

2.1 Multiplying Integers

Learning Target: Find products of integers.

- Success Criteria:**
- I can explain the rules for multiplying integers.
 - I can find products of integers with the same sign.
 - I can find products of integers with different signs.

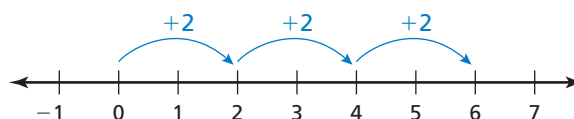
EXPLORATION 1

Understanding Products Involving Negative Integers

Work with a partner.



- a. The number line and integer counters model the product $3 \cdot 2$. How can you find $3 \cdot (-2)$? Explain.



- b. Use the tables to find $-3 \cdot 2$ and $-3 \cdot (-2)$. Explain your reasoning.

2	•	2	=	4
1	•	2	=	2
0	•	2	=	0
-1	•	2	=	<input type="text"/>
-2	•	2	=	<input type="text"/>
-3	•	2	=	<input type="text"/>

-3	•	3	=	-9
-3	•	2	=	-6
-3	•	1	=	-3
-3	•	0	=	<input type="text"/>
-3	•	-1	=	<input type="text"/>
-3	•	-2	=	<input type="text"/>

- c. **INDUCTIVE REASONING** Complete the table. Then write general rules for multiplying (i) two integers with the same sign and (ii) two integers with different signs.

Expression	Type of Product	Product	Product: Positive or Negative
$3 \cdot 2$	Integers with the same sign		
$3 \cdot (-2)$			
$-3 \cdot 2$			
$-3 \cdot (-2)$			
$6 \cdot 3$			
$2 \cdot (-5)$			
$-6 \cdot 5$			
$-5 \cdot (-3)$			

Math Practice

Construct Arguments

Construct an argument that you can use to convince a friend of the rules you wrote in Exploration 1(c).

2.1 Lesson

Consider the following methods for evaluating $3(-2 + 4)$.

Evaluate in parentheses:

$$\begin{aligned} 3(-2 + 4) &= 3(2) \\ &= 6 \end{aligned}$$

Use the Distributive Property:

$$\begin{aligned} 3(-2 + 4) &= 3(-2) + 3(4) \\ &= ? + 12 \end{aligned}$$

For the Distributive Property to be true, $3(-2)$ must equal -6 . This leads to the following rules for multiplying integers.

Key Ideas

Multiplying Integers with the Same Sign

Words The product of two integers with the same sign is positive.

Numbers $2 \cdot 3 = 6$ $-2 \cdot (-3) = 6$

Multiplying Integers with Different Signs

Words The product of two integers with different signs is negative.

Numbers $2 \cdot (-3) = -6$ $-2 \cdot 3 = -6$

EXAMPLE 1 Multiplying Integers

Find each product.

a. $-5 \cdot (-6)$

The integers have the same sign.

$$-5 \cdot (-6) = 30$$

The product is positive.

▶ The product is 30.

b. $3(-4)$

The integers have different signs.

$$3(-4) = -12$$

The product is negative.

▶ The product is -12 .

Try It Find the product.

1. $5 \cdot 5$

2. $-1(-9)$

3. $-7 \cdot (-8)$

4. $12 \cdot (-2)$

5. $4(-6)$

6. $-25(0)$

EXAMPLE 2

Evaluating Expressions

The expression $(-2)^2$ indicates to multiply the number in parentheses, -2 , by itself. The expression -2^2 , however, indicates to find the opposite of 2^2 .

Remember

Use order of operations when evaluating an expression.



- a. Find $(-2)^2$.

$$\begin{aligned}(-2)^2 &= (-2) \cdot (-2) \\ &= 4\end{aligned}$$

Write $(-2)^2$ as repeated multiplication.
Multiply.

- b. Find -2^2 .

$$\begin{aligned}-2^2 &= -(2 \cdot 2) \\ &= -4\end{aligned}$$

Write 2^2 as repeated multiplication.
Multiply 2 and 2.

- c. Find $-2 \cdot 17 \cdot (-5)$.

$$\begin{aligned}-2 \cdot 17 \cdot (-5) &= -2 \cdot (-5) \cdot 17 \\ &= 10 \cdot 17 \\ &= 170\end{aligned}$$

Commutative Property of Multiplication
Multiply -2 and -5 .
Multiply 10 and 17.

- d. Find $-6(-3 + 4) + 6$.

$$\begin{aligned}-6(-3 + 4) + 6 &= -6(1) + 6 \\ &= -6 + 6 \\ &= 0\end{aligned}$$

Perform operation in parentheses.
Multiplication Property of 1
Additive Inverse Property

Try It Evaluate the expression.

7. $8 \cdot (-15) \cdot 0$

8. $24 - 3^3$

9. $10 - 7(3 - 5)$



Self-Assessment for Concepts & Skills

Solve each exercise. Then rate your understanding of the success criteria in your journal.

10. **WRITING** What can you conclude about two integers whose product is (a) positive and (b) negative?

EVALUATING AN EXPRESSION Evaluate the expression.

11. $4(-8)$

12. $-5(-7)$

13. $12 - 3^2 \cdot (-2)$

MP REASONING Tell whether the statement is *true* or *false*. Explain your reasoning.

14. The product of three positive integers is positive.

15. The product of three negative integers is positive.

EXAMPLE 3

Modeling Real Life

You solve a number puzzle on your phone. You start with 250 points. You finish the puzzle in 8 minutes 45 seconds and make 3 mistakes. What is your score?



Understand the problem.

Make a plan.

Solve and check.

You are given ways to gain points and lose points when completing a puzzle. You are asked to find your score after finishing the puzzle.

Use a verbal model to solve the problem. Find the sum of the starting points, mistake penalties, and time bonus.

$$\text{Score} = \text{Starting points} + \text{Number of mistakes} \cdot \text{Penalty per mistake} + \text{Time bonus}$$

$$= 250 + 3(-50) + 75$$

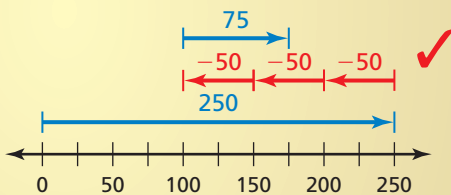
$$= 250 + (-150) + 75$$

$$= 100 + 75$$

$$= 175$$

$$10 \text{ min} - 8 \text{ min } 45 \text{ sec} = 1 \text{ min } 15 \text{ sec} \\ = 75 \text{ sec}$$

Another Method



So, your score is 175 points.



Self-Assessment for Problem Solving

Solve each exercise. Then rate your understanding of the success criteria in your journal.

16. On a mountain, the temperature decreases by 18°F for each 5000-foot increase in elevation. At 7000 feet, the temperature is 41°F . What is the temperature at 22,000 feet? Justify your answer.

Player	Coins	Time
1	31	0:02:03
2	18	0:01:55
3	24	0:01:58
4	27	0:02:01

17. Players in a racing game earn 3 points for each coin they collect. Each player loses 5 points for each second that he or she finishes after the first-place finisher. The table shows the results of a race. List the players in order from greatest to least number of points.



2.1 Practice



Go to BigIdeasMath.com to get HELP with solving the exercises.

▶ Review & Refresh

Find the distance between the two numbers on a number line.

1. -4.3 and 0.8

2. -7.7 and -6.4

3. $-2\frac{3}{5}$ and -1

Divide.

4. $27 \div 9$

5. $48 \div 6$

6. $56 \div 4$

7. $153 \div 8$

8. What is the prime factorization of 84?

A. $2^2 \times 3^2$

B. $2^3 \times 7$

C. $3^3 \times 7$

D. $2^2 \times 3 \times 7$

▶ Concepts, Skills, & Problem Solving

MP CHOOSE TOOLS Use a number line or integer counters to find the product.
(See Exploration 1, p. 49.)

9. $2(-4)$

10. $-6(3)$

11. $4(-5)$

MULTIPLYING INTEGERS Find the product.

12. $6 \cdot 4$

13. $7(-3)$

14. $-2(8)$

15. $-3(-4)$

16. $-6 \cdot 7$

17. $3 \cdot 9$

18. $8 \cdot (-5)$

19. $-1 \cdot (-12)$

20. $-5(10)$

21. $-13(0)$

22. $-9 \cdot 9$

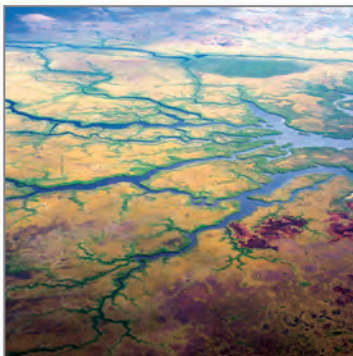
23. $15(-2)$

24. $-10 \cdot 11$

25. $-6 \cdot (-13)$

26. $7(-14)$

27. $-11 \cdot (-11)$



28. **MODELING REAL LIFE** You burn 10 calories each minute you jog. What integer represents the change in your calories after you jog for 20 minutes?

29. **MODELING REAL LIFE** In a four-year period, about 80,000 acres of coastal wetlands in the United States are lost each year. What integer represents the total change in coastal wetlands?

EVALUATING EXPRESSIONS Evaluate the expression.

30. $(-4)^2$

31. -6^2

32. $-5 \cdot 3 \cdot (-2)$

33. $3 \cdot (-12) \cdot 0$

34. $-5(-7)(-20)$

35. $5 - 8^2$

36. $-5^2 \cdot 4$

37. $-2 \cdot (-3)^3$

38. $2 + 1 \cdot (-7 + 5)$

39. $4 - (-2)^3$

40. $4 \cdot (25 \cdot 3^2)$

41. $-4(3^2 - 8) + 1$

YOU BE THE TEACHER Your friend evaluates the expression. Is your friend correct?

Explain your reasoning.

42.

$$-2(-7) = -14$$

43.

$$-10^2 = -100$$

MP PATTERNS Find the next two numbers in the pattern.

44. $-12, 60, -300, 1500, \dots$

45. $7, -28, 112, -448, \dots$

46. **MP PROBLEM SOLVING** In a scavenger hunt, each team earns 25 points for each item that they find. Each team loses 15 points for every minute after 4:00 P.M. that they report to the city park. The table shows the number of items found by each team and the time that each team reported to the park. Which team wins the scavenger hunt? Justify your answer.

Team	Items	Time
A	13	4:03 P.M.
B	15	4:07 P.M.
C	11	3:56 P.M.
D	12	4:01 P.M.

47. **MP REASONING** The height of an airplane during a landing is given by $22,000 + (-480t)$, where t is the time in minutes. Estimate how many minutes it takes the plane to land. Explain your reasoning.

48. **MP PROBLEM SOLVING** The table shows the price of a bluetooth speaker each month for 4 months.

Month	Price (dollars)
June	165
July	$165 + (-12)$
August	$165 + 2(-12)$
September	$165 + 3(-12)$



- Describe the change in the price of the speaker.
- The table at the right shows the amount of money you save each month. When do you have enough money saved to buy the speaker? Explain your reasoning.

Amount Saved	
June	\$35
July	\$55
August	\$45
September	\$18

49. **DIG DEEPER!** Two integers, a and b , have a product of 24. What is the least possible sum of a and b ?

50. **MP NUMBER SENSE** Consider two integers p and q . Explain why $p \times (-q) = (-p) \times q = -pq$.