2 Multiplying and Dividing Rational Numbers

Chapter Learning Target:
Understand multiplying and dividing rational numbers.

Chapter Success Criteria:

- I can explain the rules for multiplying integers.
- I can explain the rules for dividing integers.
- I can evaluate expressions involving rational numbers.
- I can solve real-life problems involving multiplication and division of rational numbers.

STEAM Video

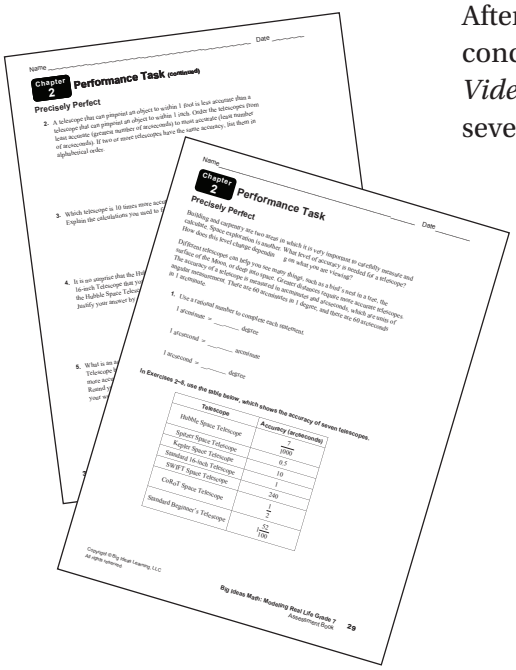
Carpenter or Joiner

Carpenters and joiners must be precise with their measurements when building structures. In what other real-life situations must measurements be precise?

Watch the STEAM Video "Carpenter or Joiner." Then answer the following questions.

1. Robert says that changes in water content cause wood to shrink or expand *across* the grain more than *along* the grain. What does this mean?
2. Describe how you can cut a log so that the pieces shrink in different ways as they dry out.

Performance Task



Name _____ Date _____

Chapter 2 Performance Task (continued)

Precisely Perfect

2. A telescope that can pinpoint an object to within 1 foot is less accurate than a telescope that can pinpoint an object to within 1 inch. Order the telescopes from least accurate (greatest number of arcseconds) to most accurate (least number of arcseconds). If two or more telescopes have the same accuracy, list them in alphabetical order.

3. Which telescope is 10 times more accurate than the Kepler Space Telescope? Explain the calculations you used to find your answer.

4. It is an empirical fact that the Hubble Space Telescope is 10 times more accurate than the 16-inch Telescope that you found in Exercise 3. Do you agree with the statement that the Hubble Space Telescope is 10 times more accurate than the 16-inch Telescope? Justify your answer by using the definition of accuracy.

5. What is the accuracy of the Standard Beginner's Telescope? Explain how you found your answer.

Name _____ Date _____

Chapter 2 Performance Task

Precisely Perfect

Building and carpentry are two areas in which it is very important to carefully measure and calculate. Space exploration is another. What level of accuracy is needed for a telescope? How does this level change depending on what you are viewing?

Different telescopes can help you see many things, such as a leaf's vein in a tree, the surface of the Moon, or deep into space. Greater distances require more accurate telescopes. The accuracy of a telescope is measured in arcseconds, which are units of angular measurement. There are 60 arcminutes in 1 degree, and there are 60 arcseconds in 1 arcminute.

1. Use a natural number to complete each statement.

1 arcminute = _____ degree

1 arcsecond = _____ arcminute

1 arcsecond = _____ degree

In Exercises 2–5, use the table below, which shows the accuracy of seven telescopes.

Telescope	Accuracy (arcseconds)
Hubble Space Telescope	$\frac{7}{1000}$
Kepler Space Telescope	0.5
Keplar Space Telescope	10
Standard 16-inch Telescope	1
SWIFT Space Telescope	240
CoRoT Space Telescope	1
Standard Beginner's Telescope	$\frac{1}{2}$
	$1\frac{52}{100}$

Precisely Perfect

After completing this chapter, you will be able to use the concepts you learned to answer the questions in the *STEAM Video Performance Task*. You will be given the accuracies of seven telescopes. For example:

Accuracy (arcseconds)

Hubble Space Telescope: $\frac{7}{1000}$

Kepler Space Telescope: 10

Standard Beginner's Telescope: $1\frac{52}{100}$

You will be asked to compare the accuracies of the telescopes. Why do different telescopes have different accuracies?

Getting Ready for Chapter 2

Chapter Exploration

1. Work with a partner. Use integer counters to find each product.

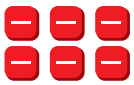
$+$ $= +1$

$-$ $= -1$

a. $(+3) \times (-2)$

$(+3) \times (-2)$

Add 3 groups of -2.

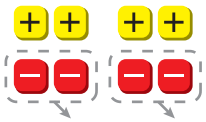


$(+3) \times (-2) =$ ____

b. $(-2) \times (-2)$

$(-2) \times (-2)$

Remove 2 groups of -2.



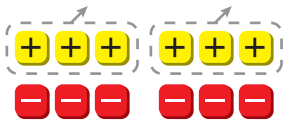
$(-2) \times (-2) =$ ____

Start with enough zero pairs so you can remove 2 groups of -2.

c. $(-2) \times (+3)$

$(-2) \times (+3)$

Remove 2 groups of 3.



$(-2) \times (+3) =$ ____

Start with enough zero pairs so you can remove 2 groups of 3.

Work with a partner. Use integer counters to find the product.

2. $(+3) \times (+2)$ **3.** $(+3) \times (-1)$ **4.** $(+2) \times (-4)$

5. $(-3) \times (+2)$ **6.** $(-2) \times (-3)$ **7.** $(-1) \times (-4)$

8. $(-1) \times (-2)$ **9.** $(+3) \times (+1)$ **10.** $(-3) \times (-2)$

11. $(-2) \times (+2)$ **12.** $(-2) \times (+4)$ **13.** $(-4) \times (-2)$

14. MAKE A CONJECTURE Use your results in Exercises 1–13 to determine the sign of each product.

a. negative integer and a positive integer

b. two negative integers

c. two positive integers

Vocabulary

The following vocabulary terms are defined in this chapter. Think about what each term might mean and record your thoughts.

terminating decimal repeating decimal complex fraction

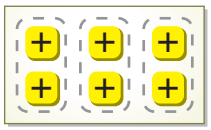
2.1 Multiplying Integers

Learning Target: Find products of integers.

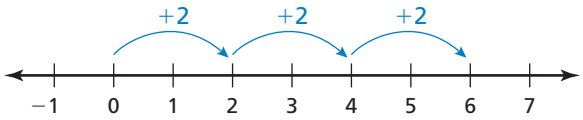
Success Criteria:
- I can explain the rules for multiplying integers.
- I can find products of integers with the same sign.
- I can find products of integers with different signs.

EXPLORATION 1 Understanding Products Involving Negative Integers

Work with a partner.



a. The number line and integer counters model the product $3 \cdot 2$. How can you find $3 \cdot (-2)$? Explain.



b. Use the tables to find $-3 \cdot 2$ and $-3 \cdot (-2)$. Explain your reasoning.

2	•	2	=	4
1	•	2	=	2
0	•	2	=	0
−1	•	2	=	
−2	•	2	=	
−3	•	2	=	

−3	•	3	=	−9
−3	•	2	=	−6
−3	•	1	=	−3
−3	•	0	=	
−3	•	−1	=	
−3	•	−2	=	

c. **INDUCTIVE REASONING** Complete the table. Then write general rules for multiplying (i) two integers with the same sign and (ii) two integers with different signs.

Expression	Type of Product	Product	Product: Positive or Negative
$3 \cdot 2$	Integers with the same sign		
$3 \cdot (-2)$			
$-3 \cdot 2$			
$-3 \cdot (-2)$			
$6 \cdot 3$			
$2 \cdot (-5)$			
$-6 \cdot 5$			
$-5 \cdot (-3)$			

Math Practice

Construct Arguments

Construct an argument that you can use to convince a friend of the rules you wrote in Exploration 1(c).

2.1 Lesson

Consider the following methods for evaluating $3(-2 + 4)$.

Evaluate in parentheses:	**Use the Distributive Property:**
$3(-2 + 4) = 3(2)$	$3(-2 + 4) = 3(-2) + 3(4)$
$= 6$	$= ? + 12$

For the Distributive Property to be true, $3(-2)$ must equal -6. This leads to the following rules for multiplying integers.

Key Ideas

Multiplying Integers with the Same Sign

Words The product of two integers with the same sign is positive.

Numbers $2 \cdot 3 = 6$ $\qquad -2 \cdot (-3) = 6$

Multiplying Integers with Different Signs

Words The product of two integers with different signs is negative.

Numbers $2 \cdot (-3) = -6$ $\qquad -2 \cdot 3 = -6$

EXAMPLE 1 Multiplying Integers

Find each product.

a. $-5 \cdot (-6)$

The integers have the same sign.

$-5 \cdot (-6) = 30$

The product is positive.

▶ The product is 30.

b. $3(-4)$

The integers have different signs.

$3(-4) = -12$

The product is negative.

▶ The product is -12.

Try It **Find the product.**

1. $5 \cdot 5$ **2.** $-1(-9)$ **3.** $-7 \cdot (-8)$

4. $12 \cdot (-2)$ **5.** $4(-6)$ **6.** $-25(0)$

EXAMPLE 2 Evaluating Expressions

The expression $(-2)^2$ indicates to multiply the number in parentheses, -2, by itself. The expression -2^2, however, indicates to find the opposite of 2^2.

a. Find $(-2)^2$.

$(-2)^2 = (-2) \cdot (-2)$ Write $(-2)^2$ as repeated multiplication.

$= 4$ Multiply.

b. Find -2^2.

$-2^2 = -(2 \cdot 2)$ Write 2^2 as repeated multiplication.

$= -4$ Multiply 2 and 2.

c. Find $-2 \cdot 17 \cdot (-5)$.

$-2 \cdot 17 \cdot (-5) = -2 \cdot (-5) \cdot 17$ Commutative Property of Multiplication

$= 10 \cdot 17$ Multiply -2 and -5.

$= 170$ Multiply 10 and 17.

Remember

Use order of operations when evaluating an expression.

d. Find $-6(-3 + 4) + 6$.

$-6(-3 + 4) + 6 = -6(1) + 6$ Perform operation in parentheses.

$= -6 + 6$ Multiplication Property of 1

$= 0$ Additive Inverse Property

Try It **Evaluate the expression.**

7. $8 \cdot (-15) \cdot 0$ **8.** $24 - 3^3$ **9.** $10 - 7(3 - 5)$

Self-Assessment for Concepts & Skills

Solve each exercise. Then rate your understanding of the success criteria in your journal.

10. WRITING What can you conclude about two integers whose product is (a) positive and (b) negative?

EVALUATING AN EXPRESSION **Evaluate the expression.**

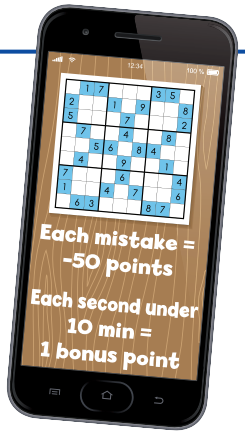
11. $4(-8)$ **12.** $-5(-7)$ **13.** $12 - 3^2 \cdot (-2)$

MP REASONING **Tell whether the statement is *true* or *false*. Explain your reasoning.**

14. The product of three positive integers is positive.

15. The product of three negative integers is positive.

EXAMPLE 3 Modeling Real Life



You solve a number puzzle on your phone. You start with 250 points. You finish the puzzle in 8 minutes 45 seconds and make 3 mistakes. What is your score?

Understand the problem.

You are given ways to gain points and lose points when completing a puzzle. You are asked to find your score after finishing the puzzle.

Make a plan.

Use a verbal model to solve the problem. Find the sum of the starting points, mistake penalties, and time bonus.

Solve and check.

$$\text{Score} = \text{Starting points} + \text{Number of mistakes} \cdot \text{Penalty per mistake} + \text{Time bonus}$$

$$= 250 + 3(-50) + 75$$

10 min − 8 min 45 sec = 1 min 15 sec
= 75 sec

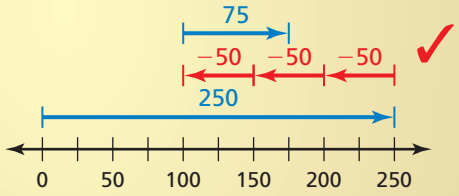
$$= 250 + (-150) + 75$$

$$= 100 + 75$$

$$= 175$$

So, your score is 175 points.

Another Method



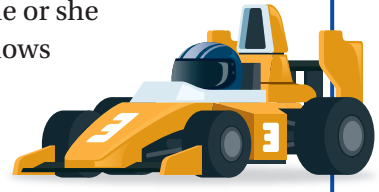
Self-Assessment for Problem Solving

Solve each exercise. Then rate your understanding of the success criteria in your journal.

16. On a mountain, the temperature decreases by 18°F for each 5000-foot increase in elevation. At 7000 feet, the temperature is 41°F. What is the temperature at 22,000 feet? Justify your answer.

17. Players in a racing game earn 3 points for each coin they collect. Each player loses 5 points for each second that he or she finishes after the first-place finisher. The table shows the results of a race. List the players in order from greatest to least number of points.

Player	Coins	Time
1	31	0:02:03
2	18	0:01:55
3	24	0:01:58
4	27	0:02:01



2.1 Practice

Go to **BigIdeasMath.com** to get HELP with solving the exercises.

Review & Refresh

Find the distance between the two numbers on a number line.

1. -4.3 and 0.8 **2.** -7.7 and -6.4 **3.** $-2\frac{3}{5}$ and -1

Divide.

4. $27 \div 9$ **5.** $48 \div 6$ **6.** $56 \div 4$ **7.** $153 \div 8$

8. What is the prime factorization of 84?

A. $2^2 \times 3^2$ **B.** $2^3 \times 7$ **C.** $3^3 \times 7$ **D.** $2^2 \times 3 \times 7$

Concepts, Skills, & Problem Solving

MP CHOOSE TOOLS **Use a number line or integer counters to find the product.**
(See Exploration 1, p. 49.)

9. $2(-4)$ **10.** $-6(3)$ **11.** $4(-5)$

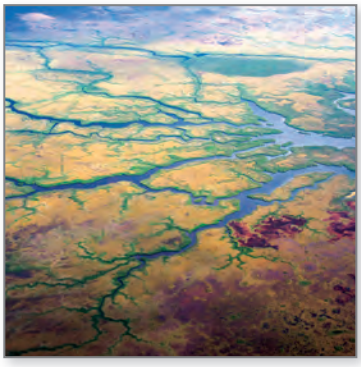
MULTIPLYING INTEGERS **Find the product.**

12. $6 \cdot 4$ **13.** $7(-3)$ **14.** $-2(8)$ **15.** $-3(-4)$

16. $-6 \cdot 7$ **17.** $3 \cdot 9$ **18.** $8 \cdot (-5)$ **19.** $-1 \cdot (-12)$

20. $-5(10)$ **21.** $-13(0)$ **22.** $-9 \cdot 9$ **23.** $15(-2)$

24. $-10 \cdot 11$ **25.** $-6 \cdot (-13)$ **26.** $7(-14)$ **27.** $-11 \cdot (-11)$



28. MODELING REAL LIFE You burn 10 calories each minute you jog. What integer represents the change in your calories after you jog for 20 minutes?

29. MODELING REAL LIFE In a four-year period, about 80,000 acres of coastal wetlands in the United States are lost each year. What integer represents the total change in coastal wetlands?

EVALUATING EXPRESSIONS **Evaluate the expression.**

30. $(-4)^2$ **31.** -6^2 **32.** $-5 \cdot 3 \cdot (-2)$

33. $3 \cdot (-12) \cdot 0$ **34.** $-5(-7)(-20)$ **35.** $5 - 8^2$

36. $-5^2 \cdot 4$ **37.** $-2 \cdot (-3)^3$ **38.** $2 + 1 \cdot (-7 + 5)$

39. $4 - (-2)^3$ **40.** $4 \cdot (25 \cdot 3^2)$ **41.** $-4(3^2 - 8) + 1$

YOU BE THE TEACHER **Your friend evaluates the expression. Is your friend correct? Explain your reasoning.**

42. $-2(-7) = -14$

43. $-10^2 = -100$

MP PATTERNS **Find the next two numbers in the pattern.**

44. $-12, 60, -300, 1500, \ldots$

45. $7, -28, 112, -448, \ldots$

46. **MP PROBLEM SOLVING** In a scavenger hunt, each team earns 25 points for each item that they find. Each team loses 15 points for every minute after 4:00 P.M. that they report to the city park. The table shows the number of items found by each team and the time that each team reported to the park. Which team wins the scavenger hunt? Justify your answer.

Team	Items	Time
A	13	4:03 P.M.
B	15	4:07 P.M.
C	11	3:56 P.M.
D	12	4:01 P.M.

47. **MP REASONING** The height of an airplane during a landing is given by $22{,}000 + (-480t)$, where t is the time in minutes. Estimate how many minutes it takes the plane to land. Explain your reasoning.

48. **MP PROBLEM SOLVING** The table shows the price of a bluetooth speaker each month for 4 months.

Month	Price (dollars)
June	165
July	$165 + (-12)$
August	$165 + 2(-12)$
September	$165 + 3(-12)$

a. Describe the change in the price of the speaker.

b. The table at the right shows the amount of money you save each month. When do you have enough money saved to buy the speaker? Explain your reasoning.

Amount Saved	
June	$35
July	$55
August	$45
September	$18

49. **DIG DEEPER!** Two integers, a and b, have a product of 24. What is the least possible sum of a and b?

50. **MP NUMBER SENSE** Consider two integers p and q. Explain why $p \times (-q) = (-p) \times q = -pq$.

2.2 Dividing Integers

Learning Target: Find quotients of integers.

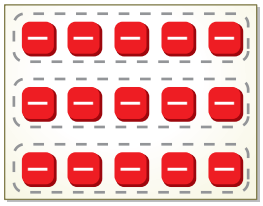
Success Criteria:
- I can explain the rules for dividing integers.
- I can find quotients of integers with the same sign.
- I can find quotients of integers with different signs.

EXPLORATION 1 Understanding Quotients Involving Negative Integers

Work with a partner.

a. Discuss the relationship between multiplication and division with your partner.

b. **INDUCTIVE REASONING** Complete the table. Then write general rules for dividing (i) two integers with the same sign and (ii) two integers with different signs.



Expression	Type of Quotient	Quotient	Quotient: Positive, Negative, or Zero
$-15 \div 3$	Integers with different signs		
$12 \div (-6)$			
$10 \div (-2)$			
$-6 \div 2$			
$-12 \div (-12)$			
$-21 \div (-7)$			
$0 \div (-15)$			
$0 \div 4$			
$-5 \div 4$			
$5 \div (-4)$			

Math Practice

Recognize Usefulness of Tools

Can you use number lines or integer counters to reach the same conclusions as in part (b)? Explain why or why not.

c. Find the values of $-\frac{8}{4}$, $\frac{-8}{4}$, and $\frac{8}{-4}$. What do you notice? Is this true for $-\frac{a}{b}$, $\frac{-a}{b}$, and $\frac{a}{-b}$ when a and b are integers? Explain.

d. Is every quotient of integers a rational number? Explain your reasoning.

2.2 Lesson

Key Ideas

Remember
Division by 0 is undefined.

Dividing Integers with the Same Sign

Words The quotient of two integers with the same sign is positive.

Numbers $8 \div 2 = 4$ $\quad -8 \div (-2) = 4$

Dividing Integers with Different Signs

Words The quotient of two integers with different signs is negative.

Numbers $8 \div (-2) = -4$ $\quad -8 \div 2 = -4$

EXAMPLE 1 Dividing Integers with the Same Sign

Find $-18 \div (-6)$.

The integers have the same sign.

$-18 \div (-6) = 3$

The quotient is positive.

The quotient is 3.

Try It **Find the quotient.**

1. $14 \div 2$ **2.** $-32 \div (-4)$ **3.** $-40 \div (-8)$

EXAMPLE 2 Dividing Integers with Different Signs

Find each quotient.

a. $75 \div (-25)$ **b.** $\frac{-54}{6}$

If a and b are integers, then $-\frac{a}{b} = \frac{-a}{b} = \frac{a}{-b}$. So, you can also think of $\frac{-54}{6}$ as $-\frac{54}{6} = -9$.

The integers have different signs.

$75 \div (-25) = -3$ $\quad \frac{-54}{6} = -9$

The quotient is negative.

The quotient is -3. The quotient is -9.

Try It **Find the quotient.**

4. $0 \div (-6)$ **5.** $\frac{-49}{7}$ **6.** $\frac{21}{-3}$

EXAMPLE 3 Evaluating Expressions

Find the value of each expression when $x = 8$ and $y = -4$.

a. $\dfrac{x}{2y}$

$$\frac{x}{2y} = \frac{8}{2(-4)}$$ Substitute 8 for x and -4 for y.

$$= \frac{8}{-8}$$ Multiply 2 and -4.

$$= -1$$ Divide 8 by -8.

▶ The value of the expression is -1.

b. $-x^2 + 12 \div y$

$$-x^2 + 12 \div y = -8^2 + 12 \div (-4)$$ Substitute 8 for x and -4 for y.

$$= -(8 \cdot 8) + 12 \div (-4)$$ Write 8^2 as repeated multiplication.

$$= -64 + 12 \div (-4)$$ Multiply 8 and 8.

$$= -64 + (-3)$$ Divide 12 by -4.

$$= -67$$ Add.

▶ The value of the expression is -67.

Try It **Evaluate the expression when $a = -18$ and $b = -6$.**

7. $a \div b$ **8.** $\dfrac{a + 6}{3}$ **9.** $\dfrac{b^2}{a} + 4$

Self-Assessment for Concepts & Skills

Solve each exercise. Then rate your understanding of the success criteria in your journal.

10. WRITING What can you conclude about two integers whose quotient is (a) positive, (b) negative, or (c) zero?

DIVIDING INTEGERS **Find the quotient.**

11. $-12 \div 4$ **12.** $\dfrac{-6}{-2}$ **13.** $15 \div (-3)$

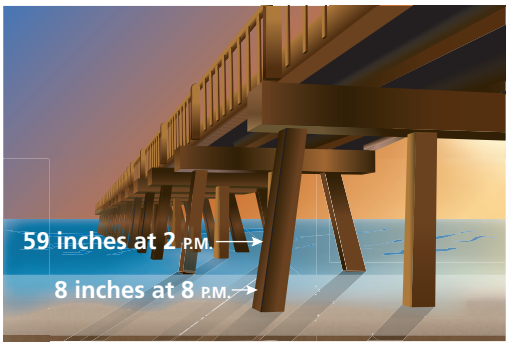
14. WHICH ONE DOESN'T BELONG? Which expression does *not* belong with the other three? Explain your reasoning.

$\dfrac{10}{-5}$ $\dfrac{-10}{5}$ $\dfrac{-10}{-5}$ $-\dfrac{10}{5}$

EXAMPLE 4 Modeling Real Life

You measure the height of the tide using the support beams of a pier. What is the mean hourly change in the height?

To find the mean hourly change in the height of the tide, divide the change in the height by the elapsed time.



$$\text{mean hourly change} = \frac{\text{final height} - \text{initial height}}{\text{elapsed time}}$$

The elapsed time from 2 P.M. to 8 P.M. is 6 hours.

$$= \frac{8 - 59}{6} \quad \text{Substitute.}$$

$$= \frac{-51}{6} \quad \text{Subtract.}$$

$$= -8\frac{1}{2} \quad \text{Divide.}$$

The mean change in the height of the tide is $-8\frac{1}{2}$ inches per hour.

Self-Assessment for Problem Solving

Solve each exercise. Then rate your understanding of the success criteria in your journal.

15. A female grizzly bear weighs 500 pounds. After hibernating for 6 months, she weighs only 350 pounds. What is the mean monthly change in weight?

16. The table shows the change in the number of crimes committed in a city each year for 4 years. What is the mean yearly change in the number of crimes?

Year	2014	2015	2016	2017
Change in Crimes	215	−321	−185	95

17. DIG DEEPER! At a restaurant, when a customer buys 4 pretzels, the fifth pretzel is free. Soft pretzels cost $3.90 each. You order 12 soft pretzels. What is your mean cost per pretzel?

2.2 Practice

Go to **BigIdeasMath.com** to get HELP with solving the exercises.

Review & Refresh

Find the product.

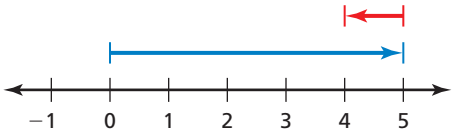
1. $8 \cdot 10$ **2.** $-6(9)$ **3.** $4(7)$ **4.** $-9(-8)$

Order the numbers from least to greatest.

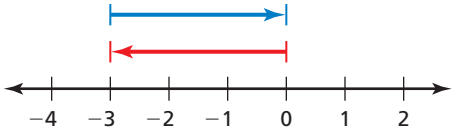
5. $28\%, \frac{1}{4}, 0.24$ **6.** $42\%, 0.45, \frac{2}{5}$ **7.** $\frac{7}{10}, 0.69, 71\%, \frac{9}{10}, 0.84$

Write an addition expression and write a subtraction expression represented by the number line. Then evaluate the expressions.

8.



9.



Concepts, Skills, & Problem Solving

MP CHOOSE TOOLS **Complete the table.** (See Exploration 1, p. 55.)

	Expression	Type of Quotient	Quotient	Quotient: Positive, Negative, or Zero
10.	$14 \div (-2)$			
11.	$-24 \div 12$			
12.	$-55 \div (-5)$			

DIVIDING INTEGERS **Find the quotient, if possible.**

13. $4 \div (-2)$ **14.** $21 \div (-7)$ **15.** $-20 \div 4$ **16.** $-18 \div (-3)$

17. $\frac{-14}{2}$ **18.** $\frac{0}{6}$ **19.** $\frac{-15}{-5}$ **20.** $\frac{54}{-9}$

21. $-\frac{33}{11}$ **22.** $-49 \div (-7)$ **23.** $0 \div (-2)$ **24.** $\frac{60}{-6}$

25. $\frac{-56}{14}$ **26.** $\frac{18}{0}$ **27.** $-\frac{65}{5}$ **28.** $\frac{-84}{-7}$

YOU BE THE TEACHER **Your friend finds the quotient. Is your friend correct? Explain your reasoning.**

29.

$\frac{-63}{-9} = -7$

30.

$0 \div (-5) = -5$

31. MODELING REAL LIFE You read 105 pages of a novel over 7 days. What is the mean number of pages you read each day?

USING ORDER OF OPERATIONS **Evaluate the expression.**

32. $-8 - 14 \div 2 + 5$

33. $24 \div (-4) + (-2) \cdot (-5)$

EVALUATING EXPRESSIONS **Evaluate the expression when $x = 10$, $y = -2$, and $z = -5$.**

34. $x \div y$

35. $12 \div 3y$

36. $\frac{2z}{y}$

37. $\frac{-x + y}{6}$

38. $100 \div (-z^2)$

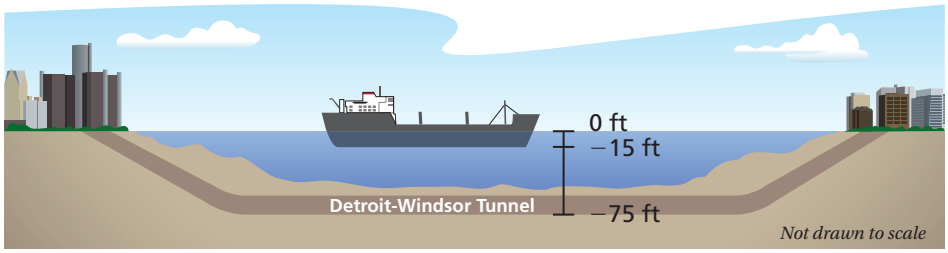
39. $\frac{10y^2}{z}$

40. $\left|\frac{xz}{-y}\right|$

41. $\frac{-x^2 + 6z}{y}$

42. MP PATTERNS Find the next two numbers in the pattern $-128, 64, -32, 16, \ldots$. Explain your reasoning.

43. MODELING REAL LIFE The Detroit-Windsor Tunnel is an underwater highway that connects the cities of Detroit, Michigan, and Windsor, Ontario. How many times deeper is the roadway than the bottom of the ship?



44. MODELING REAL LIFE A snowboarder descends from an elevation of 2253 feet to an elevation of 1011 feet in 3 minutes. What is the mean change in elevation per minute?

45. MP REASONING The table shows a golfer's scores relative to *par* for three out of four rounds of a tournament.

Scorecard	
Round 1	+1
Round 2	−4
Round 3	−3
Round 4	?

a. What was the golfer's mean score per round for the first 3 rounds?

b. The golfer's goal for the tournament is to have a mean score no greater than -3. Describe how the golfer can achieve this goal.

46. MP PROBLEM SOLVING The regular admission price for an amusement park is \$72. For a group of 15 or more, the admission price is reduced by \$25 per person. How many people need to be in a group to save \$500?

47. DIG DEEPER! Write a set of five different integers that has a mean of -10. Explain how you found your answer.

2.3 Converting Between Fractions and Decimals

Learning Target: Convert between different forms of rational numbers.

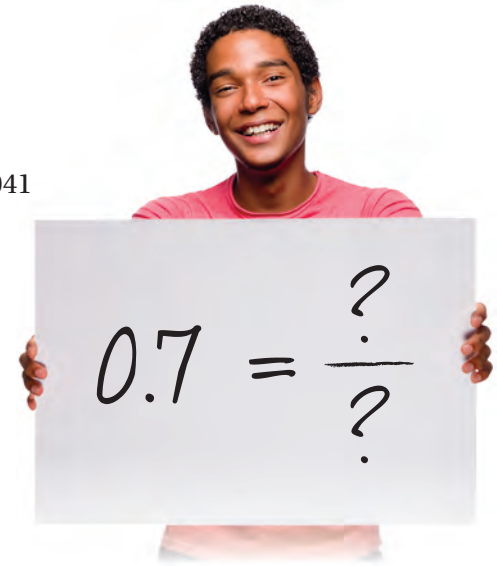
Success Criteria:
- I can explain the difference between terminating and repeating decimals.
- I can write fractions and mixed numbers as decimals.
- I can write decimals as fractions and mixed numbers.

EXPLORATION 1 Analyzing Denominators of Decimal Fractions

Work with a partner.

a. Write each decimal as a fraction or mixed number.

0.7 1.29 12.831 0.0041



b. What do the factors of the denominators of the fractions you wrote have in common? Is this always true for decimal fractions?

EXPLORATION 2 Exploring Decimal Representations

Work with a partner.

a. A fraction $\frac{a}{b}$ can be interpreted as $a \div b$. Use a calculator to convert each unit fraction to a decimal. Do some of the decimals look different than the others? Explain.

$\frac{1}{2}$ $\frac{1}{3}$ $\frac{1}{4}$ $\frac{1}{5}$

$\frac{1}{6}$ $\frac{1}{7}$ $\frac{1}{8}$

$\frac{1}{9}$ $\frac{1}{10}$ $\frac{1}{11}$ $\frac{1}{12}$

Math Practice

Use Technology to Explore

How do calculators help you learn about different types of decimals? How can you find decimal forms of fractions without using a calculator?

b. Compare and contrast the fractions in part (a) with the fractions you wrote in Exploration 1. What conclusions can you make?

c. Does every fraction have a decimal form that either *terminates* or *repeats*? Explain your reasoning.

2.3 Lesson

Key Vocabulary
terminating decimal, *p. 62*
repeating decimal, *p. 62*

Because you can divide any integer by any nonzero integer, you can use long division to write fractions and mixed numbers as decimals. These decimals are rational numbers and will either *terminate* or *repeat*.

A **terminating decimal** is a decimal that ends.

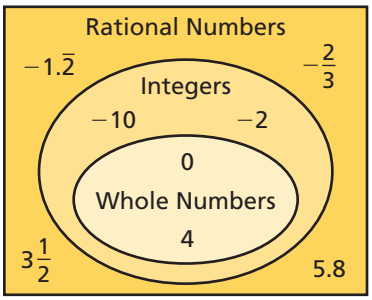
$1.5, -0.25, 10.824$

A **repeating decimal** is a decimal that has a pattern that repeats.

$-1.333\ldots = -1.\overline{3}$

$0.151515\ldots = 0.\overline{15}$

Use *bar notation* to show which of the digits repeat.



EXAMPLE 1 Writing Fractions and Mixed Numbers as Decimals

a. Write $-2\frac{1}{4}$ as a decimal.

Notice that $-2\frac{1}{4} = -\frac{9}{4}$.

Use long division to divide 9 by 4.

Divide 9 by 4.

$$4\overline{)9.00}\quad 2.25$$
$$-\ 8$$
$$1\ 0$$
$$-\ 8$$
$$20$$
$$-\ 20$$
$$0$$

The remainder is 0. So, it is a terminating decimal.

So, $-2\frac{1}{4} = -2.25$.

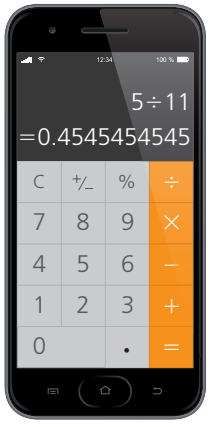
Another Method
Use equivalent fractions.

$$\frac{1}{4} = \frac{1 \times 25}{4 \times 25} = \frac{25}{100}$$

So, $-2\frac{1}{4} = -2\frac{25}{100}$

$= -2.25$. ✓

b. Write $\frac{5}{11}$ as a decimal.



Use long division to divide 5 by 11.

Divide 5 by 11.

$$11\overline{)5.0000}\quad 0.4545$$
$$-\ 44$$
$$60$$
$$-\ 55$$
$$50$$
$$-\ 44$$
$$60$$
$$-\ 55$$
$$5$$

The remainder repeats. So, it is a repeating decimal.

So, $\frac{5}{11} = 0.\overline{45}$.

Try It **Write the fraction or mixed number as a decimal.**

1. $-\frac{6}{5}$
2. $-7\frac{3}{8}$
3. $-\frac{3}{11}$
4. $1\frac{5}{27}$

This shows two representations for simplifying fractions. Use whichever you prefer.

Any terminating decimal can be written as a fraction whose denominator is a power of 10. You can often simplify the resulting fraction by *dividing out* any common factors, which is the same as removing the common factor from the numerator and denominator.

$$0.48 = \frac{48}{100} = \frac{48 \div 4}{100 \div 4} = \frac{12}{25} \quad \text{or} \quad 0.48 = \frac{48}{100} = \frac{12 \cdot \cancel{4}}{25 \cdot \cancel{4}} = \frac{12}{25}$$

EXAMPLE 2 Writing a Terminating Decimal as a Fraction

Write -0.26 as a fraction in simplest form.

$$-0.26 = -\frac{26}{100}$$

Write the digits after the decimal point in the numerator.

The last digit is in the hundredths place. So, use 100 in the denominator.

$$= -\frac{13 \cdot \cancel{2}}{50 \cdot \cancel{2}}$$

Divide out the common factor, 2.

$$= -\frac{13}{50}$$

Simplify.

So, $-0.26 = -\frac{13}{50}$.

Reading

-0.26 is read as "negative twenty-six hundredths."

Try It **Write the decimal as a fraction or mixed number in simplest form.**

5. -0.3 **6.** 0.125 **7.** -3.1 **8.** -10.25

Self-Assessment for Concepts & Skills

Solve each exercise. Then rate your understanding of the success criteria in your journal.

9. WRITING Compare and contrast terminating decimals and repeating decimals.

WRITING A FRACTION OR MIXED NUMBER AS A DECIMAL **Write the fraction or mixed number as a decimal.**

10. $\frac{3}{16}$ **11.** $-\frac{7}{15}$ **12.** $6\frac{17}{20}$

WRITING A DECIMAL AS A FRACTION OR MIXED NUMBER **Write the decimal as a fraction or mixed number in simplest form.**

13. 0.6 **14.** -12.48 **15.** 0.408

EXAMPLE 3 Modeling Real Life

Creature	Elevation (kilometers)
Anglerfish	$-\frac{13}{10}$
Shark	$-\frac{2}{11}$
Squid	$-2\frac{1}{5}$
Whale	-0.8

The table shows the elevations of four sea creatures relative to sea level. Which of the sea creatures are deeper than the whale? Explain.

One way to compare the depths of the creatures is to use a number line. First, write each fraction or mixed number as a decimal.

$$-\frac{13}{10} = -1.3$$

$$-\frac{2}{11} = -0.\overline{18}$$

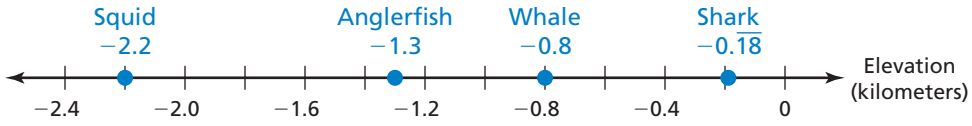
$$-2\frac{1}{5} = -2\frac{2}{10} = -2.2$$

Divide 2 by 11.

$$\begin{array}{r} 0.1818 \\ 11\overline{)2.0000} \\ -\ 11 \\ \hline 90 \\ -\ 88 \\ \hline 20 \\ -\ 11 \\ \hline 90 \\ -\ 88 \\ \hline 2 \end{array}$$

The remainder repeats. So, it is a repeating decimal.

Then graph each decimal on a number line.



Both -2.2 and -1.3 are less than -0.8. So, the squid and the anglerfish are deeper than the whale.

Self-Assessment for Problem Solving

Solve each exercise. Then rate your understanding of the success criteria in your journal.

16. A box turtle hibernates in sand at an elevation of -1.625 feet. A spotted turtle hibernates at an elevation of $-1\frac{7}{12}$ feet. Which turtle hibernates deeper in the sand? How much deeper?

Elevation (miles)	
50.6	$50\frac{13}{25}$
$50\frac{8}{15}$	$\frac{155}{3}$

17. A *red sprite* is an electrical flash that occurs in Earth's upper atmosphere. The table shows the elevations of four red sprites. What is the range of the elevations?

2.3 Practice

Go to **BigIdeasMath.com** to get HELP with solving the exercises.

Review & Refresh

Find the quotient.

1. $12 \div (-6)$ **2.** $-48 \div 8$ **3.** $-42 \div (-7)$ **4.** $-33 \div (-3)$

Find the product.

5. $\begin{array}{r} 5.88 \\ \times \quad 6 \\ \hline \end{array}$ **6.** $2.0035 \cdot 4$ **7.** 5.49×13.509 **8.** $\begin{array}{r} 1.0006 \\ \times \ 0.003 \\ \hline \end{array}$

9. Find the missing values in the ratio table. Then write the equivalent ratios.

Hours	2		$\frac{4}{3}$
Dollars Earned	18	72	

Concepts, Skills, & Problem Solving

MP **STRUCTURE** **Without dividing, determine whether the decimal form of the fraction *terminates* or *repeats*. Explain.** (See Explorations 1 & 2, p. 61.)

10. $\frac{3}{8}$ **11.** $\frac{5}{7}$ **12.** $\frac{11}{40}$ **13.** $\frac{5}{24}$

WRITING A FRACTION OR MIXED NUMBER AS A DECIMAL **Write the fraction or mixed number as a decimal.**

14. $\frac{7}{8}$ **15.** $\frac{1}{11}$ **16.** $-3\frac{1}{2}$ **17.** $-\frac{7}{9}$

18. $-\frac{17}{40}$ **19.** $1\frac{5}{6}$ **20.** $4\frac{2}{15}$ **21.** $\frac{25}{24}$

22. $-\frac{13}{11}$ **23.** $-2\frac{17}{18}$ **24.** $-5\frac{7}{12}$ **25.** $8\frac{15}{22}$

26. **YOU BE THE TEACHER** Your friend writes $-\frac{7}{11}$ as a decimal. Is your friend correct? Explain your reasoning.

$-\frac{7}{11} = -0.63$

WRITING A DECIMAL AS A FRACTION OR MIXED NUMBER **Write the decimal as a fraction or mixed number in simplest form.**

27. -0.9 **28.** 0.45 **29.** -0.258 **30.** -0.312

31. -2.32 **32.** -1.64 **33.** 6.012 **34.** -12.405

35. **MODELING REAL LIFE** You find one quarter, two dimes, and two nickels.

a. Write the dollar amount as a decimal.

b. Write the dollar amount as a fraction or mixed number in simplest form.

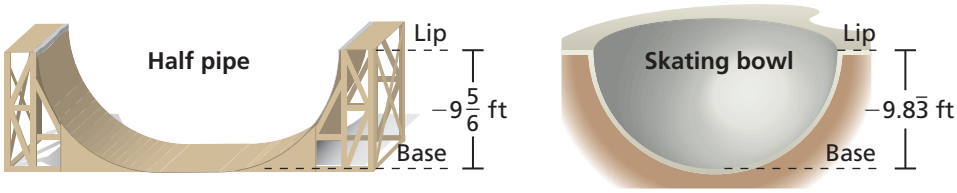
COMPARING RATIONAL NUMBERS **Copy and complete the statement using < or >.**

36. $-4\frac{6}{10}$ ▢ -4.65

37. $-5\frac{3}{11}$ ▢ $-5.\overline{2}$

38. $-2\frac{13}{16}$ ▢ $-2\frac{11}{14}$

39. **MODELING REAL LIFE** Is the half pipe deeper than the skating bowl? Explain.



Player	Hits	At Bats
1	42	90
2	38	80

40. **MODELING REAL LIFE** In softball, a batting average is the number of hits divided by the number of times at bat. Does Player 1 or Player 2 have the greater batting average?

ORDERING RATIONAL NUMBERS **Order the numbers from least to greatest.**

41. $-\frac{3}{4}, 0.5, \frac{2}{3}, -\frac{7}{3}, 1.2$

42. $\frac{9}{5}, -2.5, -1.1, -\frac{4}{5}, 0.8$

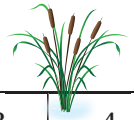
43. $-1.4, -\frac{8}{5}, 0.6, -0.9, \frac{1}{4}$

44. $2.1, -\frac{6}{10}, -\frac{9}{4}, -0.75, \frac{5}{3}$

45. $-\frac{7}{2}, -2.8, -\frac{5}{4}, \frac{4}{3}, 1.3$

46. $-\frac{11}{5}, -2.4, 1.6, \frac{15}{10}, -2.25$

47. **MODELING REAL LIFE** The table shows the changes in the water level of a pond over several weeks. Order the numbers from least to greatest.



Week	1	2	3	4
Change (inches)	$-\frac{7}{5}$	$-1\frac{5}{11}$	-1.45	$-1\frac{91}{200}$

48. **OPEN-ENDED** Find one terminating decimal and one repeating decimal between $-\frac{1}{2}$ and $-\frac{1}{3}$.

49. **MP PROBLEM SOLVING** You miss 3 out of 10 questions on a science quiz and 4 out of 15 questions on a math quiz. On which quiz did you have a greater percentage of correct answers?

50. **CRITICAL THINKING** A hackberry tree has roots that reach a depth of $6\frac{5}{12}$ meters. The top of the tree is $18.2\overline{8}$ meters above the ground. Find the total height from the bottom of the roots to the top of the tree.

51. **DIG DEEPER!** Let a and b be integers.

a. When can $-\frac{1}{a}$ be written as a positive, repeating decimal?

b. When can $\frac{1}{ab}$ be written as a positive, terminating decimal?

2.4 Multiplying Rational Numbers

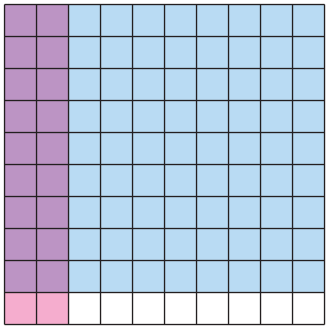
Learning Target: Find products of rational numbers.

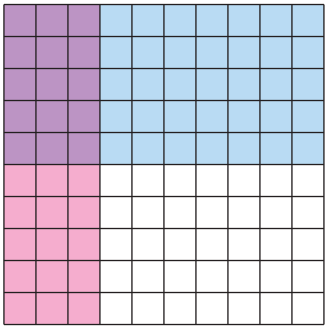
Success Criteria:
- I can explain the rules for multiplying rational numbers.
- I can find products of rational numbers with the same sign.
- I can find products of rational numbers with different signs.

EXPLORATION 1 Finding Products of Rational Numbers

Work with a partner.

a. Write a multiplication expression represented by each area model. Then find the product.





b. Complete the table.

	Expression	Product	Expression	Product
i.	0.2×0.9		-0.2×0.9	
ii.	$0.3(0.5)$		$0.3(-0.5)$	
iii.	$\frac{1}{4} \cdot \frac{1}{2}$		$\frac{1}{4} \cdot \left(-\frac{1}{2}\right)$	
iv.	$1.2(0.4)$		$-1.2(-0.4)$	
v.	$\frac{3}{10}\left(\frac{2}{5}\right)$		$-\frac{3}{10}\left(-\frac{2}{5}\right)$	
vi.	0.6×1.8		-0.6×1.8	
vii.	$1\frac{1}{4} \cdot 2\frac{1}{2}$		$-1\frac{1}{4} \cdot \left(-2\frac{1}{2}\right)$	

c. Do the rules for multiplying integers apply to all rational numbers? Explain your reasoning.

Math Practice

Consider Similar Problems

How is multiplying integers similar to multiplying other rational numbers? How is it different?

2.4 Lesson

When the signs of two numbers are different, their product is negative. When the signs of two numbers are the same, their product is positive.

Key Idea

Multiplying Rational Numbers

Words To multiply rational numbers, use the same rules for signs as you used for multiplying integers.

Numbers $-\frac{2}{7} \cdot \frac{1}{3} = -\frac{2}{21}$ $\qquad -\frac{2}{7} \cdot \left(-\frac{1}{3}\right) = \frac{2}{21}$

EXAMPLE 1 Multiplying Rational Numbers

a. Find -2.5×3.6. **Estimate** $-2.5 \cdot 4 = -10$

Because the decimals have different signs, the product is negative. So, find the opposite of the product of 2.5 and 3.6.

$$\begin{array}{r} 2.5 \quad \leftarrow \text{1 decimal place} \\ \times\ 3.6 \quad \leftarrow +\text{ 1 decimal place} \\ \hline 1\,5\,0 \\ 7\,5\,0 \\ \hline 9.0\,0 \quad \leftarrow \text{2 decimal places} \end{array}$$

So, $-2.5 \times 3.6 = -9$. **Reasonable?** $-9 \approx -10$ ✓

b. Find $-\frac{1}{3}\left(-2\frac{3}{4}\right)$. **Estimate** $-\frac{1}{3} \cdot (-3) = 1$

Because the numbers have the same sign, the product is positive. So, find the product of $\frac{1}{3}$ and $2\frac{3}{4}$.

$$\frac{1}{3}\left(2\frac{3}{4}\right) = \frac{1}{3}\left(\frac{11}{4}\right)$$ Write the mixed number as an improper fraction.

$$= \frac{11}{12}$$ Multiply the numerators and the denominators.

So, $-\frac{1}{3}\left(-2\frac{3}{4}\right) = \frac{11}{12}$. **Reasonable?** $\frac{11}{12} \approx 1$ ✓

Try It **Find the product. Write fractions in simplest form.**

1. -5.1×1.8 **2.** $-6.3(-0.6)$ **3.** $-\frac{4}{5}\left(-\frac{2}{3}\right)$ **4.** $4\frac{1}{2} \cdot \left(-2\frac{1}{3}\right)$

The properties of multiplication you have used apply to all rational numbers. You can also write $-\frac{a}{b}$ as $\frac{-a}{b}$ or $\frac{a}{-b}$ when performing operations with rational numbers.

EXAMPLE 2 Using Properties to Multiply Rational Numbers

Find $\left(-\frac{1}{7} \cdot \frac{4}{5}\right) \cdot (-7) \cdot \left(-\frac{1}{2}\right)$.

You can use properties of multiplication to find the product.

$$\left(-\frac{1}{7} \cdot \frac{4}{5}\right) \cdot (-7) \cdot \left(-\frac{1}{2}\right) = -7 \cdot \left(-\frac{1}{7} \cdot \frac{4}{5}\right) \cdot \left(-\frac{1}{2}\right)$$ Commutative Property of Multiplication

$$= \left[-7 \cdot \left(-\frac{1}{7}\right)\right] \cdot \frac{4}{5} \cdot \left(-\frac{1}{2}\right)$$ Associative Property of Multiplication

$$= 1 \cdot \frac{4}{5} \cdot \left(-\frac{1}{2}\right)$$ Multiplicative Inverse Property

$$= \frac{4}{5} \cdot \left(\frac{-1}{2}\right)$$ Multiplication Property of One

$$= \frac{{}^{2}\cancel{4} \cdot (-1)}{5 \cdot \cancel{2}_{1}}$$ Multiply. Divide out the common factor, 2.

$$= \frac{-2}{5}, \text{ or } -\frac{2}{5}$$ Simplify.

Notice that Example 2 uses different notation to demonstrate the following.

$$\frac{4 \cdot (-1)}{5 \cdot 2} = \frac{2 \cdot {}^{1}\cancel{2} \cdot (-1)}{5 \cdot \cancel{2}_{1}}$$

Try It **Find the product. Write fractions in simplest form.**

5. $-\frac{2}{3} \cdot 7\frac{7}{8} \cdot \frac{3}{2}$

6. $-7.02(0.1)(100)(-10)$

Self-Assessment for Concepts & Skills

Solve each exercise. Then rate your understanding of the success criteria in your journal.

7. WRITING Explain how to determine whether a product of two rational numbers is *positive* or *negative*.

MULTIPLYING RATIONAL NUMBERS **Find the product. Write fractions in simplest form.**

8. $-\frac{3}{10} \times \left(-\frac{8}{15}\right)$

9. $-\frac{2}{3} \cdot 1\frac{1}{3}$

10. $-2.8(-1.7)$

11. $1\frac{3}{5} \cdot \left(-3\frac{3}{4}\right)$

EXAMPLE 3 Modeling Real Life

A school record for the 40-meter dash is 15.24 seconds. Predict the school record after 15 years when the school record decreases by about 0.06 second per year.

Use a verbal model to solve the problem. Because the school record *decreases* by about 0.06 second per year, the change in the school record each year is -0.06 second.

$$\begin{array}{l} \text{School record after 15 years} \\ \text{Current school record} \\ \text{Number of years} \\ \text{Average yearly change} \end{array}$$

$$\begin{aligned} &= 15.24 + 15(-0.06) && \text{Substitute.} \\ &= 15.24 + (-0.9) && \text{Multiply 15 and } -0.06. \\ &= 14.34 && \text{Add 15.24 and } -0.9. \end{aligned}$$

You can predict that the school record will be about 14.34 seconds after 15 years.

Check Reasonableness

Because $0.06 < 0.1$, the school record decreases by less than $0.1 \cdot 15 = 1.5$ seconds. So, the school record is greater than $15.24 - 1.5 = 13.74$ seconds.

Because $14.34 > 13.74$, the answer is reasonable. ✓

Self-Assessment for Problem Solving

Solve each exercise. Then rate your understanding of the success criteria in your journal.

12. A swimmer's best time in an event is 53.87 seconds. On average, his best time decreases by 0.28 second each of the next five times he swims the event. Does he accomplish his goal of swimming the event in less than 52.5 seconds?

13. DIG DEEPER! *Terminal velocity* is the fastest speed that an object can fall through the air. A skydiver reaches a terminal velocity of 120 miles per hour. What is the change in elevation of the skydiver after falling at terminal velocity for 15 seconds? Justify your answer.

2.4 Practice

Go to **BigIdeasMath.com** to get HELP with solving the exercises.

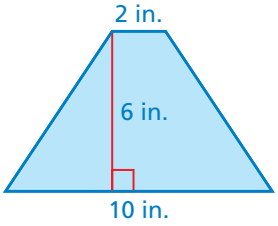
Review & Refresh

Write the fraction or mixed number as a decimal.

1. $\frac{5}{16}$ **2.** $-\frac{9}{22}$ **3.** $6\frac{8}{11}$ **4.** $-\frac{26}{24}$

Find the area of the figure.

5.



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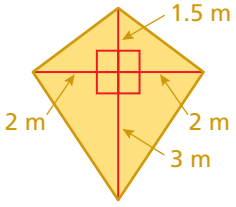

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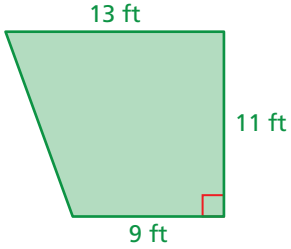
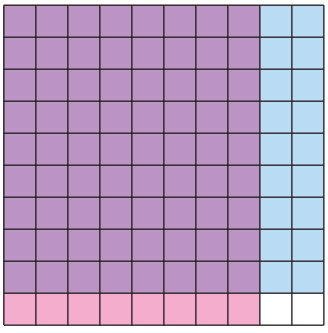


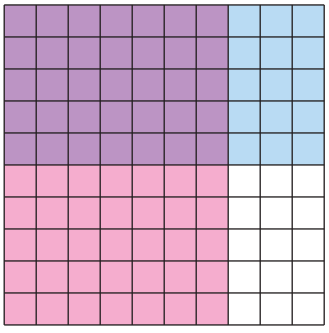
Concepts, Skills, & Problem Solving

FINDING PRODUCTS OF RATIONAL NUMBERS **Write a multiplication expression represented by the area model. Then find the product.** (See Exploration 1, p. 67.)

8.



9.



MP REASONING **Without multiplying, tell whether the value of the expression is positive or negative. Explain your reasoning.**

10. $-1\left(\frac{4}{5}\right)$ **11.** $\frac{4}{7} \cdot \left(-3\frac{1}{2}\right)$ **12.** $-0.25(-3.659)$

MULTIPLYING RATIONAL NUMBERS **Find the product. Write fractions in simplest form.**

13. $-\frac{1}{4} \times \left(-\frac{4}{3}\right)$ **14.** $\frac{5}{6}\left(-\frac{8}{15}\right)$ **15.** $-2\left(-1\frac{1}{4}\right)$

16. $-3\frac{1}{3} \cdot \left(-2\frac{7}{10}\right)$ **17.** $0.4 \times (-0.03)$ **18.** $-0.05 \times (-0.5)$

19. $-8(0.09)(-0.5)$ **20.** $\frac{5}{6} \cdot \left(-4\frac{1}{2}\right) \cdot \left(-2\frac{1}{5}\right)$ **21.** $\left(-1\frac{2}{3}\right)^3$

YOU BE THE TEACHER **Your friend evaluates the expression. Is your friend correct? Explain your reasoning.**

22.

$$-\frac{1}{4} \times \frac{3}{2} = \frac{-1}{4} \times \frac{3}{2}$$
$$= \frac{-3}{8}$$

23.

$$-2.2 \times (-3.7) = -8.14$$

24. **MODELING REAL LIFE** The hour hand of a clock moves $-30°$ every hour. How many degrees does it move in $2\frac{1}{5}$ hours?

25. **MODELING REAL LIFE** A 14.5-gallon gasoline tank is $\frac{3}{4}$ full. How many gallons will it take to fill the tank?

26. **OPEN-ENDED** Write two fractions whose product is $-\frac{3}{5}$.

USING PROPERTIES **Find the product. Write fractions in simplest form.**

27. $\frac{1}{5} \cdot \frac{3}{8} \cdot (-5)$

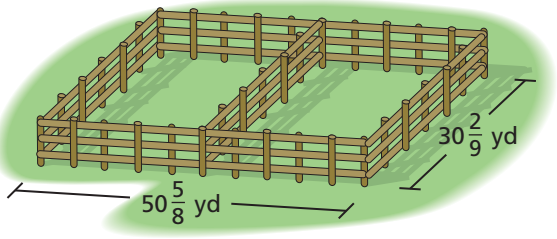
28. $0.01(4.6)(-200)$

29. $(-17.2 \times 2.5) \times 4$

30. $\left(-\frac{5}{9} \times \frac{2}{7}\right) \times \left(-\frac{7}{2}\right)$

31. $\left[-\frac{2}{3} \cdot \left(-\frac{5}{7}\right)\right] \cdot \left(-\frac{9}{4}\right)$

32. $(-4.5 \cdot 8.61) \cdot \left(-\frac{2}{9}\right)$



33. MP **PROBLEM SOLVING** Fencing costs $25.80 per yard. How much does it cost to enclose two adjacent rectangular pastures as shown? Justify your answer.

ALGEBRA **Evaluate the expression when $x = -2$, $y = 3$, and $z = -\frac{1}{5}$.**

34. $x \cdot z$

35. xyz

36. $\frac{1}{3} + x \cdot z$

37. $\frac{1}{2}z - \frac{2}{3}y$

EVALUATING AN EXPRESSION **Evaluate the expression. Write fractions in simplest form.**

38. $-4.2 + 8.1 \times (-1.9)$

39. $-3\frac{3}{4} \times \frac{5}{6} - 2\frac{1}{3}$

40. $\left(-\frac{2}{3}\right)^2 - \frac{3}{4}\left(2\frac{1}{3}\right)$

41. **DIG DEEPER!** Use positive or negative integers to fill in the blanks so that the product is $\frac{1}{4}$. Justify your answer.

$$\frac{\square}{2} \times \left(-\frac{5}{\square}\right) \times \frac{\square}{\square}$$

2.5 Dividing Rational Numbers

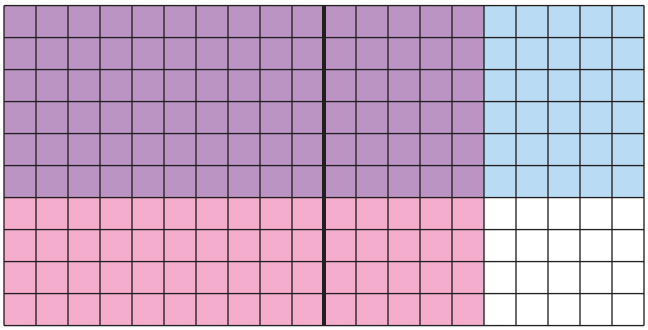
Learning Target: Find quotients of rational numbers.

Success Criteria:
- I can explain the rules for dividing rational numbers.
- I can find quotients of rational numbers with the same sign.
- I can find quotients of rational numbers with different signs.

EXPLORATION 1 Finding Quotients of Rational Numbers

Work with a partner.

a. Write two division expressions represented by the area model. Then find the quotients.



b. Complete the table.

	Expression	Quotient	Expression	Quotient
i.	$0.9 \div 1.5$		$-0.9 \div 1.5$	
ii.	$1 \div \frac{1}{2}$		$-1 \div \frac{1}{2}$	
iii.	$2 \div 0.25$		$2 \div (-0.25)$	
iv.	$0 \div \frac{4}{5}$		$0 \div \left(-\frac{4}{5}\right)$	
v.	$1\frac{1}{2} \div 3$		$-1\frac{1}{2} \div (-3)$	
vi.	$0.8 \div 0.1$		$-0.8 \div (-0.1)$	

c. Do the rules for dividing integers apply to all rational numbers? Explain your reasoning.

d. Write a real-life story involving the quotient $-0.75 \div 3$. Interpret the quotient in the context of the story.

Math Practice

Applying Mathematics

How does interpreting a division expression in a real-life story help you make sense of the quotient?

2.5 Lesson

Key Vocabulary
complex fraction, *p. 75*

Key Idea

Dividing Rational Numbers

Words To divide rational numbers, use the same rules for signs as you used for dividing integers.

Numbers $-\frac{1}{2} \div \frac{4}{9} = -\frac{1}{2} \cdot \frac{9}{4} = -\frac{9}{8}$ $\quad -\frac{1}{2} \div \left(-\frac{4}{9}\right) = -\frac{1}{2} \cdot \left(-\frac{9}{4}\right) = \frac{9}{8}$

EXAMPLE 1 Dividing Rational Numbers

a. Find $-8.4 \div (-3.6)$.

Because the decimals have the same sign, the quotient is positive. Use long division to divide 8.4 by 3.6.

$$3.6\overline{)8.4} \longrightarrow 36\overline{)84.00}$$

$$\begin{array}{r} 2.33 \\ -\ 72 \\ \hline 12\ 0 \\ -\ 10\ 8 \\ \hline 1\ 20 \\ -\ 1\ 08 \\ \hline 12 \end{array}$$

The remainder repeats. So, it is a repeating decimal.

So, $-8.4 \div (-3.6) = 2.\overline{3}$.

Another Method Write the division expression as a fraction.

$$\frac{-8.4}{-3.6} = \frac{84}{36} = \frac{7}{3} = 2\frac{1}{3}, \text{ or } 2.\overline{3}$$ ✓

b. Find $\frac{6}{5} \div \left(-\frac{4}{3}\right)$.

$$\frac{6}{5} \div \left(-\frac{4}{3}\right) = \frac{6}{5} \cdot \left(-\frac{3}{4}\right)$$ Multiply by the reciprocal of $-\frac{4}{3}$.

$$= \frac{{}^{3}\cancel{6} \cdot (-3)}{5 \cdot \cancel{4}_{2}}$$ Multiply the numerators and the denominators. Divide out the common factor, 2.

$$= \frac{-9}{10}, \text{ or } -\frac{9}{10}$$ Simplify.

So, $\frac{6}{5} \div \left(-\frac{4}{3}\right) = -\frac{9}{10}$.

Remember
The *reciprocal* of $\frac{a}{b}$ is $\frac{b}{a}$.

Try It **Find the quotient. Write fractions in simplest form.**

1. $-2.4 \div 3.2$
2. $-6 \div (-1.1)$
3. $-\frac{6}{5} \div \left(-\frac{1}{2}\right)$
4. $-\frac{1}{3} \div 2\frac{2}{3}$

You can represent division involving fractions using *complex fractions*. A **complex fraction** has at least one fraction in the numerator, denominator, or both.

EXAMPLE 2 Evaluating a Complex Fraction

Evaluate $\dfrac{-\frac{10}{9}}{-\frac{1}{6}+1}$.

Rewrite the complex fraction as a division expression.

Notice how $-\frac{1}{6}=\frac{-1}{6}$ is used to find the sum in parentheses.

$$-\frac{10}{9}\div\left(-\frac{1}{6}+1\right)=-\frac{10}{9}\div\left(\frac{-1}{6}+\frac{6}{6}\right)$$ Rewrite $-\frac{1}{6}$ as $\frac{-1}{6}$ and 1 as $\frac{6}{6}$.

$$=-\frac{10}{9}\div\frac{5}{6}$$ Add fractions.

$$=-\frac{10}{9}\cdot\frac{6}{5}$$ Multiply by the reciprocal of $\frac{5}{6}$.

$$=-\frac{\overset{2}{\cancel{10}}\cdot\overset{2}{\cancel{6}}}{\underset{3}{\cancel{9}}\cdot\underset{1}{\cancel{5}}}$$ Multiply. Divide out common factors.

$$=-\frac{4}{3}$$ Simplify.

Try It **Evaluate the expression. Write fractions in simplest form.**

5. $\dfrac{-\frac{1}{2}}{6}$ **6.** $\dfrac{-2\frac{1}{2}}{-\frac{3}{4}}$ **7.** $\dfrac{-1\frac{2}{3}\cdot\left(-\frac{3}{5}\right)}{\left(\frac{1}{3}\right)^2}$

Self-Assessment for Concepts & Skills

Solve each exercise. Then rate your understanding of the success criteria in your journal.

8. WRITING Explain how to determine whether a quotient of two rational numbers is *positive* or *negative*.

EVALUATING AN EXPRESSION **Evaluate the expression. Write fractions in simplest form.**

9. $\frac{3}{8}\div\left(-\frac{9}{5}\right)$ **10.** $-6.8\div(-3.6)$ **11.** $\dfrac{-\frac{2}{9}}{2\frac{2}{5}}$

EXAMPLE 3 Modeling Real Life

A restaurant launches a mobile app that allows customers to rate their food on a scale from −5 to 5. So far, customers have given the lasagna scores of 2.25, −3.5, 0, −4.5, 1.75, −1, 3.5, and −2.5. Should the restaurant consider changing the recipe? Explain.

Understand the problem.

You are given eight scores for lasagna. You are asked to determine whether the restaurant should make changes to the lasagna recipe.

Make a plan.

Use the mean score to determine whether people generally like the lasagna. Then decide whether the recipe should change.

Solve and check.

Divide the sum of the scores by the number of scores. Group together scores that are convenient to add.

$$\text{mean} = \frac{0 + (-3.5 + 3.5) + (2.25 + 1.75) + [(-4.5) + (-2.5) + (-1)]}{8}$$

$$= \frac{0 + 0 + 4 + (-8)}{8}$$

$$= \frac{-4}{8}, \text{ or } -0.5$$

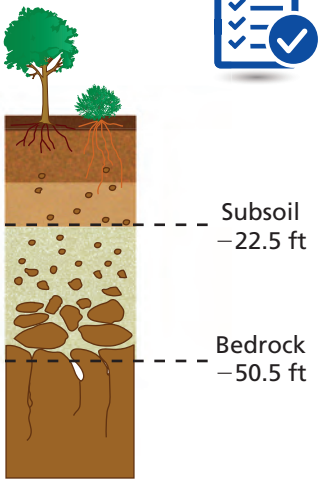
The mean score is below the "mediocre" score of 0.

So, the restaurant should consider changing the recipe.

Look Back
Only 3 of the 8 scores were better than "mediocre." So, it makes sense to conclude that the restaurant should change the recipe. ✓

Self-Assessment for Problem Solving

Solve each exercise. Then rate your understanding of the success criteria in your journal.



12. DIG DEEPER! Soil is composed of several layers. A geologist measures the depths of the *subsoil* and the *bedrock*, as shown. Find and interpret two quotients involving the depths of the subsoil and the bedrock.

13. The restaurant in Example 3 receives additional scores of −0.75, −1.5, −1.25, 4.75, −0.25, −0.5, 5, and −0.5 for the lasagna. Given the additional data, should the restaurant consider changing the recipe? Explain.

2.5 Practice

Go to **BigIdeasMath.com** to get HELP with solving the exercises.

Review & Refresh

Find the product. Write fractions in simplest form.

1. $-0.5(1.31)$

2. $\frac{9}{10}\left(-1\frac{1}{4}\right)$

3. $-\frac{7}{12}\left(-\frac{3}{14}\right)$

Identify the terms, coefficients, and constants in the expression.

4. $3b + 12$

5. $14 + z + 6f$

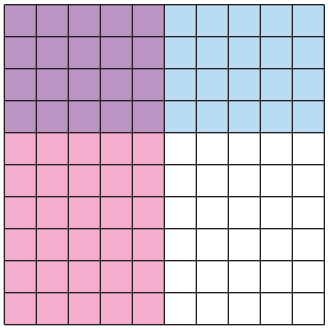
6. $8g + 14 + 5c + 7$

7. $42m + 18 + 12c^2$

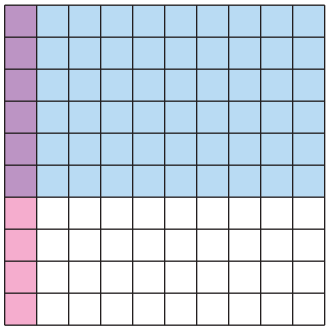
Concepts, Skills, & Problem Solving

USING TOOLS **Write two division expressions represented by the area model. Then find the quotients.** (See Exploration 1, p. 73.)

8.



9.



DIVIDING RATIONAL NUMBERS **Find the quotient. Write fractions in simplest form.**

10. $-\frac{7}{10} \div \frac{2}{5}$

11. $-0.18 \div 0.03$

12. $-3.45 \div (-15)$

13. $-8 \div (-2.2)$

14. $\frac{1}{4} \div \left(-\frac{3}{8}\right)$

15. $8.722 \div (-3.56)$

16. $12.42 \div (-4.8)$

17. $-2\frac{4}{5} \div (-7)$

18. $-10\frac{2}{7} \div \left(-4\frac{4}{11}\right)$

YOU BE THE TEACHER **Your friend evaluates the expression. Is your friend correct? Explain your reasoning.**

19.

$$-\frac{2}{3} \div \frac{4}{5} = \frac{-3}{2} \times \frac{4}{5}$$
$$= \frac{-12}{10}$$
$$= -\frac{6}{5}$$

20.

$$-4.25 \div 1.7 = 2.5$$

21. **MODELING REAL LIFE** How many 0.75-pound packages can you make with 4.5 pounds of sunflower seeds?

EVALUATING AN EXPRESSION **Evaluate the expression. Write fractions in simplest form.**

22. $\dfrac{\frac{14}{9}}{-\frac{1}{3}-\frac{1}{6}}$

23. $\dfrac{-\frac{12}{5}+\frac{3}{10}}{\frac{11}{14}-\left(-\frac{9}{14}\right)}$

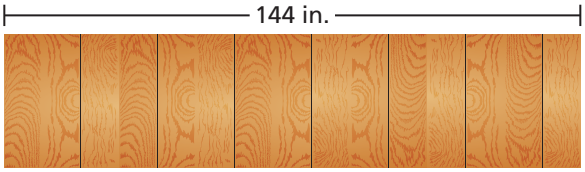
24. $-0.42 \div 0.8 + 0.2$

25. $2.85 - 6.2 \div 2^2$

26. $\frac{3}{4}+\frac{7}{10}-\frac{1}{8} \div \left(-\frac{1}{2}\right)$

27. $\dfrac{\frac{7}{6}}{\left(-\frac{11}{5}\right)\left(10\frac{1}{2}\right)\left(-\frac{5}{11}\right)}$

28. MP **PROBLEM SOLVING** The section of the boardwalk shown is made using boards that are each $9\frac{1}{4}$ inches wide. The spacing between each board is equal. What is the width of the spacing between each board?



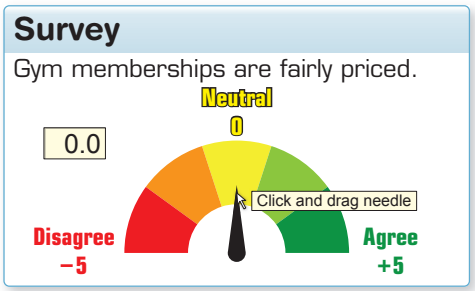
Day	Change in pressure
Monday	−0.05
Tuesday	0.09
Wednesday	−0.04
Thursday	−0.08

29. MP **REASONING** The table shows the daily changes in the barometric pressure (in inches of mercury) for four days.

a. What is the mean change?

b. The mean change for Monday through Friday is −0.01 inch. What is the change in the barometric pressure on Friday? Explain.

30. MP **LOGIC** In an online survey, gym members react to the statement shown by adjusting the position of the needle. The responses have values of −4.2, 1.6, 0.4, 0, 2.1, −5.0, −4.7, 0.6, 1.1, 0.8, 0.4, and 2.1. Explain how two people can use the results of the survey to reach different conclusions about whether the gym should adjust its membership prices.



31. **CRITICAL THINKING** Determine whether the statement is *sometimes*, *always*, or *never* true. Explain your reasoning.

a. The product of two terminating decimals is a terminating decimal.

b. The quotient of two terminating decimals is a terminating decimal.

2 Connecting Concepts

Using the Problem-Solving Plan

1. You feed several adult hamsters equal amounts of a new food recipe over a period of 1 month. You record the changes in the weights of the hamsters in the table. Use the data to answer the question "What is the typical weight change of a hamster that is fed the new recipe?"

Weight Change (ounces)				
−0.07	−0.03	−0.11	−0.04	−0.08
0.02	−0.08	−0.08	−0.06	−0.05
−0.11	−0.1	0	−0.07	−0.08

Understand the problem. You know the weight changes of 15 hamsters. You want to use this information to find the typical weight change.

Make a plan. Display the data in a dot plot to see the distribution of the data. Then use the distribution to determine the most appropriate measure of center.

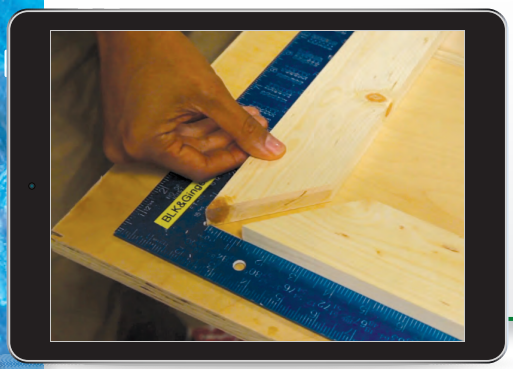
Solve and check. Use the plan to solve the problem. Then check your solution.

2. Evaluate the expression shown at the right. Write your answer in simplest form.

$$\frac{-\frac{1}{2}+\frac{2}{3}}{\frac{3}{5}\left(\frac{3}{4}-\frac{11}{8}\right)}$$

3. You drop a racquetball from a height of 60 inches. On each bounce, the racquetball bounces to a height that is 70% of its previous height. What is the change in the height of the racquetball after 3 bounces?

Performance Task



Precisely Perfect

At the beginning of this chapter, you watched a STEAM Video called "Carpenter or Joiner." You are now ready to complete the performance task related to this video, available at ***BigIdeasMath.com***. Be sure to use the problem-solving plan as you work through the performance task.

2 Chapter Review

Go to *BigIdeasMath.com* to download blank graphic organizers.

Review Vocabulary

Write the definition and give an example of each vocabulary term.

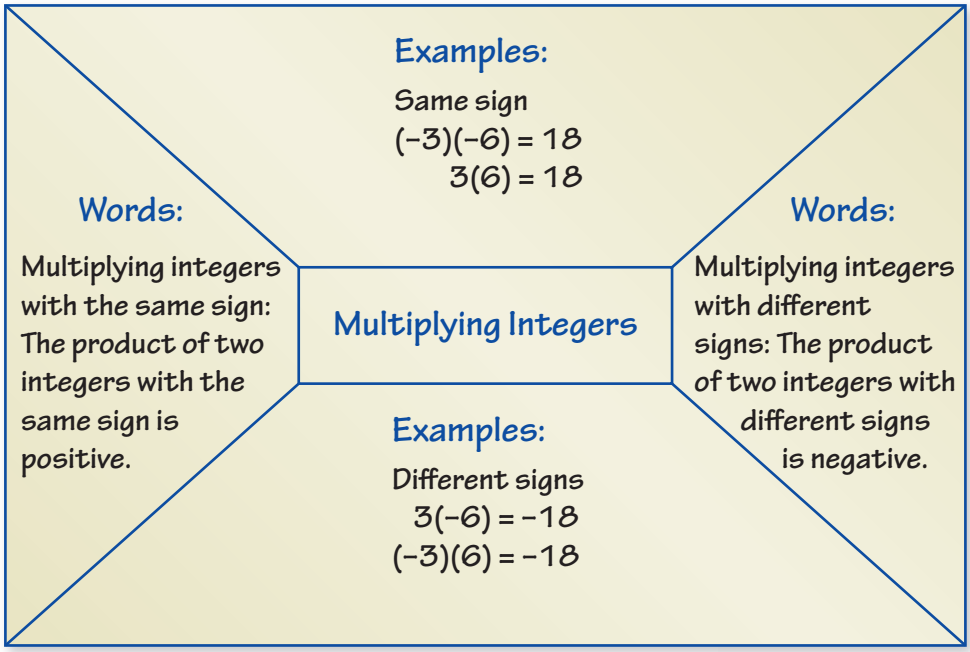
terminating decimal, *p. 62*

repeating decimal, *p. 62*

complex fraction, *p. 75*

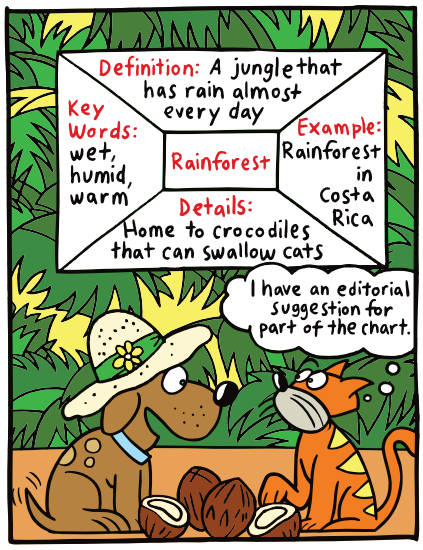
Graphic Organizers

You can use an **Information Frame** to help organize and remember a concept. Here is an example of an Information Frame for ***multiplying integers***.



Choose and complete a graphic organizer to help you study the concept.

1. dividing integers
2. writing fractions or mixed numbers as decimals
3. writing decimals as fractions or mixed numbers
4. multiplying rational numbers
5. dividing rational numbers



"I finished my Information Frame about rainforests. It makes me want to visit Costa Rica. How about you?"

Chapter Self-Assessment

As you complete the exercises, use the scale below to rate your understanding of the success criteria in your journal.

1	2	3	4
I do not understand.	I can do it with help.	I can do it on my own.	I can teach someone else.

2.1 Multiplying Integers *(pp. 49–54)*

Learning Target: Find products of integers.

Find the product.

1. $-8 \cdot 6$

2. $10(-7)$

3. $-3 \cdot (-6)$

4. You and a group of friends participate in a game where you must use clues to escape from a room. You have a limited amount of time to escape and are allowed 3 free clues. Additional clues may be requested, but each removes 5 minutes from your remaining time. What integer represents the total change in the time when you use 5 clues?

Evaluate the expression.

5. $(-3)^3$

6. $(-3)(-4)(10)$

7. $24 - 3(2 - 4^2)$

8. Write three integers whose product is negative.

9. You are playing laser tag. The table shows how many points you gain or lose when you tag or are tagged by another player in different locations. You are tagged three times on the back, twice on the shoulder, and twice on the laser. You tag two players on the front, four players on the back, and one player on the laser. What is your score?

Tag Locations	Points Gained	Points Lost
Front	200	50
Back	100	25
Shoulder	50	12
Laser	50	12

10. The product of three integers is positive. How many of the integers can be negative? Explain.

11. Two integers, c and d, have a product of -6. What is the *greatest* possible sum of c and d?

2.2 Dividing Integers *(pp. 55–60)*

Learning Target: Find quotients of integers.

Find the quotient.

12. $-18 \div 9$ **13.** $\frac{-42}{-6}$ **14.** $\frac{-30}{6}$ **15.** $84 \div (-7)$

Evaluate the expression when $x = 3$, $y = -4$, and $z = -6$.

16. $z \div x$ **17.** $\frac{xy}{z}$ **18.** $\frac{z - 2x}{y}$

Find the mean of the integers.

19. $-3, -8, 12, -15, 9$ **20.** $-54, -32, -70, -25, -65, -42$

21. The table shows the weekly profits of a fruit vendor. What is the mean profit for these weeks?

Week	1	2	3	4
Profit	−$125	−$86	$54	−$35

2.3 Converting Between Fractions and Decimals *(pp. 61–66)*

Learning Target: Convert between different forms of rational numbers.

Write the fraction or mixed number as a decimal.

22. $-\frac{8}{15}$ **23.** $\frac{5}{8}$ **24.** $-\frac{13}{6}$ **25.** $1\frac{7}{16}$

Write the decimal as a fraction or mixed number in simplest form.

26. -0.6 **27.** -0.35 **28.** -5.8 **29.** 24.23

30. The table shows the changes in the average yearly precipitation (in inches) in a city for several months. Order the numbers from least to greatest.

February	March	April	May
-1.75	$\frac{3}{11}$	0.3	$-1\frac{7}{9}$

2.4 Multiplying Rational Numbers *(pp. 67–72)*

Learning Target: Find products of rational numbers.

Find the product. Write fractions in simplest form.

31. $-\frac{4}{9}\left(-\frac{7}{9}\right)$

32. $\frac{8}{15}\left(-\frac{2}{3}\right)$

33. $-5.9(-9.7)$

34. $4.5(-5.26)$

35. $-\frac{2}{3}\left(2\frac{1}{2}\right)(-3)$

36. $-1.6(0.5)(-20)$

37. The elevation of a sunken ship is -120 feet. You are in a submarine at an elevation that is $\frac{5}{8}$ of the ship's elevation. What is your elevation?

38. Write two fractions whose product is between $\frac{1}{5}$ and $\frac{1}{2}$, and whose sum is negative.

2.5 Dividing Rational Numbers *(pp. 73–78)*

Learning Target: Find quotients of rational numbers.

Find the quotient. Write fractions in simplest form.

39. $\frac{9}{10} \div \left(-\frac{6}{5}\right)$

40. $-\frac{4}{11} \div \frac{2}{7}$

41. $-\frac{7}{8} \div \left(-\frac{5}{12}\right)$

42. $6.4 \div (-3.2)$

43. $-15.4 \div (-2.5)$

44. $-23.8 \div 5.6$

45. You use a debit card to purchase several shirts. Your account balance after buying the shirts changes by $-\$30.60$. For each shirt you purchased, the change in your account balance was $-\$6.12$. How many shirts did you buy?

46. Evaluate $\dfrac{z}{y - \frac{3}{4} + x}$ when $x = 4$, $y = -3$, and $z = -\frac{1}{8}$.

2 Practice Test

Evaluate the expression. Write fractions in simplest form.

1. $-9 \cdot 2$

2. $-72 \div (-3)$

3. $3\frac{9}{10} \times \left(-\frac{8}{3}\right)$

4. $-1\frac{5}{6} \div 4\frac{1}{6}$

5. $-4.4 \times (-6.02)$

6. $-5 \div 1.5$

Write the fraction or mixed number as a decimal.

7. $\frac{7}{40}$

8. $-\frac{1}{9}$

9. $-1\frac{5}{16}$

Write the decimal as a fraction or mixed number in simplest form.

10. -0.122

11. 0.33

12. -7.09

Evaluate the expression when $x = 5$, $y = -3$, and $z = -2$.

13. $\frac{y+z}{x}$

14. $\frac{x-5z}{y}$

15. $\frac{\frac{1}{3}x}{\frac{y}{z}}$

16. Find the mean of 11, -7, -14, 10, and -5.

17. A driver receives -25 points for each rule violation. What integer represents the change in points after 4 rule violations?

18. How many 2.25-pound containers can you fill with 24.75 pounds of almonds?

19. In a recent 10-year period, the change in the number of visitors to U.S. national parks was about $-11{,}150{,}000$ visitors.

a. What was the mean yearly change in the number of visitors?

b. During the seventh year, the change in the number of visitors was about 10,800,000. Explain how the change for the 10-year period can be negative.

20. You have a \$50 gift card to go shopping for school supplies. You buy 2 packs of pencils, 5 notebooks, 6 folders, 1 pack of pens, 3 packs of paper, 1 pack of highlighters, and 2 binders.

a. What number represents the change in the value of the gift card after buying your school supplies?

b. What percentage of the value remains on your gift card?

2 Cumulative Practice

1. When José and Sean were each 5 years old, José was $1\frac{1}{2}$ inches taller than Sean. Then José grew at an average rate of $2\frac{3}{4}$ inches per year until he was 13 years old. José was 63 inches tall when he was 13 years old. How tall was Sean when he was 5 years old?

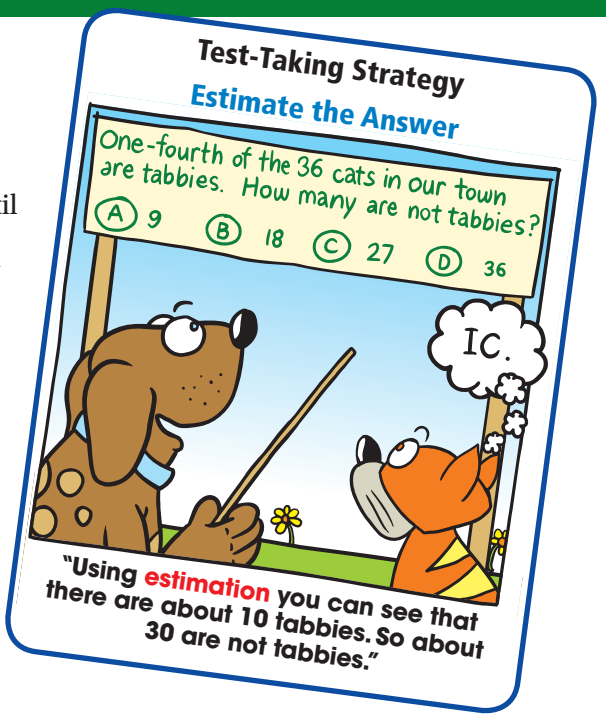
A. $39\frac{1}{2}$ in.

B. $42\frac{1}{2}$ in.

C. $44\frac{3}{4}$ in.

D. $47\frac{3}{4}$ in.

Test-Taking Strategy
Estimate the Answer



2. What is the value of $-5 + (-7)$?

F. -12

G. -2

H. 2

I. 12

3. What is the value of the expression?

$$-\frac{9}{16} + \frac{9}{8}$$

4. What is the value of $\left| a^2 - 2ac + 5b \right|$ when $a = -2$, $b = 3$, and $c = -5$?

A. -9

B. -1

C. 1

D. 9

5. Your friend evaluated the expression.

$$\begin{aligned} 2 - 3 - (-5) &= -5 - (-5) \\ &= -5 + 5 \\ &= 0 \end{aligned}$$

What should your friend do to correct the error that he made?

F. Subtract 5 from -5 instead of adding.

G. Rewrite $2 - 3$ as -1.

H. Subtract -5 from 3 before subtracting 3 from 2.

I. Rewrite $-5 + 5$ as -10.

6. What is the value of $-1\frac{1}{2} - \left(-1\frac{3}{4}\right)$?

A. $-3\frac{1}{4}$ **B.** $\frac{1}{4}$

C. $\frac{6}{7}$ **D.** $2\frac{5}{8}$

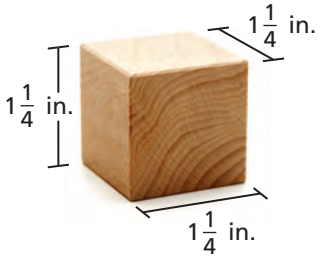
7. What is the value of the expression when $q = -2$, $r = -12$, and $s = 8$?

$$\frac{-q^2 - r}{s}$$

F. -2 **G.** -1

H. 1 **I.** 2

8. You are stacking wooden blocks with the dimensions shown. How many blocks do you need to stack vertically to build a block tower that is $7\frac{1}{2}$ inches tall?



9. Your friend evaluated an expression.

$$\begin{aligned} -4\frac{3}{4} + 2\frac{1}{5} &= -\frac{19}{4} + \frac{11}{5} \\ &= -\frac{95}{20} + \frac{44}{20} \\ &= \frac{-95 + 44}{20} \\ &= \frac{-139}{20} \\ &= -6\frac{19}{20} \end{aligned}$$

What should your friend do to correct the error that she made?

A. Rewrite $-\frac{19}{4} + \frac{11}{5}$ as $\frac{-19 + 11}{4 + 5}$.

B. Rewrite $-95 + 44$ as -51.

C. Rewrite $\frac{-95 + 44}{20}$ as $\frac{51}{20}$.

D. Rewrite $-4\frac{3}{4}$ as $-\frac{13}{4}$.

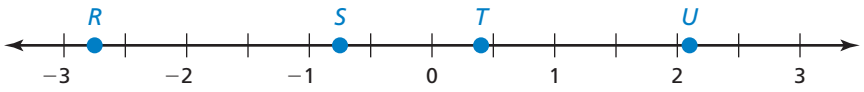
10. Which expression has the greatest value when $x = -2$ and $y = -3$?

F. $-xy$ **G.** xy

H. $x - y$ **I.** $-x - y$

11. Four points are graphed on the number line.

Think Solve Explain



Part A Choose the two points whose values have the greatest sum. Approximate this sum. Explain your reasoning.

Part B Choose the two points whose values have the greatest difference. Approximate this difference. Explain your reasoning.

Part C Choose the two points whose values have the greatest product. Approximate this product. Explain your reasoning.

Part D Choose the two points whose values have the greatest quotient. Approximate this quotient. Explain your reasoning.

12. What number belongs in the box to make the equation true?

$$\frac{-0.4}{\square} + 0.8 = -1.2$$

A. -1 **B.** -0.2

C. 0.2 **D.** 1

13. Which expression has a negative value when $x = -4$ and $y = 2$?

F. $-x + y$ **G.** $y - x$

H. $x - y$ **I.** $-x - y$

14. What is the area of a triangle with a base of $2\frac{1}{2}$ inches and a height of 2 inches?

A. $2\frac{1}{4}\text{ in.}^2$ **B.** $2\frac{1}{2}\text{ in.}^2$

C. $4\frac{1}{2}\text{ in.}^2$ **D.** 5 in.^2

15. Which decimal is equivalent to $\frac{2}{9}$?

F. 0.2 **G.** $0.\overline{2}$

H. 0.29 **I.** 4.5