3.2 Using Tape Diagrams

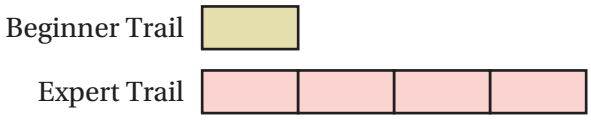
Learning Target: Use tape diagrams to model and solve ratio problems.

Success Criteria:
- I can interpret tape diagrams that represent ratio relationships.
- I can draw tape diagrams to model ratio relationships.
- I can find the value of one part of a tape diagram.
- I can use tape diagrams to solve ratio problems.

You can use a visual model, called a *tape diagram*, to represent the relationship between two quantities in a ratio.

Exploration 1 Using a Tape Diagram

Work with a partner. The tape diagram models the lengths of two snowboarding trails.



a. Compare the lengths of the two trails.

b. The Beginner Trail is 300 yards long. How long is the Expert Trail. How do you know?

2 MTR MAKE A CONNECTION
How does the tape diagram help you make sense of the quantities and the relationship between the quantities?

Exploration 2 Using a Tape Diagram

Work with a partner. The Expert Trail is 5 times longer than the Beginner Trail.

a. Draw a tape diagram to compare the lengths of the two trails.

b. The combined lengths of the two trails is 1500 yards. How long is the Beginner Trail? How long is the Expert Trail?

Algebraic Reasoning

MA.6.AR.3.1 Given a real-world context, write and interpret ratios to show the relative sizes of two quantities using appropriate notation: $\frac{a}{b}$, a to b, or $a : b$ where $b \neq 0$.

MA.6.AR.3.5 Solve mathematical and real-world problems involving ratios, rates and unit rates, including comparisons, mixtures, ratios of lengths and conversions within the same measurement system.

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3.2 Lesson

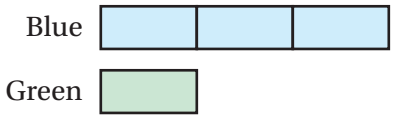
You can use tape diagrams to represent ratios and solve ratio problems.

Example 1 Interpreting a Tape Diagram

The tape diagram represents the ratio of blue monsters to green monsters you caught in a game. You caught 10 green monsters. How many blue monsters did you catch?

Reading

The tape diagram shows that the ratio of blue monsters to green monsters is 3 : 1.

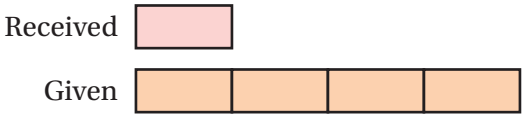


The 1 part for green represents 10 monsters. So, the 3 parts for blue represents $3 \times 10 = 30$ monsters.

▶ You caught 30 blue monsters.

Try It

1. The tape diagram represents the ratio of gifts received to gifts given. You received 4 gifts. How many gifts did you give?



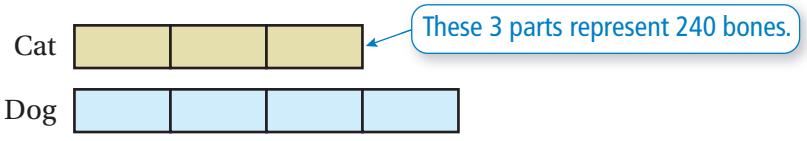
Example 2 Drawing a Tape Diagram

There are 3 bones in a cat for every 4 bones in a dog. The cat has 240 bones. How many bones does the dog have?

4 MTR COMPARE METHODS

Represent the ratio using counters or other objects. Compare this method with drawing a tape diagram. Which do you prefer?

The ratio of bones in the cat to bones in the dog is 3 : 4. Represent the ratio using a tape diagram.



One part represents $240 \div 3 = 80$ bones. So, 4 parts represent $4 \times 80 = 320$ bones.

▶ The dog has 320 bones.

Try It

2. There are 8 bones in a large snake for every 3 bones in a small snake. The small snake has 150 bones. How many bones does the large snake have?

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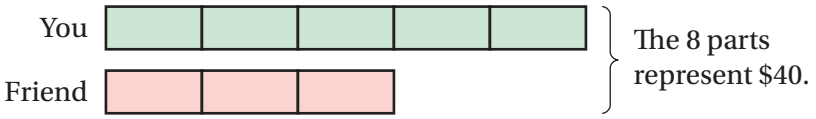
Example 3 Using a Tape Diagram to Solve a Ratio Problem

The ratio of your monthly allowance to your friend's monthly allowance is 5 : 3. The monthly allowances total $40. How much is each allowance?

Represent the ratio 5 : 3 using a tape diagram.



1 part represents $40 ÷ 8 = $5.

5 parts represent 5 × $5 = $25.

3 parts represent 3 × $5 = $15.

▶ So, your allowance is $25, and your friend's allowance is $15.

Check Verify that the ratio of allowances is 5 : 3.

5 : 3

× 5 ( ) × 5 ✓

25 : 15

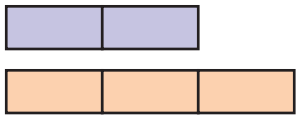
Try It

3. WHAT IF? Repeat Example 3 when the ratio of your monthly allowance to your friend's monthly allowance is 2 to 3.

In-Class Practice

1 I don't understand yet. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

5 MTR **4. STRUCTURE** What ratio is represented by the tape diagram? Can you use the tape diagram to model the ratio 6 : 9? Can you use the tape diagram to model the ratio 8 : 16? Explain your reasoning.



5. REASONING You are given a tape diagram and the total value of the parts. How can you find the value of 1 part?

2 MTR **6. MODEL A PROBLEM** Describe two ways that you can represent the ratio 12 : 4 using a tape diagram.

USING A TAPE DIAGRAM You are given the number of tickets in a bag and the ratio of winning tickets to losing tickets. How many of each kind of ticket are in the bag?

7. 35 tickets; 1 to 4

8. 80 tickets; 2 : 8

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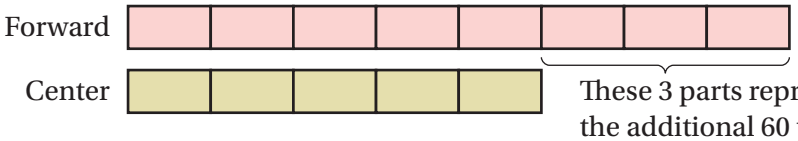
Example 4 Modeling Real Life

7 MTR

In a seven-game basketball series, a team's power forward scores 8 points for every 5 points the center scores. The forward scores 60 more points than the center in the series. How many points does each player score in the series?

The ratio of the forward's points to the center's points is 8 : 5. Represent the ratio using a tape diagram.



These 3 parts represent the additional 60 points the forward scores.

1 part represents $60 \div 3 = 20$ points.

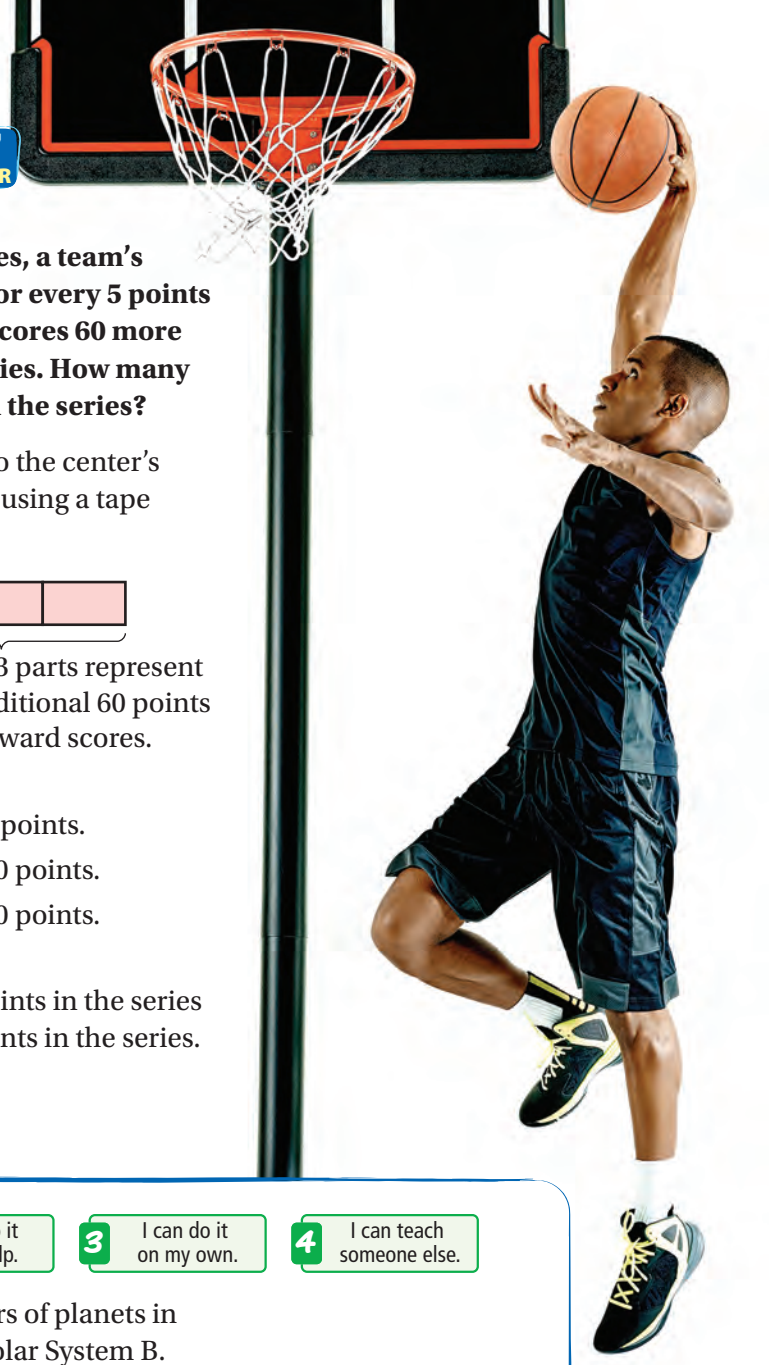
8 parts represent $8 \times 20 = 160$ points.

5 parts represent $5 \times 20 = 100$ points.

▶ So, the forward scores 160 points in the series and the center scores 100 points in the series.

6 MTR ASSESS REASONABLENESS

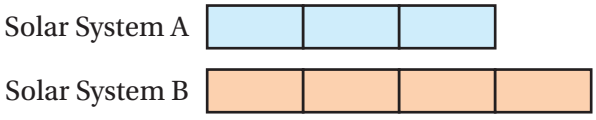
How can you use the value of a ratio to verify that 8 : 5 and 160 : 100 are equivalent?



In-Class Practice

1 I don't understand yet. | 2 I can do it with help. | 3 I can do it on my own. | 4 I can teach someone else.

9. The tape diagram represents the ratio of the numbers of planets in two different solar systems. There are 8 planets in Solar System B. How many planets are in Solar System A?



10. You and your friend play an arcade game. You score 5 points for every 9 points that your friend scores. You score 320 points less than your friend. How many points do you each score?

11. **Dig Deeper** Your team wins 18 medals at a track meet that are either gold, silver, or bronze. The ratio of silver medals to total medals is 2 : 9 and the ratio of gold medals to silver medals is 1 : 1. How many of each medal were won by your team?

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3.2 Practice WITH CalcChat® AND CalcView®

Review & Refresh

Determine whether the ratios are equivalent.

1. 11 : 4 and 22 : 8

2. 12 : 18 and 2 : 3

3. 56 : 81 and 7 : 9

4. 2 : 12 and 6 : 24

Multiply. Write the answer in simplest form.

5. $\frac{7}{10} \cdot \frac{5}{7}$

6. $2\frac{1}{3} \cdot \frac{3}{4}$

7. $5\frac{3}{8} \cdot 2\frac{1}{2}$

8. Your cousin earns $7.40 per hour working at a grocery store. She works 14.25 hours this week. Find out how much your cousin earns this week.

Concepts, Skills, & Problem Solving

USING A TAPE DIAGRAM **Use the tape diagram in Exploration 1 to answer the question.** (See Exploration 1.)

9. The beginner trail is 200 meters long. How long is the expert trail?

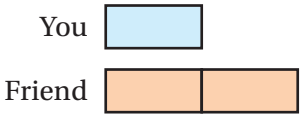
10. The expert trail is 1200 meters long. How long is the beginner trail?

11. The combined length of the trails is 2000 meters. How long is each trail?

12. The expert trail is 750 meters longer than the beginner trail. How long is each trail?

INTERPRETING A TAPE DIAGRAM **The tape diagram represents the ratio of the time you spend tutoring to the time your friend spends tutoring. You tutor for 3 hours. How many hours does your friend spend tutoring?** (See Example 1.)

13.



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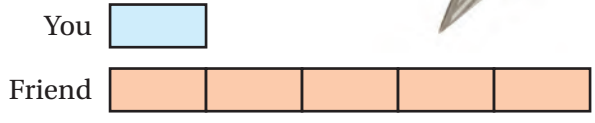

DRAWING A TAPE DIAGRAM **A bag contains red marbles and blue marbles. You are given the number of red marbles in the bag and the ratio of red marbles to blue marbles. Find the number of blue marbles in the bag.** (See Example 2.)

15. 10 red marbles; 5 to 1

16. 3 red marbles; 3 : 7

17. 12 red marbles; 4 : 3

18. 6 red marbles; 2 for every 5

19. 18 red marbles; 6 to 9

20. 12 red marbles; 3 : 4

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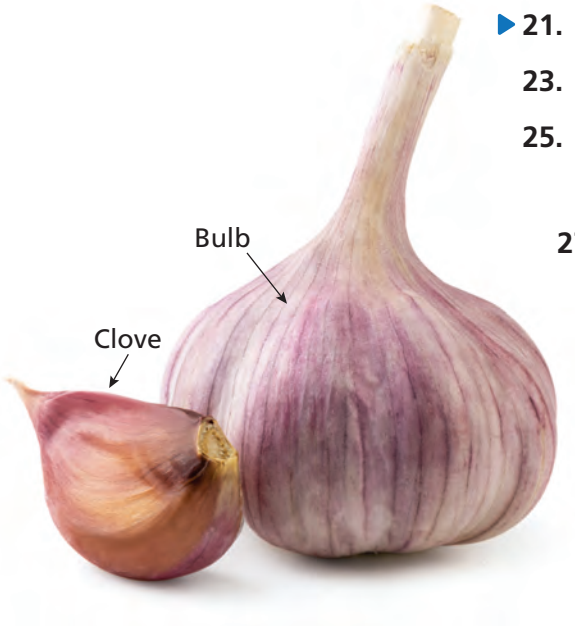
USING A TAPE DIAGRAM **A bowl contains blueberries and strawberries. You are given the total number of berries in the bowl and the ratio of blueberries to strawberries. How many of each berry are in the bowl?** (See Example 3.)



21. 16 berries; 3 : 1

22. 10 berries; 2 for every 3

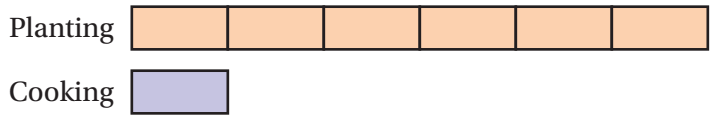
23. 12 berries; 1 to 2

24. 20 berries; 4 : 1

25. 48 berries; 9 to 3

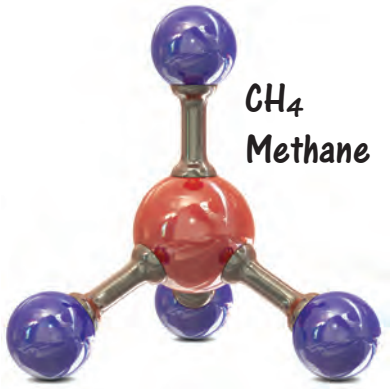
26. 46 berries; 11 for every 12

27. **PROBLEM SOLVING** You separate bulbs of garlic into two groups: one for planting and one for cooking. The tape diagram represents the ratio of bulbs for planting to bulbs for cooking. You use 6 bulbs for cooking. Each bulb has 8 cloves. How many cloves of garlic will you plant?



7 MTR

28. **MODELING REAL LIFE** Methane gas contains carbon atoms and hydrogen atoms in the ratio of 1 : 4. A sample of methane gas contains 92 hydrogen atoms. How many carbon atoms are in the sample? How many total atoms are in the sample? (See Example 4.)

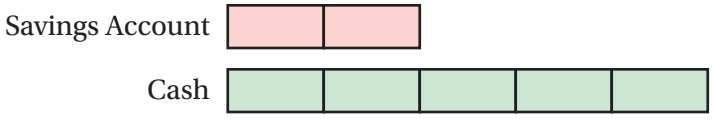


7 MTR

29. **MODELING REAL LIFE** There are 8 more girls than boys in a school play. The ratio of boys to girls is 5 : 7. How many boys and how many girls are in the play?

30. **Dig Deeper** A baseball team sells tickets for two games. The ratio of sold tickets to unsold tickets for the first game was 7 : 3. For the second game, the ratio was 13 : 2. There were 240 unsold tickets for the second game. How many tickets were sold for the first game?

31. **PROBLEM SOLVING** You have $150 in a savings account and you have some cash. The tape diagram represents the ratio of the amounts of money. You want to have twice the amount of money in your savings account as you have in cash. How much of your cash should you deposit into your savings account?



32. **Dig Deeper** A fish tank contains tetras, guppies, and minnows. The ratio of tetras to guppies is 4 : 2. The ratio of minnows to guppies is 1 : 3. There are 60 fish in the tank. How many more tetras are there than minnows? Justify your answer.

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