## 72 <br> Representations of Functions

Learning Target: Represent functions in a variety of ways.
Success Criteria: - I can write a function rule that describes a relationship.

- I can evaluate functions for given inputs.
- I can represent functions using tables and graphs.


## EXPLORATION 1 Using a Table to Describe Relationships

Work with a partner. Make a table that shows the relationship between the figure number $x$ and the area $A$ of each figure. Then use an equation to find which figure has an area of 81 square units when the pattern continues.
a.


Figure 1 Figure 2
b.


Figure 1 Figure 2


Figure 3


Figure 4


Figure 3


Figure 4

## EXPLORATION 2 Using a Graph

## Math Practice

## Construct <br> Arguments

How does the graph help you determine whether the statement is true?

Work with a partner. Use a graph to test the truth of each statement. If the statement is true, write an equation that shows how to obtain one measurement from the other.
a. "You can find the horsepower of a race-car engine if you know its volume in cubic inches."


| Volume (cubic inches), $\boldsymbol{x}$ | 200 | 350 | 350 | 500 |
| :--- | :--- | :--- | :--- | :--- |
| Horsepower, $\boldsymbol{y}$ | 375 | 650 | 250 | 600 |

b. "You can find the volume of a race-car engine in cubic centimeters if you know its volume in cubic inches."

| Volume (cubic inches), $\boldsymbol{x}$ | 100 | 200 | 300 |
| :--- | :---: | :---: | :---: |
| Volume (cubic centimeters), $\boldsymbol{y}$ | 1640 | 3280 | 4920 |

### 7.2 Lesson

Key Vocabulary
function rule, p. 282

## Remember

An independent variable represents a quantity that can change freely. A dependent variable depends on the independent variable.

## Key Idea

## Functions as Equations

A function rule is an equation that describes the relationship between inputs (independent variable) and outputs (dependent variable).


## EXAMPLE 1 Writing Function Rules

a. Write a function rule for "The output is five less than the input."

Words The output is five less than the input.
Equation $y=x-5$
A function rule is $y=x-5$.
b. Write a function rule for "The output is the square of the input."

Words The output is the square of the input.
Equation $y=x^{2}$
A function rule is $y=x^{2}$.
Try It

1. Write a function rule for "The output is one-fourth of the input."

## EXAMPLE 2 Evaluating a Function

What is the value of $y=2 x+5$ when $x=3$ ?

$$
\begin{aligned}
y & =2 x+5 & & \text { Write the equation. } \\
& =2(3)+5 & & \text { Substitute } 3 \text { for } x . \\
& =11 & & \text { Simplify. }
\end{aligned}
$$

Try It Find the value of $\boldsymbol{y}$ when $\boldsymbol{x}=5$.
2. $y=4 x-1$
3. $y=10 x$
4. $y=7-3 x$

## Functions as Tables and Graphs

A function can be represented by an input-output table and by a graph. The table and graph below represent the function $y=x+2$.

| Input, <br> $\boldsymbol{x}$ | Output, <br> $\boldsymbol{y}$ | Ordered Pair, <br> $(\boldsymbol{x}, \boldsymbol{y})$ |
| :---: | :---: | :---: |
| 1 | 3 | $(1,3)$ |
| 2 | 4 | $(2,4)$ |
| 3 | 5 | $(3,5)$ |



By drawing a line through the points, you graph all of the solutions of the function $y=x+2$.

## EXAMPLE 3 Graphing a Function

Graph the function $\boldsymbol{y}=\mathbf{- 2 x}+\mathbf{1}$.
Make an input-output table using inputs of $-1,0,1$, and 2 .

| Input, $\boldsymbol{x}$ | $\mathbf{- 2 \boldsymbol { x } + \mathbf { 1 }}$ | Output, $\boldsymbol{y}$ | Ordered Pair, $(\boldsymbol{x}, \boldsymbol{y})$ |
| :---: | :---: | :---: | :---: |
| -1 | $-2(-1)+1$ | 3 | $(-1,3)$ |
| 0 | $-2(0)+1$ | 1 | $(0,1)$ |
| 1 | $-2(1)+1$ | -1 | $(1,-1)$ |
| 2 | $-2(2)+1$ | -3 | $(2,-3)$ |

Plot the ordered pairs and draw a line through the points.


## Try It Graph the function.

5. $y=x+1$
6. $y=-3 x$
7. $y=3 x+2$

## Summary

## Representations of Functions

Words The output is 2 more than the input.

Equation $y=x+2$

Input-Output Table

| Input, $x$ | Output, $y$ |
| :---: | :---: |
| -1 | 1 |
| 0 | 2 |
| 1 | 3 |
| 2 | 4 |

Mapping Diagram
Graph



## Self-Assessment for Concepts \& skills

Solve each exercise. Then rate your understanding of the success criteria in your journal.

## WRITING FUNCTION RULES Write a function rule for the statement.

8. The output is three times the input.
9. The output is eight more than one-seventh of the input.

EVALUATING A FUNCTION Find the value of $y$ when $x=-5$.
10. $y=6 x$
11. $y=11-x$
12. $y=\frac{1}{5} x+1$

GRAPHING A FUNCTION Graph the function.
13. $y=-2 x$
14. $y=x-3$
15. $y=9-3 x$
16. DIFFERENT WORDS, SAME QUESTION Which is different?

Find "both" answers.

What output is 4 more than twice the input 3 ?

What output is the sum of 2 times the input 3 and 4 ?

What output is twice the sum of the input 3 and 4 ?

What output is 4 increased by twice the input 3 ?


A car produces 20 pounds of carbon dioxide for every gallon of gasoline burned. Write and graph a function that describes the relationship. Use a verbal model to write a function rule.

Verbal Model | Carbon dioxide |
| :---: |
| (pounds) |\(\left|=\begin{array}{c|c}Pounds <br>

per gallon\end{array}\right|\)| Gasoline used |
| :---: |
| (gallons) |

Variable Let $p$ represent the number of pounds of carbon dioxide, and let $g$ represent the number of gallons of gasoline used.

$$
\text { Equation } p \quad=\quad 20 \quad \text { • } g
$$

Make an input-output table that represents the function $p=20 g$.

| Input, $\boldsymbol{g}$ | $\mathbf{2 0 g}$ | Output, $\boldsymbol{p}$ | Ordered Pair, ( $\boldsymbol{g}, \boldsymbol{p}$ ) |
| :---: | :---: | :---: | :---: |
| 1 | $20(1)$ | 20 | $(1,20)$ |
| 2 | $20(2)$ | 40 | $(2,40)$ |
| 3 | $20(3)$ | 60 | $(3,60)$ |

Plot the ordered pairs and draw a line through the points.

Because you cannot burn a negative number of gallons of gasoline, use only positive values of $g$.


## Self-ASSeSSMent for Problem Solving

Solve each exercise. Then rate your understanding of the success criteria in your journal.
17. The World Health Organization (WHO) suggests having 23 health-care workers for every 10,000 people. How many health-care workers are needed to meet the WHO suggestion for a population of 250,000 people? Justify your answer using a graph.
18. DIG DEEPER A truck produces 22 pounds of carbon dioxide for every gallon of diesel fuel burned. The fuel economy of the truck is 18 miles per gallon. Write and graph a function that describes the relationship between carbon dioxide produced and distance traveled.

## Review \& Refresh

## Determine whether the relation is a function.

1. Input Output


2. Input Output


Find the slope of the line.
4.

5.

6.


## Concepts, Skills, \& Problem Solving

USING A GRAPH Use a graph to test the truth of the statement. If the statement is true, write an equation that shows how to obtain one measurement from the other measurement. (See Exploration 2, p. 281.)
7. "You can find the weight of a cell phone in ounces if you know its screen size in inches."

| Screen Size (inches), $\boldsymbol{x}$ | 4 | 4.7 | 5 | 5.5 |
| :--- | :--- | :--- | :---: | :---: |
| Weight (ounces), $\boldsymbol{y}$ | 4 | 4.8 | 4.8 | 6.4 |

8. "You can find the age of a child in years if you know the age of the child in months."

| Age (months), $\boldsymbol{x}$ | 9 | 12 | 15 | 24 |
| :--- | :---: | :---: | :---: | :---: |
| Age (years), $\boldsymbol{y}$ | 0.75 | 1 | 1.25 | 2 |

## WRITING FUNCTION RULES Write a function rule for the statement.

9. The output is half of the input.
10. The output is eleven more than the input.
11. The output is three less than the input.
12. The output is the cube of the input.
13. The output is six times the input.
14. The output is one more than twice the input.

EVALUATING A FUNCTION Find the value of $\boldsymbol{y}$ for the given value of $\boldsymbol{x}$.
15. $y=x+5 ; x=3$
16. $y=7 x ; x=-5$
17. $y=1-2 x ; x=9$
18. $y=3 x+2 ; x=0.5$
19. $y=2 x^{3} ; x=3$
20. $y=\frac{x}{2}+9 ; x=-12$

GRAPHING A FUNCTION Graph the function.
21. $y=x+4$
22. $y=2 x$
23. $y=-5 x+3$
24. $y=\frac{x}{4}$
25. $y=\frac{3}{2} x+1$
26. $y=1+0.5 x$

MATCHING Match the graph with the function it represents.

A. $y=\frac{x}{3}$
28.

B. $y=x+1$
29.

C. $y=-2 x+6$
30. YOU BE THE TEACHER Your friend graphs the function represented by the input-output table. Is your friend correct? Explain your reasoning.

| Input, $\boldsymbol{x}$ | -4 | -2 | 0 | 2 |
| :--- | :---: | :---: | :---: | :---: |
| Output, $\boldsymbol{y}$ | -1 | 1 | 3 | 5 |


31. MODELING REAL LIFE A dolphin eats 30 pounds of fish per day.
a. Write and graph a function that relates the number $p$ of pounds of fish that a dolphin eats in $d$ days.
b. How many total pounds of fish does a dolphin eat in 30 days?
32. MODELING REAL LIFE You fill a fish tank with 55 gallons of water on Saturday. The water evaporates at a rate of 1.5 gallons per day. You plan to add water when the tank reaches 49 gallons. When will you add water? Justify your answer.

USING AN EQUATION Find the value of $\boldsymbol{x}$ for the given value of $\boldsymbol{y}$.
33. $y=5 x-7 ; y=-22$
34. $y=9-7 x ; y=37$
35. $y=\frac{x}{4}-7 ; y=2$
36. PROBLEM SOLVING You decide to make and sell bracelets. The cost of your materials is $\$ 84.00$. You charge $\$ 3.50$ for each bracelet.
a. Write a function that represents the profit $P$ for selling $b$ bracelets.
b. Which variable is independent? dependent? Explain.
c. You will break even when the cost of your materials equals your income. How many bracelets must you sell to break even?
37. MODELING REAL LIFE A furniture store is having a sale where everything is $40 \%$ off.
a. Write and graph a function that represents the amount of discount on an item at regular price.
b. You buy a bookshelf that has a regular price of $\$ 85$. What is the sale price of the bookshelf?
38. REASONING You want to take a two-hour airboat tour. Which is a better deal, Snake Tours or Gator Tours? Use functions to justify your answer.

39. REASONING The graph of a function is a line that passes through the points $(3,2),(5,8)$, and $(8, y)$. What is the value of $y$ ?
40. CRITICAL THINKING Make a table where the independent variable is the side length of a square and the dependent variable is the perimeter. Make a second table where the independent variable is the side length of a square and the dependent variable is the area. Graph both functions in the same coordinate plane. Compare the functions.
41. PUZZLE The blocks that form the diagonals of each square are shaded. Each block has an area of one square unit. Find the "green area" of Square 20. Find the "green area" of Square 21. Explain your reasoning.


Square 1


Square 2


Square 3


Square 4


Square 5

