students' Heights (inches)					
61	74	66	59	63	72
64	60	68	70	62	61

What is the mean height of the class?

Ⓐ 61 in. Ⓑ 63.5 in. Ⓒ 65 in. Ⓓ 74 in.