1 Adding and Subtracting Rational Numbers

1.1 Rational Numbers

1.2 Adding Integers

1.3 Adding Rational Numbers

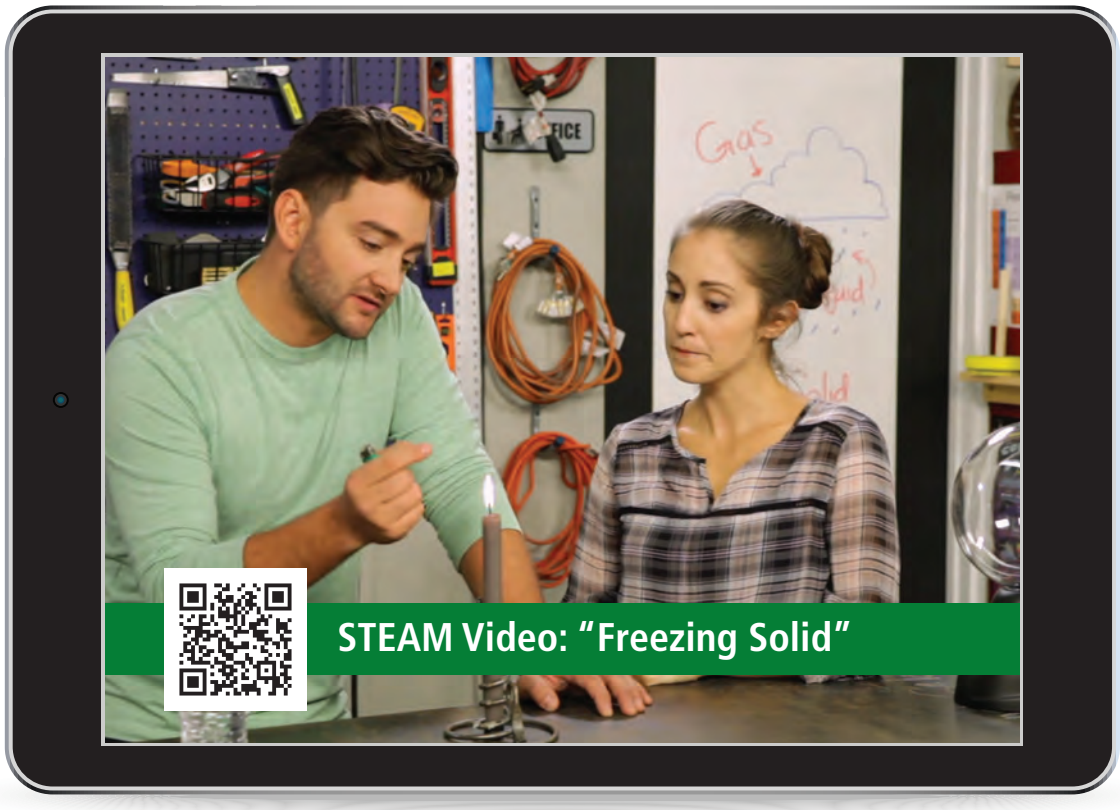
1.4 Subtracting Integers

1.5 Subtracting Rational Numbers

Chapter Learning Target:
Understand adding and subtracting rational numbers.

Chapter Success Criteria:

- I can represent rational numbers on a number line.
- I can explain the rules for adding and subtracting integers using absolute value.
- I can apply addition and subtraction with rational numbers to model real-life problems.
- I can solve problems involving addition and subtraction of rational numbers.



STEAM Video

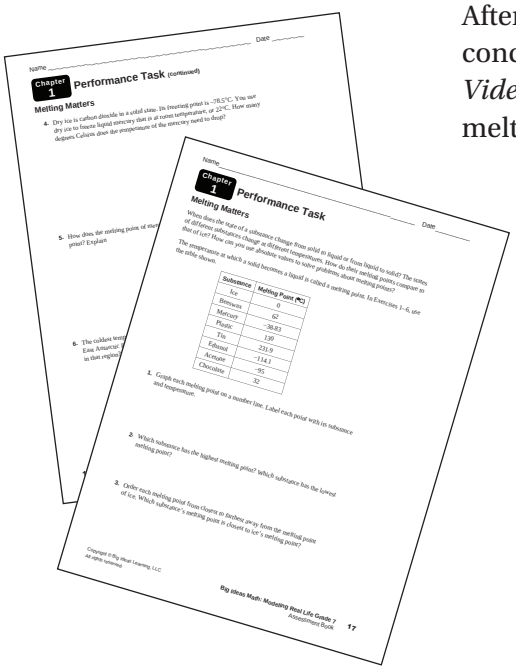
Freezing Solid

The Celsius temperature scale is defined using the freezing point, 0°C, and the boiling point, 100°C, of water. Why do you think the scale is defined using these two points?

Watch the STEAM Video "Freezing Solid." Then answer the following questions.

1. In the video, Tony says that the freezing point of wax is 53°C and the boiling point of wax is 343°C.
   - **a.** Describe the temperature of wax that has just changed from liquid form to solid form. Explain your reasoning.
   - **b.** After Tony blows out the candle, he demonstrates that there is still gas in the smoke. What do you know about the temperature of the gas that is in the smoke?
   - **c.** In what form is wax when the temperature is at 100°C, the boiling point of water?
2. Consider wax in solid, liquid, and gaseous forms. Which is hottest? coldest?

Performance Task


Name _____ Date _____

Chapter 1 Performance Task (continued)

Melting Matters

4. Dry ice is carbon dioxide in a solid state. Its freezing point is −78.5°C. You use dry ice to freeze liquid mercury that is at room temperature, or 22°C. How many degrees Celsius does the temperature of the mercury need to drop?

5. How does the melting point of water compare to the melting point of mercury? Explain.

6. The coldest temperature ever recorded in the United States was −80°F in Prospect Creek, Alaska, on January 23, 1971. The coldest temperature ever recorded in the European Union was −67.6°F in Ust'Shchugor, Russia, on January 31, 1978. Which of these temperatures is colder? Explain your reasoning.

Name _____ Date _____

Chapter 1 Performance Task

Melting Matters

When does the state of a substance change from solid to liquid or from liquid to solid? The states of different substances change at different temperatures. How do their melting points compare to one another? How can you use absolute values to solve problems about melting points?

The temperature at which a solid becomes a liquid is called a melting point. In Exercises 1–6, use the table shown.

Substance	Melting Point (°C)
Ice	0
Beeswax	62
Mercury	−38.83
Plastic	130
Tin	231.9
Ethanol	−114.1
Acetone	−95
Chocolate	32

1. Graph each melting point on a number line. Label each point with its substance and temperature.

2. Which substance has the highest melting point? which substance has the lowest melting point?

3. Order each melting point from closest to zero to farthest away from the melting point of ice. Which substance's melting point is closest to ice's melting point?

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Melting Matters

After completing this chapter, you will be able to use the concepts you learned to answer the questions in the *STEAM Video Performance Task*. You will answer questions using the melting points of the substances below.

Ice	**Tin**
Beeswax	**Ethanol**
Mercury	**Acetone**
Plastic	**Chocolate**

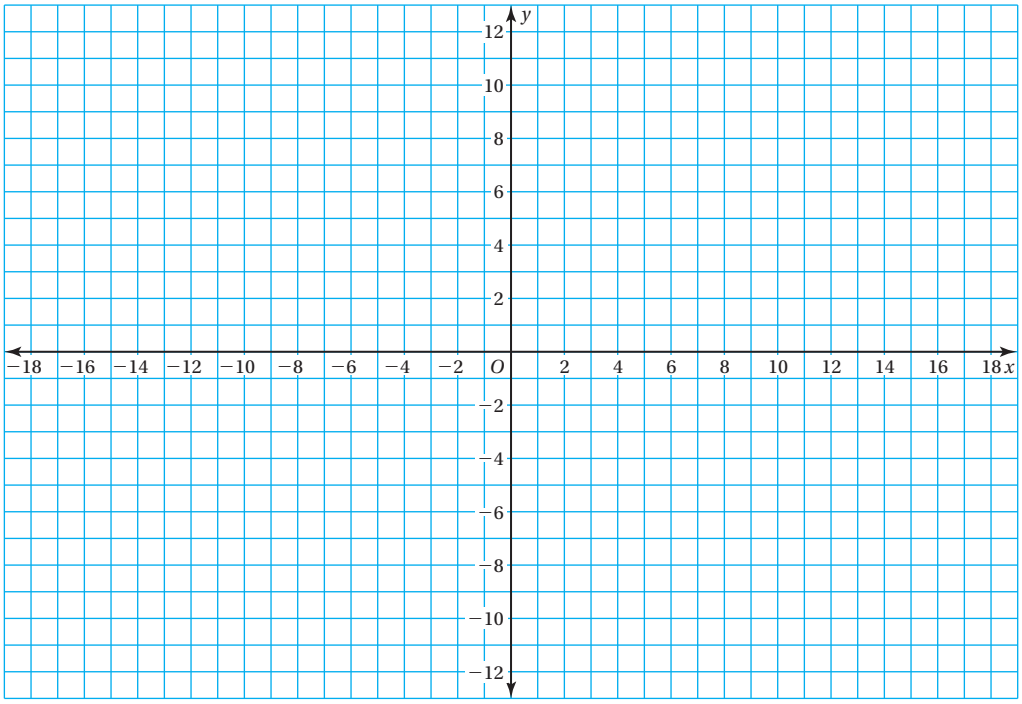
You will graph the melting points of the substances on a number line to make comparisons. How is the freezing point of a substance related to its melting point? What is meant when someone says it is below freezing outside? Explain.

Getting Ready for Chapter 1

Chapter Exploration

1. Work with a partner. Plot and connect the points to make a picture.

1(1, 11) **2**(4, 10) **3**(7, 10) **4**(11, 9) **5**(13, 8)
6(15, 5) **7**(15, 3) **8**(16, 1) **9**(16, −1) **10**(15, −1)
11(11, 1) **12**(9, 2) **13**(7, 1) **14**(5, −1) **15**(1, −1)
16(0, 0) **17**(3, 1) **18**(1, 1) **19**(−2, 0) **20**(−6, −2)
21(−9, −6) **22**(−9, −7) **23**(−7, −9) **24**(−7, −11) **25**(−8, −12)
26(−9, −11) **27**(−11, −10) **28**(−13, −11) **29**(−15, −11) **30**(−17, −12)
31(−17, −10) **32**(−15, −7) **33**(−12, −6) **34**(−11, −6) **35**(−10, −3)
36(−8, 2) **37**(−5, 6) **38**(−3, 9) **39**(−4, 10) **40**(−5, 10)
41(−2, 12)



2. Create your own "dot-to-dot" picture. Use at least 20 points.

Vocabulary

The following vocabulary terms are defined in this chapter. Think about what each term might mean and record your thoughts.

integers absolute value rational number additive inverse

1.1 Rational Numbers

Learning Target: Understand absolute values and ordering of rational numbers.

Success Criteria:
- I can graph rational numbers on a number line.
- I can find the absolute value of a rational number.
- I can use a number line to compare rational numbers.

Recall that **integers** are the set of whole numbers and their opposites. A **rational number** is a number that can be written as $\frac{a}{b}$, where a and b are integers and $b \neq 0$.

EXPLORATION 1 Using a Number Line

Work with a partner. Make a number line on the floor. Include both negative numbers and positive numbers.

a. Stand on an integer. Then have your partner stand on the opposite of the integer. How far are each of you from 0? What do you call the distance between a number and 0 on a number line?

b. Stand on a rational number that is not an integer. Then have your partner stand on any other number. Which number is greater? How do you know?

c. Stand on any number other than 0 on the number line. Can your partner stand on a number that is:

- greater than your number and farther from 0?
- greater than your number and closer to 0?
- less than your number and the same distance from 0?
- less than your number and farther from 0?

For each case in which it was not possible to stand on a number as directed, explain why it is not possible. In each of the other cases, how can you decide where your partner can stand?

Math Practice

Find Entry Points
What are some ways to determine which of two numbers is greater?

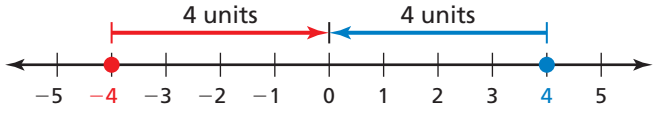
1.1 Lesson

Key Vocabulary
integers, *p. 3*
rational number, *p. 3*
absolute value, *p. 4*

Key Idea

Absolute Value

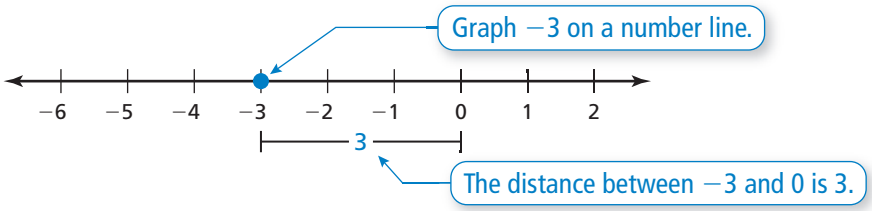
Words The **absolute value** of a number is the distance between the number and 0 on a number line. The absolute value of a number a is written as $|a|$.



Numbers $|-4| = 4$ $|4| = 4$

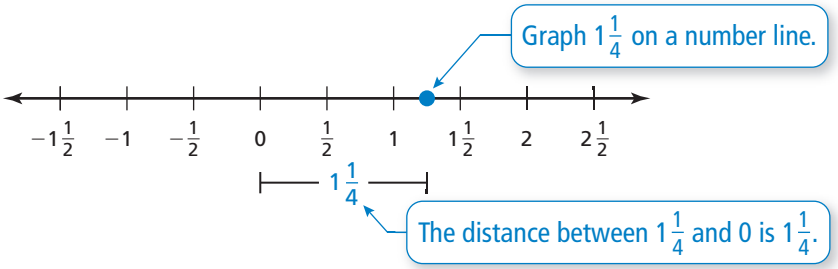
EXAMPLE 1 Finding Absolute Values of Rational Numbers

a. Find the absolute value of -3.



So, $|-3| = 3$.

b. Find the absolute value of $1\frac{1}{4}$.



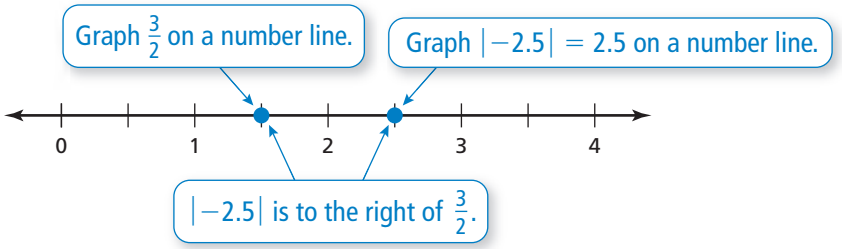
So, $\left|1\frac{1}{4}\right| = 1\frac{1}{4}$.

Try It **Find the absolute value.**

1. $|7|$
2. $\left|-\frac{5}{3}\right|$
3. $|-2.6|$

EXAMPLE 2 Comparing Rational Numbers

Compare $|-2.5|$ and $\frac{3}{2}$.



So, $|-2.5| > \frac{3}{2}$.

Remember

Two numbers that are the same distance from 0 on a number line, but on opposite sides of 0, are called *opposites*. The opposite of a number a is $-a$.

Try It **Copy and complete the statement using <, >, or =.**

4. $|9| \square |-9|$ **5.** $-\left|\frac{1}{2}\right| \square -\frac{1}{4}$ **6.** $7 \square -|-4.5|$

Self-Assessment for Concepts & Skills

Solve each exercise. Then rate your understanding of the success criteria in your journal.

7. VOCABULARY Which of the following numbers are integers?

$$9,\ 3.2,\ -1,\ \frac{1}{2},\ -0.25,\ 15$$

8. VOCABULARY What is the absolute value of a number?

COMPARING RATIONAL NUMBERS **Copy and complete the statement using <, >, or =. Use a number line to justify your answer.**

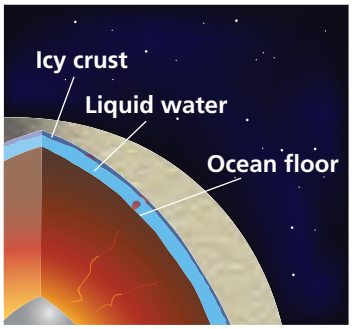
9. $3.5 \square \left|-\frac{7}{2}\right|$ **10.** $\left|\frac{11}{4}\right| \square |-2.8|$

11. WRITING You compare two numbers, a and b. Explain how $a > b$ and $|a| < |b|$ can both be true statements.

12. WHICH ONE DOESN'T BELONG? Which expression does *not* belong with the other three? Explain your reasoning.

$|6|$ $\quad$ 6 $\quad$ -6 $\quad$ $|-6|$

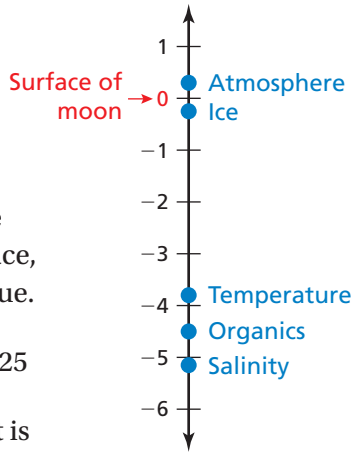
EXAMPLE 3 Modeling Real Life



A moon has an ocean underneath its icy surface. Scientists run tests above and below the surface. The table shows the elevations of each test. Which test is deepest? Which test is closest to the surface?

Test	Temperature	Salinity	Atmosphere	Organics	Ice
Elevation (miles)	−3.8	−5.15	0.3	−4.5	−0.25

To determine which test is deepest, find the least elevation. Graph the elevations on a vertical number line.



The number line shows that the salinity test is deepest. The number line also shows that the atmosphere test and the ice test are closest to the surface. To determine which is closer to the surface, identify which elevation has a lesser absolute value.

Atmosphere: $|0.3| = 0.3$ **Ice:** $|-0.25| = 0.25$

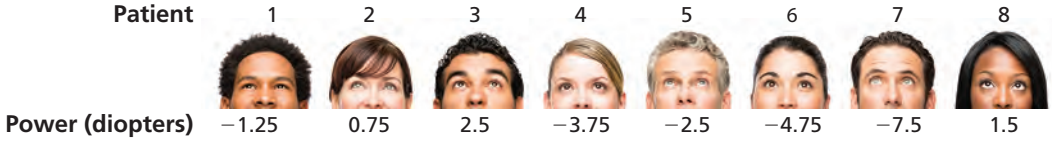
So, the salinity test is deepest and the ice test is closest to the surface.

Self-Assessment for Problem Solving

Solve each exercise. Then rate your understanding of the success criteria in your journal.

13. An airplane is at an elevation of 5.5 miles. A submarine is at an elevation of −10.9 kilometers. Which is closer to sea level? Explain.

14. The image shows the corrective powers (in *diopters*) of contact lenses for eight people. The farther the number of diopters is from 0, the greater the power of the lens. Positive diopters correct *farsightedness* and negative diopters correct *nearsightedness*. Who is the most nearsighted? the most farsighted? Who has the best eyesight?



1.1 Practice

Go to **BigIdeasMath.com** to get HELP with solving the exercises.

Review & Refresh

Write the ratio.

1. deer to bears
2. bears to deer
3. bears to animals
4. animals to deer

Find the GCF of the numbers.

5. 8, 20
6. 12, 30
7. 7, 28
8. 48, 72

Concepts, Skills, & Problem Solving

NUMBER SENSE **Determine which number is greater and which number is farther from 0. Explain your reasoning.** (See Exploration 1, p. 3.)

9. $4, -6$
10. $-3.25, \frac{7}{2}$
11. $-\frac{4}{5}, -1.3$

FINDING ABSOLUTE VALUES **Find the absolute value.**

12. $|8|$
13. $|-2|$
14. $|-10|$
15. $|10|$
16. $|0|$
17. $\left|\frac{1}{3}\right|$
18. $\left|\frac{7}{8}\right|$
19. $\left|-\frac{5}{9}\right|$
20. $\left|\frac{11}{8}\right|$
21. $|3.8|$
22. $|-5.3|$
23. $\left|-\frac{15}{4}\right|$
24. $|7.64|$
25. $|-18.26|$
26. $\left|4\frac{2}{5}\right|$
27. $\left|-5\frac{1}{6}\right|$

COMPARING RATIONAL NUMBERS **Copy and complete the statement using <, >, or =.**

28. $2 \square |-5|$
29. $|-1| \square |-8|$
30. $|5| \square |-5|$
31. $|-2| \square 0$
32. $0.4 \square \left|-\frac{7}{8}\right|$
33. $|4.9| \square |-5.3|$
34. $-|4.7| \square \frac{1}{2}$
35. $\left|-\frac{3}{4}\right| \square -\left|\frac{3}{4}\right|$
36. $-\left|1\frac{1}{4}\right| \square -\left|-1\frac{3}{8}\right|$

YOU BE THE TEACHER **Your friend compares two rational numbers. Is your friend correct? Explain your reasoning.**

37. $|-10| = -10$

38. $\left|-\frac{4}{5}\right| > -\left|\frac{1}{2}\right|$

39. OPEN-ENDED Write a negative number whose absolute value is greater than 3.

40. MODELING REAL LIFE The *summit elevation* of a volcano is the elevation of the top of the volcano relative to sea level. The summit elevation of Kilauea, a volcano in Hawaii, is 1277 meters. The summit elevation of Loihi, an underwater volcano in Hawaii, is −969 meters. Which summit is higher? Which summit is closer to sea level?

41. MODELING REAL LIFE The *freezing point* of a liquid is the temperature at which the liquid becomes a solid.

a. Which liquid in the table has the lowest freezing point?

b. Is the freezing point of mercury or butter closer to the freezing point of water, 0°C?

Liquid	Freezing Point (°C)
Butter	35
Airplane fuel	−53
Honey	−3
Mercury	−39
Candle wax	53

ORDERING RATIONAL NUMBERS **Order the values from least to greatest.**

42. $8, |3|, -5, |-2|, -2$

43. $|-6.3|, -7.2, 8, |5|, -6.3$

44. $|3.5|, |-1.8|, 4.6, 3\frac{2}{5}, |2.7|$

45. $\left|-\frac{3}{4}\right|, \frac{5}{8}, \left|\frac{1}{4}\right|, -\frac{1}{2}, \left|-\frac{7}{8}\right|$

46. MP PROBLEM SOLVING The table shows golf scores, relative to *par*.

a. The player with the lowest score wins. Which player wins?

b. Which player is closest to par?

c. Which player is farthest from par?

Player	Score
1	+5
2	0
3	−4
4	−1
5	+2

47. DIG DEEPER! You use the table below to record the temperature at the same location each hour for several hours. At what time is the temperature coldest? At what time is the temperature closest to the freezing point of water, 0°C?

Time	10:00 A.M.	11:00 A.M.	12:00 P.M.	1:00 P.M.	2:00 P.M.	3:00 P.M.
Temperature (°C)	−2.6	−2.7	−0.15	1.6	−1.25	−3.4

MP REASONING **Determine whether $n \geq 0$ or $n \leq 0$.**

48. $n + |-n| = 2n$

49. $n + |-n| = 0$

TRUE OR FALSE? **Determine whether the statement is *true* or *false*. Explain your reasoning.**

50. If $x < 0$, then $|x| = -x$.

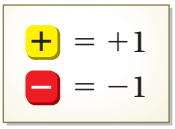
51. The absolute value of every rational number is positive.

1.2 Adding Integers

Learning Target: Find sums of integers.

Success Criteria:
- I can explain how to model addition of integers on a number line.
- I can find sums of integers by reasoning about absolute values.
- I can explain why the sum of a number and its opposite is 0.

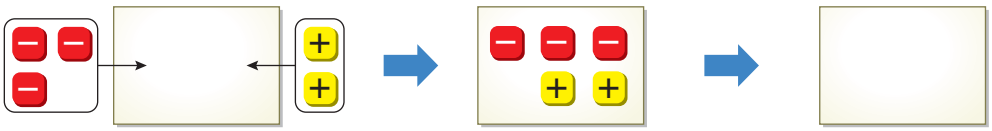
EXPLORATION 1 Using Integer Counters to Find Sums



Work with a partner. You can use the integer counters shown at the left to find sums of integers.

a. How can you use integer counters to model a sum? a sum that equals 0?

b. What expression is being modeled below? What is the value of the sum?



c. **INDUCTIVE REASONING** Use integer counters to complete the table.

Expression	Type of Sum	Sum	Sum: Positive, Negative, or Zero
$-3 + 2$	Integers with different signs		
$-4 + (-3)$			
$5 + (-3)$			
$7 + (-7)$			
$2 + 4$			
$-6 + (-2)$			
$-5 + 9$			
$15 + (-9)$			
$-10 + 10$			
$-6 + (-6)$			
$13 + (-13)$			

Math Practice

Make Conjectures
How can absolute values be used to write a rule about the sum of two integers?

d. How can you tell whether the sum of two integers is *positive*, *negative*, or *zero*?

e. Write rules for adding (i) two integers with the same sign, (ii) two integers with different signs, and (iii) two opposite integers.

1.2 Lesson

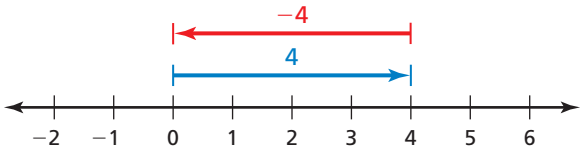
Key Vocabulary
additive inverse, *p. 11*

You have used number lines to find sums of positive numbers, which involve movement to the right. Now you will find sums with negative numbers, which involve movement to the left.

EXAMPLE 1 Using Number Lines to Find Sums

a. Find $4 + (-4)$.

Draw an arrow from 0 to 4 to represent 4. Then draw an arrow 4 units to the left to represent adding -4.

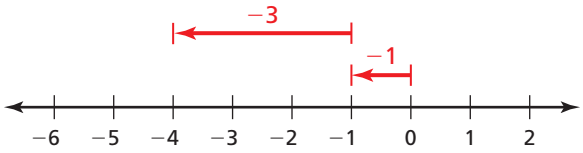


So, $4 + (-4) = 0$.

The length of each arrow is the absolute value of the number it represents.

b. Find $-1 + (-3)$.

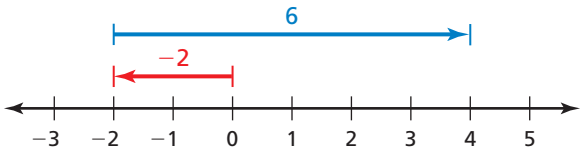
Draw an arrow from 0 to -1 to represent -1. Then draw an arrow 3 units to the left to represent adding -3.



So, $-1 + (-3) = -4$.

c. Find $-2 + 6$.

Draw an arrow from 0 to -2 to represent -2. Then draw an arrow 6 units to the right to represent adding 6.



So, $-2 + 6 = 4$.

Try It **Use a number line to find the sum.**

1. $-2 + 2$ **2.** $4 + (-5)$ **3.** $-3 + (-3)$

Using integer counters and number lines leads to the following rules for adding integers.

Key Ideas

Adding Integers with the Same Sign

Words Add the absolute values of the integers. Then use the common sign.

Numbers $2 + 5 = 7$ $\qquad -2 + (-5) = -7$

Adding Integers with Different Signs

Words Subtract the lesser absolute value from the greater absolute value. Then use the sign of the integer with the greater absolute value.

Numbers $8 + (-10) = -2$ $\qquad -13 + 17 = 4$

Additive Inverse Property

Words The sum of a number and its **additive inverse,** or opposite, is 0.

Numbers $6 + (-6) = 0$ $\qquad -25 + 25 = 0$

Algebra $a + (-a) = 0$

Notice that Example 1(a) shows the Additive Inverse Property.

EXAMPLE 2 Adding Integers with the Same Sign

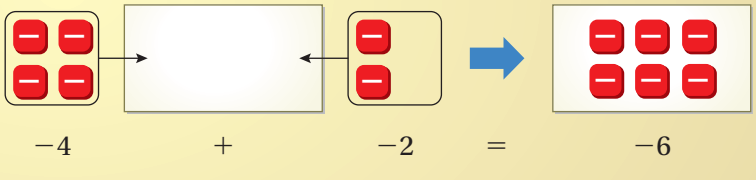
Find $-4 + (-2)$.

$-4 + (-2) = -6$ Add $|-4|$ and $|-2|$.

Use the common sign.

The sum is -6.

Check Use integer counters.



Try It **Find the sum.**

4. $7 + 13$ **5.** $-8 + (-5)$ **6.** $-2 + (-15)$

EXAMPLE 3 Adding Integers with Different Signs

a. Find $5 + (-10)$.

$5 + (-10) = -5$

$|-10| > |5|$. So, subtract $|5|$ from $|-10|$.

Use the sign of -10.

The sum is -5.

b. Find $-3 + 7$.

$-3 + 7 = 4$

$|7| > |-3|$. So, subtract $|-3|$ from $|7|$.

Use the sign of 7.

The sum is 4.

c. Find $-12 + 12$.

$-12 + 12 = 0$

The sum is 0 by the Additive Inverse Property.

-12 and 12 are opposites.

The sum is 0.

Try It **Find the sum.**

7. $-2 + 11$ **8.** $9 + (-10)$ **9.** $-31 + 31$

Self-Assessment for Concepts & Skills

Solve each exercise. Then rate your understanding of the success criteria in your journal.

10. WRITING Explain how to use a number line to find the sum of two integers.

ADDING INTEGERS Find the sum. Use a number line to justify your answer.

11. $-8 + 20$ **12.** $30 + (-30)$ **13.** $-10 + (-18)$

14. MP NUMBER SENSE Is $3 + (-4)$ the same as $-4 + 3$? Explain.

MP LOGIC Tell whether the statement is *true* or *false*. Explain your reasoning.

15. The sum of two negative integers is always negative.

16. The sum of an integer and its absolute value is always 0.

You can use the Commutative and Associative Properties of Addition to find sums of integers.

EXAMPLE 4 Modeling Real Life

The list shows four account transactions. Find the change in the account balance.

JULY TRANSACTIONS	
Withdrawal	-$40
Deposit	$50
Deposit	$75
Withdrawal	-$50

Understand the problem.

You are given amounts of two withdrawals and two deposits. You are asked to find how much the balance in the account changed.

Make a plan.

Find the sum of the transactions. Notice that 50 and -50 are opposites and combine to make 0. So, use properties of addition to first group those terms.

Solve and check.

$$\begin{aligned} -40 + 50 + 75 + (-50) &= -40 + 75 + 50 + (-50) && \text{Comm. Prop. of Add.} \\ &= -40 + 75 + [50 + (-50)] && \text{Assoc. Prop. of Add.} \\ &= -40 + 75 + 0 && \text{Add. Inv. Prop.} \\ &= 35 + 0 && \text{Add } -40 \text{ and } 75. \\ &= 35 && \text{Add. Prop. of Zero} \end{aligned}$$

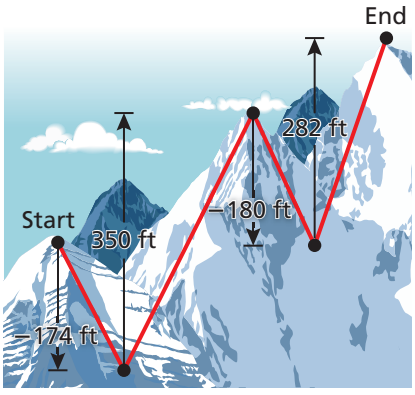
So, the account balance increased $35.

Another Method Find the sum by grouping the first two terms and the last two terms.

$$\begin{aligned} -40 + 50 + 75 + (-50) &= (-40 + 50) + [75 + (-50)] \\ &= 10 + 25 = 35 \quad ✓ \end{aligned}$$

Self-Assessment for Problem Solving

Solve each exercise. Then rate your understanding of the success criteria in your journal.



17. At 12:00 P.M., the water pressure on a submarine is 435 pounds per square inch. From 12:00 P.M. to 12:30 P.M., the water pressure increases 58 pounds per square inch. From 12:30 P.M. to 1:00 P.M., the water pressure decreases 116 pounds per square inch. What is the water pressure at 1:00 P.M.?

18. DIG DEEPER! The diagram shows the elevation changes between checkpoints on a trail. The trail begins at an elevation of 8136 feet. What is the elevation at the end of the trail?

1.2 Practice

Go to **BigIdeasMath.com** to get HELP with solving the exercises.

Review & Refresh

Copy and complete the statement using <, >, or =.

1. $5 \square |-7|$ **2.** $|-2.6| \square |-2.06|$ **3.** $\left|-\frac{3}{5}\right| \square -\left|\frac{5}{8}\right|$

Add.

4. $8.43 + 5.21$ **5.** $2.316 + 4.09$

6. $\frac{5}{9} + \frac{3}{9}$ **7.** $\frac{1}{2} + \frac{1}{8}$

8. The regular price of a photograph printed on a canvas is $18. You have a coupon for 15% off. How much is the discount?

A. $2.70 **B.** $3 **C.** $15 **D.** $15.30

9. Represent the ratio relationship using a graph.

Time (hours)	1	2	3
Distance (miles)	55	110	165

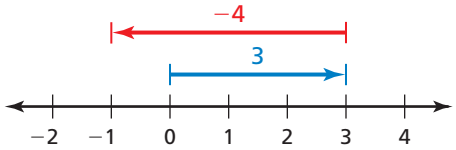
Concepts, Skills, & Problem Solving

USING INTEGER COUNTERS **Use integer counters to complete the table.**
(See Exploration 1, p. 9.)

	Expression	Type of Sum	Sum	Sum: Positive, Negative, or Zero
10.	$-5 + 8$			
11.	$-3 + (-7)$			

USING NUMBER LINES **Write an addition expression represented by the number line. Then find the sum.**

12.



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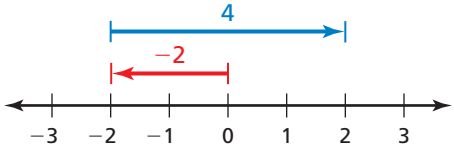

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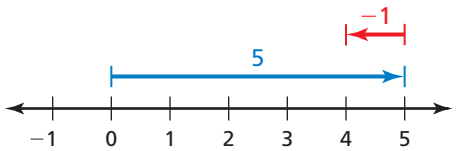

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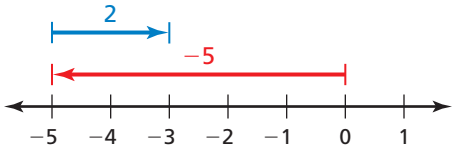


ADDING INTEGERS **Find the sum. Use integer counters or a number line to verify your answer.**

16. $6 + 4$ **17.** $-4 + (-6)$ **18.** $-2 + (-3)$ **19.** $-5 + 12$

20. $5 + (-7)$ **21.** $8 + (-8)$ **22.** $9 + (-11)$ **23.** $-3 + 13$

24. $-4 + (-16)$ **25.** $-3 + (-1)$ **26.** $14 + (-5)$ **27.** $0 + (-11)$

28. $-10 + (-15)$ **29.** $-13 + 9$ **30.** $18 + (-18)$ **31.** $-25 + (-9)$

YOU BE THE TEACHER **Your friend finds the sum. Is your friend correct? Explain your reasoning.**

32.

$9 + (-6) = 3$

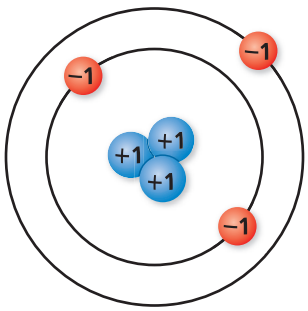
33.

$-10 + (-10) = 0$

34. **MODELING REAL LIFE** The temperature is $-3°$F at 7:00 A.M. During the next 4 hours, the temperature increases $21°$F. What is the temperature at 11:00 A.M.?

35. **MODELING REAL LIFE** Your bank account has a balance of $-\$12$. You deposit $\$60$. What is your new balance?

Lithium Atom



36. **MP PROBLEM SOLVING** A lithium atom has positively charged protons and negatively charged electrons. The sum of the charges represents the charge of the lithium atom. Find the charge of the atom.

37. **OPEN-ENDED** Write two integers with different signs that have a sum of -25. Write two integers with the same sign that have a sum of -25.

USING PROPERTIES **Tell how the Commutative and Associative Properties of Addition can help you find the sum using mental math. Then find the sum.**

38. $9 + 6 + (-6)$ **39.** $-8 + 13 + (-13)$ **40.** $9 + (-17) + (-9)$

41. $7 + (-12) + (-7)$ **42.** $-12 + 25 + (-15)$ **43.** $6 + (-9) + 14$

ADDING INTEGERS **Find the sum.**

44. $13 + (-21) + 16$ **45.** $22 + (-14) + (-12)$ **46.** $-13 + 27 + (-18)$

47. $-19 + 26 + 14$ **48.** $-32 + (-17) + 42$ **49.** $-41 + (-15) + (-29)$

DESCRIBING A SUM **Describe the location of the sum, relative to p, on a number line.**

50. $p + 3$ **51.** $p + (-7)$ **52.** $p + 0$ **53.** $p + q$

ALGEBRA **Evaluate the expression when $a = 4$, $b = -5$, and $c = -8$.**

54. $a + b$ **55.** $-b + c$ **56.** $|a + b + c|$

57. MODELING REAL LIFE The table shows the income and expenses for a school carnival. The school's goal was to raise $1100. Did the school reach its goal? Explain.

Games	Concessions	Donations	Flyers	Decorations
$650	$530	$52	−$28	−$75

OPEN-ENDED Write a real-life story using the given topic that involves the sum of an integer and its additive inverse.

58. income and expenses

59. the amount of water in a bottle

60. the elevation of a blimp

MENTAL MATH Use mental math to solve the equation.

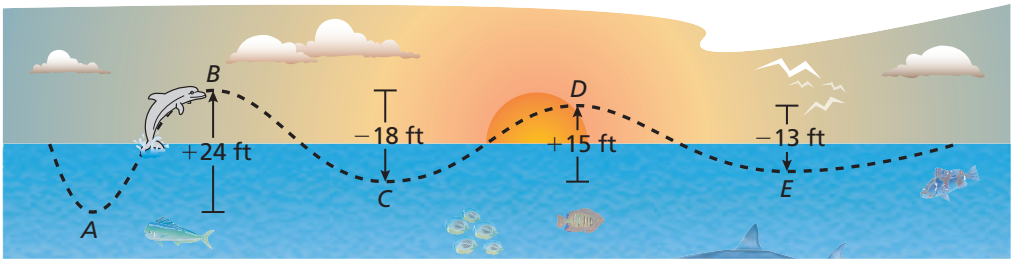
61. $d + 12 = 2$ **62.** $b + (-2) = 0$ **63.** $-8 + m = -15$

64. DIG DEEPER! Starting at point A, the path of a dolphin jumping out of the water is shown.

a. Is the dolphin deeper at point C or point E? Explain your reasoning.

b. Is the dolphin higher at point B or point D? Explain your reasoning.

c. What is the change in elevation of the dolphin from point A to point E?

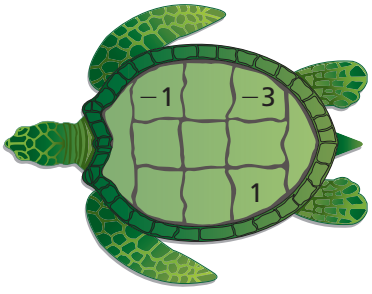


65. MP NUMBER SENSE Consider the integers p and q. Describe all of the possible values of p and q for each circumstance. Justify your answers.

a. $p + q = 0$ **b.** $p + q < 0$ **c.** $p + q > 0$

66. PUZZLE According to a legend, the Chinese Emperor Yu-Huang saw a magic square on the back of a turtle. In a *magic square*, the numbers in each row and in each column have the same sum. This sum is called the *magic sum*.

Copy and complete the magic square so that each row and each column has a magic sum of 0. Use each integer from −4 to 4 exactly once.



1.3 Adding Rational Numbers

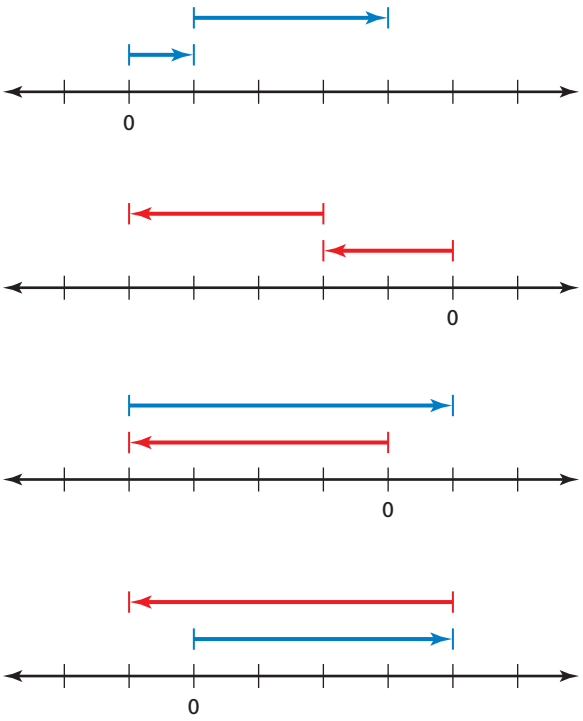
Learning Target: Find sums of rational numbers.

Success Criteria:
- I can explain how to model addition of rational numbers on a number line.
- I can find sums of rational numbers by reasoning about absolute values.
- I can use properties of addition to efficiently add rational numbers.

EXPLORATION 1 Adding Rational Numbers

Work with a partner.

a. Choose a unit fraction to represent the space between the tick marks on each number line. What addition expressions are being modeled? What are the sums?



Math Practice

Look for Structure
How do the lengths and directions of the arrows determine the sign of the sum?

b. Do the rules for adding integers apply to all rational numbers? Explain your reasoning.

c. You have used the following properties to add integers. Do these properties apply to all rational numbers? Explain your reasoning.

- Commutative Property of Addition
- Associative Property of Addition
- Additive Inverse Property

1.3 Lesson

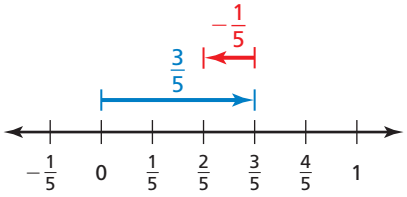
Key Idea

Adding Rational Numbers

Words To add rational numbers, use the same rules as you used for adding integers.

Numbers
$$\frac{3}{5} + \left(-\frac{1}{5}\right) = \left|\frac{3}{5}\right| - \left|-\frac{1}{5}\right|$$
$$= \frac{3}{5} - \frac{1}{5}$$
$$= \frac{2}{5}$$

Model



EXAMPLE 1 Adding Rational Numbers

Find $-\frac{8}{3} + \frac{5}{6}$. **Estimate $-3 + 1 = -2$**

Because the signs are different and $\left|-\frac{8}{3}\right| > \left|\frac{5}{6}\right|$, subtract $\left|\frac{5}{6}\right|$ from $\left|-\frac{8}{3}\right|$.

$\left|-\frac{8}{3}\right| - \left|\frac{5}{6}\right| = \frac{8}{3} - \frac{5}{6}$ Find the absolute values.

$= \frac{16}{6} - \frac{5}{6}$ Rewrite $\frac{8}{3}$ as $\frac{16}{6}$.

$= \frac{16 - 5}{6}$ Write the difference of the numerators over the common denominator.

$= \frac{11}{6}$, or $1\frac{5}{6}$ Simplify.

Because $\left|-\frac{8}{3}\right| > \left|\frac{5}{6}\right|$, use the sign of $-\frac{8}{3}$.

So, $-\frac{8}{3} + \frac{5}{6} = -1\frac{5}{6}$. **Reasonable? $-1\frac{5}{6} \approx -2$** ✓

Try It **Find the sum. Write your answer in simplest form.**

1. $-\frac{1}{2} + \left(-\frac{3}{2}\right)$
2. $-1\frac{3}{8} + \frac{3}{4}$
3. $4 + \left(-\frac{7}{2}\right)$

EXAMPLE 2 Adding Rational Numbers

Find $-0.75 + (-1.5)$. **Estimate $-1 + (-1.5) = -2.5$**

Because the signs are the same, add $|-0.75|$ and $|-1.5|$.

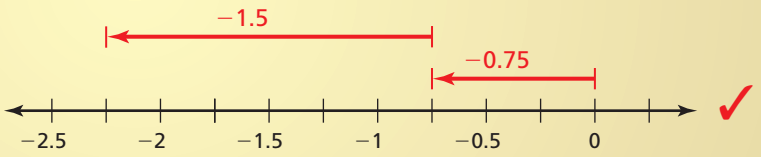
$$|-0.75| + |-1.5| = 0.75 + 1.5 \qquad \text{Find the absolute values.}$$

$$= 2.25 \qquad \text{Add.}$$

Because -0.75 and -1.5 are both negative, use a negative sign in the sum.

So, $-0.75 + (-1.5) = -2.25$. **Reasonable? $-2.25 \approx -2.5$** ✓

Check



Try It

Find the sum.

4. $-3.3 + (-2.7)$ **5.** $-5.35 + 4$ **6.** $1.65 + (-0.9)$

Self-Assessment for Concepts & Skills

Solve each exercise. Then rate your understanding of the success criteria in your journal.

7. WRITING Explain how to use a number line to find the sum of two rational numbers.

ADDING RATIONAL NUMBERS **Find the sum.**

8. $-\frac{7}{10} + \frac{1}{5}$ **9.** $-\frac{3}{4} + \left(-\frac{1}{3}\right)$ **10.** $-2.6 + 4.3$

11. DIFFERENT WORDS, SAME QUESTION Which is different? Find "both" answers.

Add -4.5 and 3.5.

What is the distance between -4.5 and 3.5?

What is -4.5 increased by 3.5?

Find the sum of -4.5 and 3.5.

EXAMPLE 3 Modeling Real Life

Year	Profit (millions of dollars)
2013	-1.7
2014	-4.75
2015	1.7
2016	0.8
2017	3.2

The table shows the annual profits (in millions of dollars) of an online gaming company from 2013 to 2017. Positive numbers represent *gains*, and negative numbers represent *losses*. Which statement describes the profit over the five-year period?

A. gain of \$0.75 million **B.** gain of \$75,000

C. loss of \$75,000 **D.** loss of \$750,000

To determine the amount of the gain or loss, find the sum of the profits.

$$\begin{aligned}
\text{five-year profit} &= -1.7 + (-4.75) + 1.7 + 0.8 + 3.2 && \text{Write the sum.}\\
&= -1.7 + 1.7 + (-4.75) + 0.8 + 3.2 && \text{Comm. Prop. of Add.}\\
&= 0 + (-4.75) + 0.8 + 3.2 && \text{Additive Inv. Prop.}\\
&= -4.75 + 0.8 + 3.2 && \text{Add. Prop. of Zero}\\
&= -4.75 + (0.8 + 3.2) && \text{Assoc. Prop. of Add.}\\
&= -4.75 + 4 && \text{Add 0.8 and 3.2.}\\
&= -0.75 && \text{Add } -4.75 \text{ and 4.}
\end{aligned}$$

The five-year profit is $-\$0.75$ million. So, the company has a five-year loss of \$0.75 million, or \$750,000.

The correct answer is **D**.

The Commutative and Associative Properties of Addition are true for all rational numbers.

Self-Assessment for Problem Solving

Solve each exercise. Then rate your understanding of the success criteria in your journal.

Day	Change in elevation (miles)
1	$-\frac{1}{4}$
2	$\frac{1}{2}$
3	$-\frac{1}{5}$
4	?

12. A bottle contains 10.5 cups of orange juice. You drink 1.2 cups of the juice each morning and 0.9 cup of the juice each afternoon. How much total juice do you drink each day? When will you run out of juice?

13. DIG DEEPER! The table shows the changes in elevation of a hiker each day for three days. How many miles of elevation must the hiker gain on the fourth day to gain $\frac{1}{4}$ mile of elevation over the four days?

1.3 Practice

Go to **BigIdeasMath.com** to get HELP with solving the exercises.

Review & Refresh

Find the sum. Use a number line to verify your answer.

1. $3 + 12$ **2.** $5 + (-7)$ **3.** $-4 + (-1)$ **4.** $-6 + 6$

Subtract.

5. $69 - 38$ **6.** $82 - 74$ **7.** $177 - 63$ **8.** $451 - 268$

9. What is the range of the numbers below?

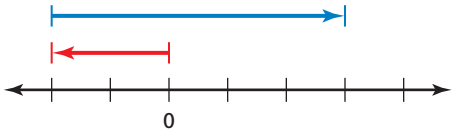
12, 8, 17, 12, 15, 18, 30

A. 12 **B.** 15 **C.** 18 **D.** 22

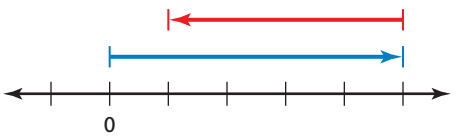
Concepts, Skills, & Problem Solving

USING TOOLS **Choose a unit fraction to represent the space between the tick marks on the number line. Write the addition expression being modeled. Then find the sum.** (See Exploration 1, p. 17.)

10.



11.



ADDING RATIONAL NUMBERS **Find the sum. Write fractions in simplest form.**

12. $\frac{11}{12} + \left(-\frac{7}{12}\right)$ **13.** $-1\frac{1}{5} + \left(-\frac{3}{5}\right)$ **14.** $-4.2 + 3.3$

15. $-\frac{9}{14} + \frac{2}{7}$ **16.** $12.48 + (-10.636)$ **17.** $-2\frac{1}{6} + \left(-\frac{2}{3}\right)$

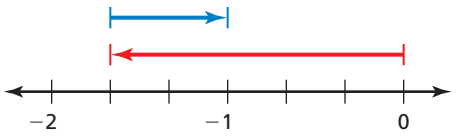
18. $-20.25 + 15.711$ **19.** $-32.306 + (-24.884)$ **20.** $\frac{15}{4} + \left(-4\frac{1}{3}\right)$

21. **YOU BE THE TEACHER** Your friend finds the sum. Is your friend correct? Explain your reasoning.

$$-3.7 + (-0.25) = |-3.7| + |-0.25|$$
$$= 3.7 + 0.25$$
$$= 3.95$$

OPEN-ENDED **Describe a real-life situation that can be represented by the addition expression modeled on the number line.**

22.



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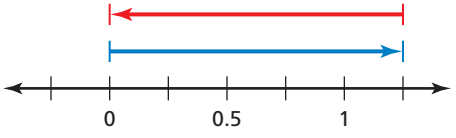


24. **MODELING REAL LIFE** You eat $\frac{3}{10}$ of a coconut. Your friend eats $\frac{1}{5}$ of the coconut. What fraction of the coconut do you and your friend eat?

25. **MODELING REAL LIFE** Your bank account balance is −$20.85. You deposit $15.50. What is your new balance?

26. **NUMBER SENSE** When is the sum of two negative mixed numbers an integer?

June	July	August
$-2\frac{1}{8}$	$1\frac{1}{4}$	$-\frac{7}{8}$

27. **WRITING** You are adding two rational numbers with different signs. How can you tell if the sum will be *positive*, *negative*, or *zero*?

28. **DIG DEEPER!** The table at the left shows the water level (in inches) of a reservoir for three months compared to the yearly average. Is the water level for the three-month period greater than or less than the yearly average? Explain.

USING PROPERTIES **Tell how the Commutative and Associative Properties of Addition can help you find the sum using mental math. Then find the sum.**

29. $4.5 + (-6.21) + (-4.5)$ **30.** $\frac{1}{3} + \left(\frac{2}{3} + \frac{5}{8}\right)$ **31.** $8\frac{1}{2} + \left[4\frac{1}{10} + \left(-8\frac{1}{2}\right)\right]$

ADDING RATIONAL NUMBERS **Find the sum. Explain each step.**

32. $6 + 4\frac{3}{4} + (-2.5)$ **33.** $-4.3 + \frac{4}{5} + 12$ **34.** $5\frac{1}{3} + 7.5 + \left(-3\frac{1}{6}\right)$

35. **PROBLEM SOLVING** The table at the right shows the annual profits (in thousands of dollars) of a county fair from 2013 to 2016. What must the 2017 profit be (in hundreds of dollars) to break even over the five-year period?

Year	Profit (thousands of dollars)
2013	2.5
2014	1.4
2015	−3.3
2016	−1.4
2017	?

36. **REASONING** Is $|a + b| = |a| + |b|$ true for all rational numbers a and b? Explain.

37. **REPEATED REASONING** Evaluate the expression.

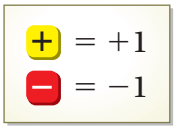
$$\frac{19}{20} + \left(-\frac{18}{20}\right) + \frac{17}{20} + \left(-\frac{16}{20}\right) + \cdots + \left(-\frac{4}{20}\right) + \frac{3}{20} + \left(-\frac{2}{20}\right) + \frac{1}{20}$$

1.4 Subtracting Integers

Learning Target: Find differences of integers.

Success Criteria:
- I can explain how subtracting integers is related to adding integers.
- I can explain how to model subtraction of integers on a number line.
- I can find differences of integers by reasoning about absolute values.

EXPLORATION 1 Using Integer Counters to Find Differences



Work with a partner.

a. Use integer counters to find the following sum and difference. What do you notice?

$$4 + (-2) \qquad 4 - 2$$

b. In part (a), you *removed* zero pairs to find the sums. How can you use integer counters and zero pairs to find $-3 - 1$?

c. **INDUCTIVE REASONING** Use integer counters to complete the table.

Expression	Operation: Add or Subtract	Answer
$4 - 2$	Subtract 2.	
$4 + (-2)$		
$-3 - 1$		
$-3 + (-1)$		
$3 - 8$		
$3 + (-8)$		
$9 - 13$		
$9 + (-13)$		
$-6 - (-3)$		
$-6 + 3$		
$-5 - (-12)$		
$-5 + 12$		

Math Practice

Interpret Results
What do the results tell you about the relationship between subtracting integers and adding integers?

d. Write a general rule for subtracting integers.

1.4 Lesson

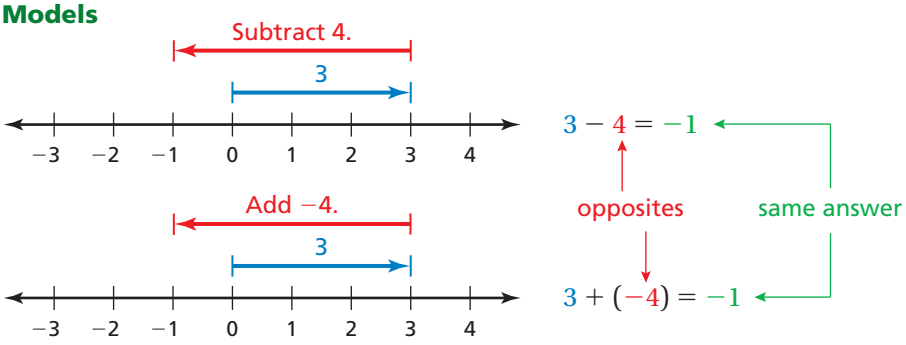
Key Idea

Subtracting Integers

Words To subtract an integer, add its opposite.

Numbers $3 - 4 = 3 + (-4) = -1$

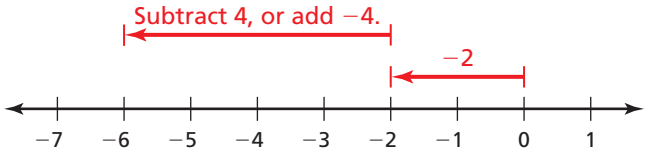
Models



EXAMPLE 1 Using Number Lines to Find Differences

a. Find $-2 - 4$.

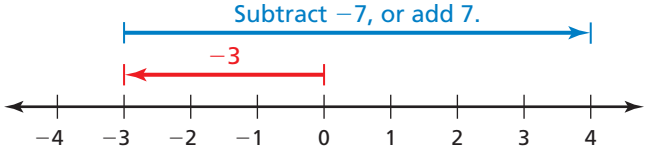
Draw an arrow from 0 to -2 to represent -2. Then draw an arrow 4 units to the left to represent subtracting 4, or adding -4.



So, $-2 - 4 = -6$.

b. Find $-3 - (-7)$.

Draw an arrow from 0 to -3 to represent -3. Then draw an arrow 7 units to the right to represent subtracting -7, or adding 7.



So, $-3 - (-7) = 4$.

Try It

Use a number line to find the difference.

1. $1 - 4$ **2.** $-5 - 2$ **3.** $6 - (-5)$

EXAMPLE 2 Subtracting Integers

a. Find $3 - 12$.

$3 - 12 = 3 + (-12)$ Add the opposite of 12.

$= -9$ Add.

▶ The difference is -9.

b. Find $-8 - (-13)$.

$-8 - (-13) = -8 + 13$ Add the opposite of -13.

$= 5$ Add.

▶ The difference is 5.

c. Find $5 - (-4)$.

$5 - (-4) = 5 + 4$ Add the opposite of -4.

$= 9$ Add.

▶ The difference is 9.

Try It **Find the difference.**

4. $8 - 3$ **5.** $9 - 17$ **6.** $-3 - 3$

7. $-14 - 9$ **8.** $10 - (-8)$ **9.** $-12 - (-12)$

Self-Assessment for Concepts & Skills

Solve each exercise. Then rate your understanding of the success criteria in your journal.

10. WRITING Explain how to use a number line to find the difference of two integers.

MATCHING Match the subtraction expression with the corresponding addition expression. Explain your reasoning.

11. $9 - (-5)$ **12.** $-9 - 5$ **13.** $-9 - (-5)$ **14.** $9 - 5$

A. $-9 + 5$ **B.** $9 + (-5)$ **C.** $-9 + (-5)$ **D.** $9 + 5$

SUBTRACTING INTEGERS Find the difference. Use a number line to justify your answer.

15. $10 - 12$ **16.** $6 - (-8)$ **17.** $-7 - (-4)$

EXAMPLE 3 Modeling Real Life

Which continent has the greater range of elevations?

	North America	Africa
Highest Elevation	6198 m	5895 m
Lowest Elevation	−86 m	−155 m

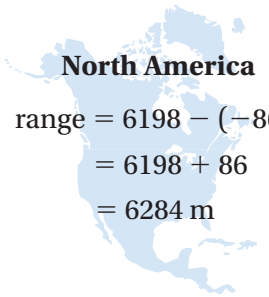
Understand the problem.

You are given the highest and lowest elevations in North America and Africa. You are asked to find the continent with the greater difference between its highest and lowest elevations.

Make a plan.

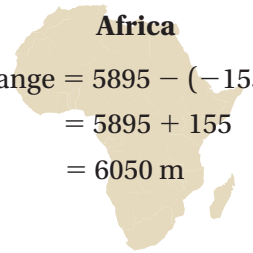
Find the range of elevations for each continent by subtracting the lowest elevation from the highest elevation. Then compare the ranges.

Solve and check.



North America

$$\begin{aligned}\text{range} &= 6198 - (-86)\\ &= 6198 + 86\\ &= 6284 \text{ m}\end{aligned}$$



Africa

$$\begin{aligned}\text{range} &= 5895 - (-155)\\ &= 5895 + 155\\ &= 6050 \text{ m}\end{aligned}$$

Because 6284 meters is greater than 6050 meters, North America has the greater range of elevations.

Another Method North America's highest elevation is $6198 - 5895 = 303$ meters higher than Africa's highest elevation. Africa's lowest elevation is $|-155| - |-86| = 69$ meters lower than North America's lowest elevation. Because $303 > 69$, North America has the greater range of elevations. ✓

Self-Assessment for Problem Solving

Solve each exercise. Then rate your understanding of the success criteria in your journal.

18. A polar vortex causes the temperature to decrease from 3°C at 3:00 P.M. to −2°C at 4:00 P.M. The temperature continues to change by the same amount each hour until 8:00 P.M. Find the total change in temperature from 3:00 P.M. to 8:00 P.M.

19. DIG DEEPER! While on vacation, you map several locations using a coordinate plane in which each unit represents 1 mile. A cove is at $(3, -7)$, an island is at $(-5, 4)$, and you are currently at $(3, 4)$. Are you closer to the cove or the island? Justify your answer.

1.4 Practice

Go to **BigIdeasMath.com** to get HELP with solving the exercises.

Review & Refresh

Find the sum. Write fractions in simplest form.

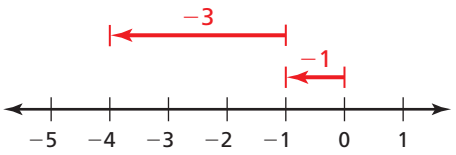
1. $\frac{5}{9} + \left(-\frac{2}{9}\right)$ **2.** $-8.75 + 2.43$ **3.** $-3\frac{1}{8} + \left(-2\frac{3}{8}\right)$

Add.

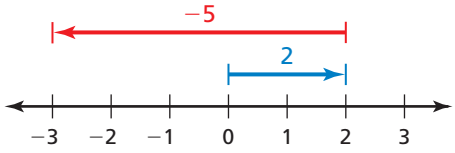
4. $2.48 + 6.711$ **5.** $12.807 + 7.116$ **6.** $18.7126 + 14.033$

Write an addition expression represented by the number line. Then find the sum.

7.



8.



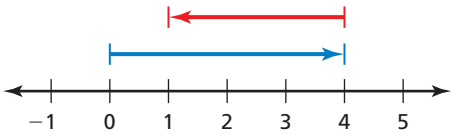
Concepts, Skills, & Problem Solving

USING INTEGER COUNTERS **Use integer counters to find the difference.**
(See Exploration 1, p. 23.)

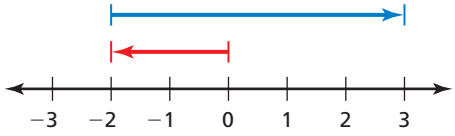
9. $5 - 3$ **10.** $1 - 4$ **11.** $-2 - (-6)$

USING NUMBER LINES **Write an addition expression and write a subtraction expression represented by the number line. Then evaluate the expressions.**

12.



13.



SUBTRACTING INTEGERS **Find the difference. Use a number line to verify your answer.**

14. $4 - 7$ **15.** $8 - (-5)$ **16.** $-6 - (-7)$ **17.** $-2 - 3$

18. $5 - 8$ **19.** $-4 - 6$ **20.** $-8 - (-3)$ **21.** $10 - 7$

22. $-8 - 13$ **23.** $15 - (-2)$ **24.** $-9 - (-13)$ **25.** $-7 - (-8)$

26. $-6 - (-6)$ **27.** $-10 - 12$ **28.** $32 - (-6)$ **29.** $0 - (-20)$

30. **YOU BE THE TEACHER** Your friend finds the difference. Is your friend correct? Explain your reasoning.

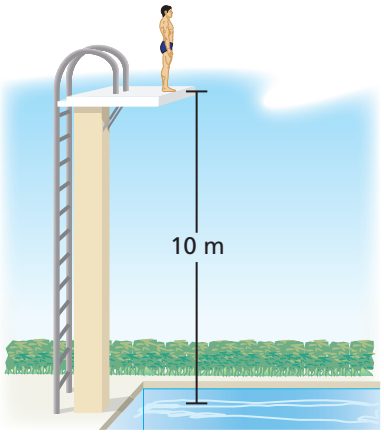
$7 - (-12) = 7 + 12 = 19$

31. **STRUCTURE** A scientist records the water temperature and the air temperature in Antarctica. The water temperature is -2°C. The air is 9°C colder than the water. Which expression can be used to find the air temperature? Explain your reasoning.

$-2 + 9$ $\quad$ $-2 - 9$ $\quad$ $9 - 2$

32. **MODELING REAL LIFE** A shark is 80 feet below the surface of the water. It swims up and jumps out of the water to a height of 15 feet above the surface. Find the vertical distance the shark travels. Justify your answer.

33. **MODELING REAL LIFE** The figure shows a diver diving from a platform. The diver reaches a depth of 4 meters. What is the change in elevation of the diver?



34. **OPEN-ENDED** Write two different pairs of negative integers, x and y, that make the statement $x - y = -1$ true.

USING PROPERTIES **Tell how the Commutative and Associative Properties of Addition can help you evaluate the expression using mental math. Then evaluate the expression.**

35. $2 - 7 + (-2)$ $\quad$ **36.** $-6 - 8 + 6$ $\quad$ **37.** $8 + (-8 - 5)$

38. $-39 + 46 - (-39)$ $\quad$ **39.** $[13 + (-28)] - 13$ $\quad$ **40.** $-2 + (-47 - 8)$

ALGEBRA **Evaluate the expression when $k = -3$, $m = -6$, and $n = 9$.**

41. $4 - n$ $\quad$ **42.** $m - (-8)$ $\quad$ **43.** $-5 + k - n$ $\quad$ **44.** $|m - k|$

45. **MODELING REAL LIFE** The table shows the record monthly high and low temperatures for a city in Alaska.

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
High (°F)	56	57	56	72	82	92	84	85	73	64	62	53
Low (°F)	−35	−38	−24	−15	1	29	34	31	19	−6	−21	−36

a. Which month has the greatest range of temperatures?

b. What is the range of temperatures for the year?

REASONING **Tell whether the difference of the two integers is *always*, *sometimes*, or *never* positive. Explain your reasoning.**

46. two positive integers

47. a positive integer and a negative integer

48. two negative integers

49. a negative integer and a positive integer

NUMBER SENSE **For what values of a and b is the statement true?**

50. $|a - b| = |b - a|$

51. $|a - b| = |a| - |b|$

1.5 Subtracting Rational Numbers

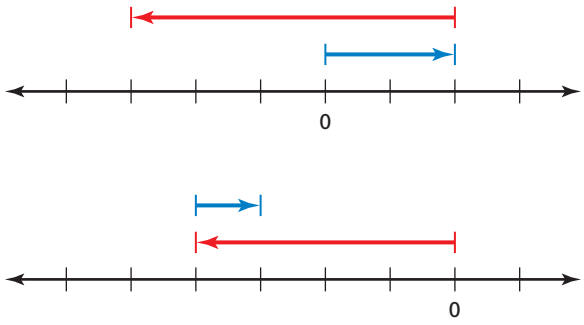
Learning Target: Find differences of rational numbers and find distances between numbers on a number line.

Success Criteria:
- I can explain how to model subtraction of rational numbers on a number line.
- I can find differences of rational numbers by reasoning about absolute values.
- I can find distances between numbers on a number line.

EXPLORATION 1 Subtracting Rational Numbers

Work with a partner.

a. Choose a unit fraction to represent the space between the tick marks on each number line. What expressions involving subtraction are being modeled? What are the differences?



b. Do the rules for subtracting integers apply to all rational numbers? Explain your reasoning.

c. You have used the commutative and associative properties to add integers. Do these properties apply in expressions involving subtraction? Explain your reasoning.

EXPLORATION 2 Finding Distances on a Number Line

Work with a partner.

Math Practice

Find General Methods

How can you find the distance between any two rational numbers on a number line?

a. Find the distance between 3 and -2 on a number line.

b. The distance between 3 and 0 is the absolute value of 3, because $|3 - 0| = |3| = 3$. How can you use absolute values to find the distance between 3 and -2? Justify your answer.

c. Choose any two rational numbers. Use your method in part (b) to find the distance between the numbers. Use a number line to check your answer.

1.5 Lesson

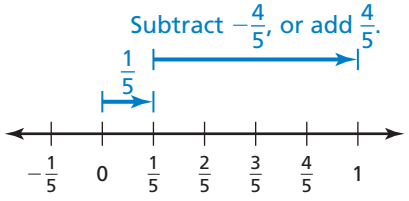
Key Idea

Subtracting Rational Numbers

Words To subtract rational numbers, use the same rules as you used for subtracting integers.

Numbers
$$\frac{1}{5} - \left(-\frac{4}{5}\right) = \frac{1}{5} + \frac{4}{5}$$
$$= \frac{5}{5}$$
$$= 1$$

Model



EXAMPLE 1 Subtracting Rational Numbers

Find $-4\frac{1}{7} - \frac{5}{7}$. **Estimate** $-4 - 1 = -5$

Rewrite the difference as a sum by adding the opposite.

$$-4\frac{1}{7} - \frac{5}{7} = -4\frac{1}{7} + \left(-\frac{5}{7}\right)$$

Because the signs are the same, add $\left|-4\frac{1}{7}\right|$ and $\left|-\frac{5}{7}\right|$.

$$\left|-4\frac{1}{7}\right| + \left|-\frac{5}{7}\right| = 4\frac{1}{7} + \frac{5}{7}$$ Find the absolute values.

$$= 4 + \frac{1}{7} + \frac{5}{7}$$ Write $4\frac{1}{7}$ as $4 + \frac{1}{7}$.

$$= 4 + \frac{6}{7}, \text{ or } 4\frac{6}{7}$$ Add fractions and simplify.

Because $-4\frac{1}{7}$ and $-\frac{5}{7}$ are both negative, use a negative sign in the difference.

So, $-4\frac{1}{7} - \frac{5}{7} = -4\frac{6}{7}$. **Reasonable?** $-4\frac{6}{7} \approx -5$ ✓

Try It **Find the difference. Write your answer in simplest form.**

1. $\frac{1}{3} - \left(-\frac{1}{3}\right)$
2. $-3\frac{1}{3} - \frac{2}{3}$
3. $4 - 5\frac{1}{2}$

EXAMPLE 2 Subtracting Rational Numbers

Find $2.4 - 5.6$.

Rewrite the difference as a sum by adding the opposite.

$$2.4 - 5.6 = 2.4 + (-5.6)$$

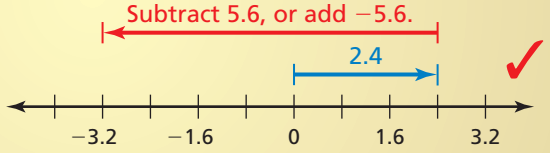
Because the signs are different and $|-5.6| > |2.4|$, subtract $|2.4|$ from $|-5.6|$.

$$|-5.6| - |2.4| = 5.6 - 2.4 \qquad \text{Find the absolute values.}$$
$$= 3.2 \qquad \text{Subtract.}$$

Because $|-5.6| > |2.4|$, use the sign of -5.6.

So, $2.4 - 5.6 = -3.2$.

Check



Try It **Find the difference.**

4. $-2.1 - 3.9$ **5.** $-8.8 - (-8.8)$ **6.** $0.45 - (-0.05)$

EXAMPLE 3 Using Properties of Addition

Evaluate $-1\frac{3}{8} - 8\frac{1}{2} - \left(-6\frac{7}{8}\right)$.

Use properties of addition to group the mixed numbers that include fractions with the same denominator.

$$-1\tfrac{3}{8} - 8\tfrac{1}{2} - \left(-6\tfrac{7}{8}\right) = -1\tfrac{3}{8} + \left(-8\tfrac{1}{2}\right) + 6\tfrac{7}{8} \qquad \text{Rewrite as a sum of terms.}$$
$$= -1\tfrac{3}{8} + 6\tfrac{7}{8} + \left(-8\tfrac{1}{2}\right) \qquad \text{Comm. Prop. of Add.}$$
$$= 5\tfrac{1}{2} + \left(-8\tfrac{1}{2}\right) \qquad \text{Add } -1\tfrac{3}{8} \text{ and } 6\tfrac{7}{8}.$$
$$= -3 \qquad \text{Add } 5\tfrac{1}{2} \text{ and } -8\tfrac{1}{2}.$$

So, $-1\frac{3}{8} - 8\frac{1}{2} - \left(-6\frac{7}{8}\right) = -3$.

Try It **Evaluate the expression. Write fractions in simplest form.**

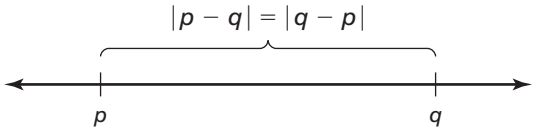
7. $-2 - \frac{2}{5} + 1\frac{3}{5}$ **8.** $7.8 - 3.3 - (-1.2) + 4.3$

Key Idea

Distance between Numbers on a Number Line

Words The distance between any two numbers on a number line is the absolute value of the difference of the numbers.

Model



EXAMPLE 4 Finding Distance on a Number Line

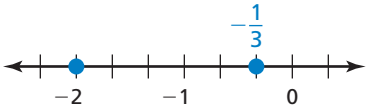
Find the distance between $-\frac{1}{3}$ and -2 on a number line.

To find the distance, find the absolute value of the difference of the numbers.

$$\left|-2-\left(-\frac{1}{3}\right)\right| = \left|-2+\frac{1}{3}\right| \qquad \text{Add the opposite of } -\frac{1}{3}.$$

$$= \left|-1\frac{2}{3}\right| \qquad \text{Add } -2 \text{ and } \frac{1}{3}.$$

$$= 1\frac{2}{3} \qquad \text{Find the absolute value.}$$



So, the distance between $-\frac{1}{3}$ and -2 is $1\frac{2}{3}$.

Try It **Find the distance between the two numbers on a number line.**

9. -3 and 9 **10.** -7.5 and -15.3 **11.** $1\frac{1}{2}$ and $-\frac{2}{3}$

Self-Assessment for Concepts & Skills

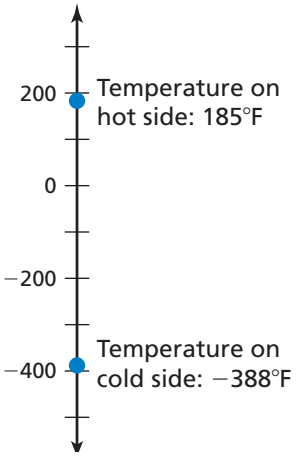
Solve each exercise. Then rate your understanding of the success criteria in your journal.

12. WRITING Explain how to use a number line to find the difference of two rational numbers.

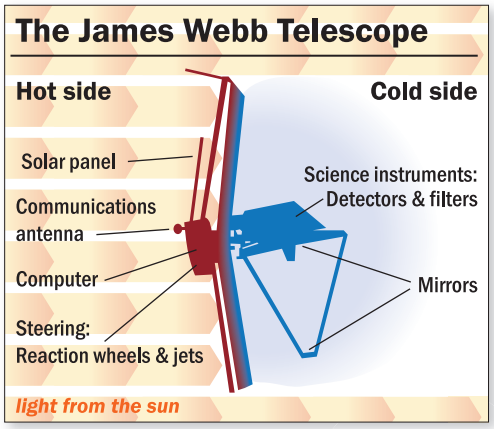
SUBTRACTING RATIONAL NUMBERS **Find the difference. Use a number line to justify your answer.**

13. $4.9 - 1.6$ **14.** $\frac{7}{8} - \left(-\frac{3}{4}\right)$ **15.** $\frac{1}{3} - 2\frac{1}{6}$

EXAMPLE 5 Modeling Real Life



The number line shows the temperatures on each side of the James Webb telescope when in Earth's orbit. Find and interpret the distance between the points.



The number line shows that the temperature on the hot side is 185°F and the temperature on the cold side is −388°F.

To find the distance between the points, find the absolute value of the difference of the numbers.

$$|185 - (-388)| = |185 + 388| \qquad \text{Add the opposite of } -388.$$

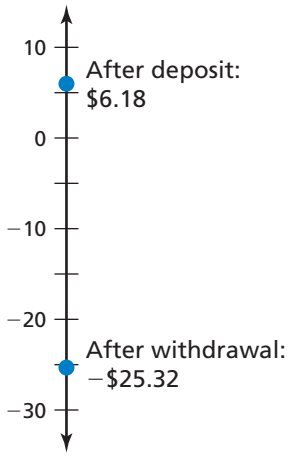
$$= |573| \qquad \text{Add 185 and 388.}$$

$$= 573 \qquad \text{Find the absolute value.}$$

The temperatures are 573°F apart on the number line. So, the hot side is 573°F hotter than the cold side.

Self-Assessment for Problem Solving

Solve each exercise. Then rate your understanding of the success criteria in your journal.



16. A parasail is $\frac{3}{100}$ mile above the water. After 5 minutes, the parasail is $\frac{1}{50}$ mile above the water. Find and interpret the change in height of the parasail.

17. DIG DEEPER! You withdraw $55 from a bank account to purchase a game. Then you make a deposit. The number line shows the balances of the account after each transaction.

a. Find and interpret the distance between the points.

b. How much money was in your account before buying the game?

1.5 Practice

Go to **BigIdeasMath.com** to get HELP with solving the exercises.

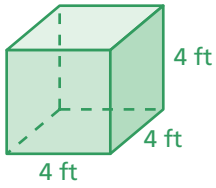
Review & Refresh

Find the difference. Use a number line to verify your answer.

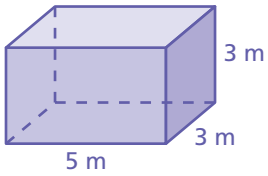
1. $9 - 5$ **2.** $-8 - (-8)$ **3.** $-12 - 7$ **4.** $12 - (-3)$

Find the volume of the prism.

5.



6.



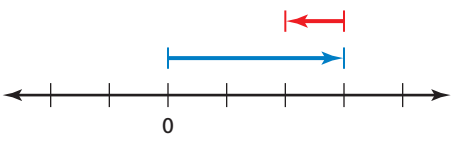
Order the values from least to greatest.

7. $6, |3|, |-4|, 1, -2$ **8.** $|4.5|, -3.6, 2, |-1.8|, 1.2$

Concepts, Skills, & Problem Solving

USING TOOLS **Choose a unit fraction to represent the space between the tick marks on the number line. Write an expression involving subtraction that is being modeled. Then find the difference.** (See Exploration 1, p. 29.)

9.



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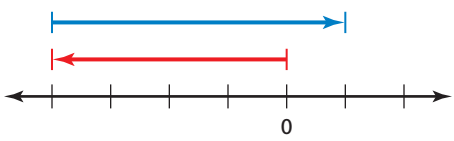


SUBTRACTING RATIONAL NUMBERS **Find the difference. Write fractions in simplest form.**

11. $\frac{5}{8} - \left(-\frac{7}{8}\right)$ **12.** $-1\frac{1}{3} - 1\frac{2}{3}$ **13.** $-1 - 2.5$

14. $\frac{4}{5} - \left(-\frac{3}{10}\right)$ **15.** $5.5 - 8.1$ **16.** $-5 - \frac{5}{3}$

17. $-8\frac{3}{8} - 10\frac{1}{6}$ **18.** $-4.62 - 3.51$ **19.** $-\frac{1}{2} - \left(-\frac{5}{9}\right)$

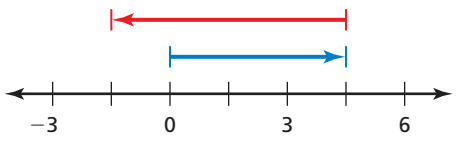
20. $-7.34 - (-5.51)$ **21.** $6.673 - (-8.29)$ **22.** $12\frac{2}{5} - 17\frac{1}{3}$

23. **YOU BE THE TEACHER** Your friend finds the difference. Is your friend correct? Explain your reasoning.

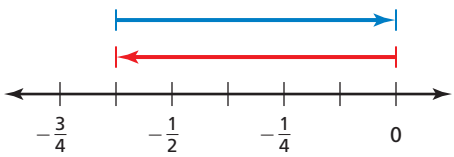
$$\frac{3}{2} - \frac{9}{2} = \left|\frac{3}{2}\right| + \left|\frac{9}{2}\right| = \frac{12}{2} = 6$$

OPEN-ENDED **Describe a real-life situation that can be represented by the subtraction expression modeled on the number line.**

24.



25.



26. **MODELING REAL LIFE** Your water bottle is $\frac{5}{6}$ full. After tennis practice, the bottle is $\frac{3}{8}$ full. How much of the water did you drink?

27. **MODELING REAL LIFE** You have $2\frac{2}{3}$ ounces of sodium chloride. You want to replicate an experiment that uses $2\frac{3}{4}$ ounces of sodium chloride. Do you have enough sodium chloride? If not, how much more do you need?

28. **REASONING** When is the difference of two decimals an integer? Explain.

USING PROPERTIES **Tell how the Commutative and Associative Properties of Addition can help you evaluate the expression. Then evaluate the expression.**

29. $\frac{3}{4} + \frac{2}{3} - \frac{3}{4}$

30. $\frac{2}{5} - \frac{7}{10} - \left(-\frac{3}{5}\right)$

31. $8.5 + 3.4 - 6.5 - (-1.6)$

32. $-1\frac{3}{4} - \left(-8\frac{1}{3}\right) - \left(-4\frac{1}{4}\right)$

33. $2.1 + (5.8 - 4.1)$

34. $2\frac{3}{8} - 4\frac{1}{2} + 3\frac{1}{8} - \left(-\frac{1}{2}\right)$

FINDING DISTANCE ON A NUMBER LINE **Find the distance between the two numbers on a number line.**

35. 2.7 and 5.9

36. $-\frac{7}{9}$ and $-\frac{2}{9}$

37. -2.2 and 8.4

38. $\frac{3}{4}$ and $\frac{1}{8}$

39. -1.85 and 7.36

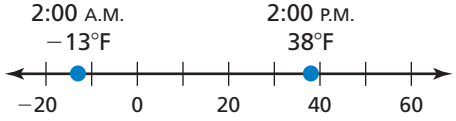
40. -7 and $-3\frac{2}{3}$

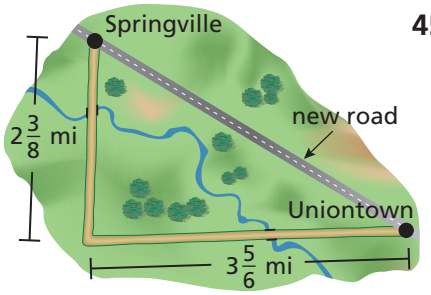
41. 2.491 and -3.065

42. $-2\frac{1}{2}$ and $-5\frac{3}{4}$

43. $-1\frac{1}{3}$ and $12\frac{7}{12}$

44. **MODELING REAL LIFE** The number line shows the temperatures at 2:00 A.M. and 2:00 P.M. in the Gobi Desert. Find and interpret the distance between the points.





45. MP **PROBLEM SOLVING** A new road that connects Uniontown to Springville is $4\frac{1}{3}$ miles long. What is the change in distance when using the new road instead of the dirt roads?

FINDING DISTANCE IN A COORDINATE PLANE **Find the distance between the points in a coordinate plane.**

46. $(-4, 7.8), (-4, -3.5)$

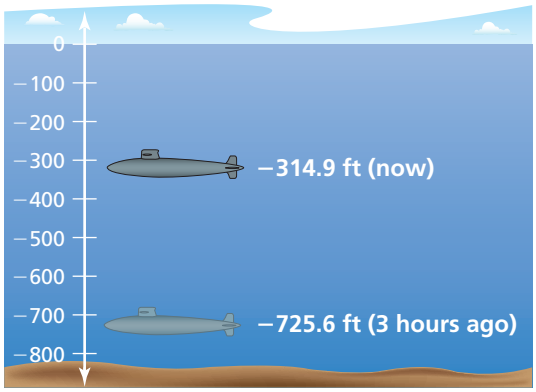
47. $(-2.63, 7), (1.85, 7)$

48. $\left(-\frac{1}{2}, -1\right), \left(\frac{5}{8}, -1\right)$

49. $\left(6, 2\frac{1}{3}\right), \left(6, -5\frac{2}{9}\right)$

50. $(-6.2, 1.4), (8.9, 1.4)$

51. $\left(7\frac{1}{7}, 1\frac{4}{5}\right), \left(7\frac{1}{7}, -\frac{9}{10}\right)$



52. DIG DEEPER! The figure shows the elevations of a submarine.

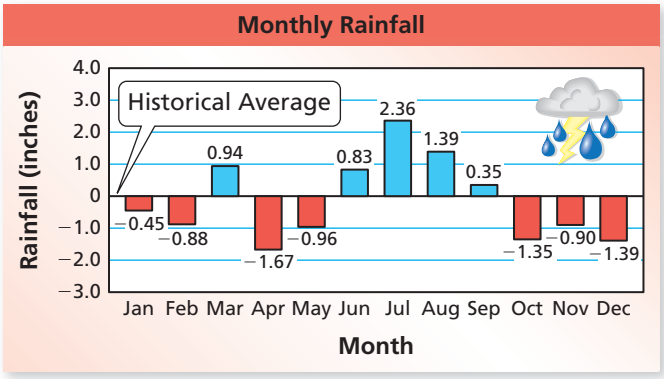
a. Find the vertical distance traveled by the submarine.

b. Find the mean hourly vertical distance traveled by the submarine.

53. MP **LOGIC** The bar graph shows how each month's rainfall compares to the historical average.

a. What is the difference in rainfall of the wettest month and the driest month?

b. What do you know about the total amount of rainfall for the year?



54. **OPEN-ENDED** Write two different pairs of negative decimals, x and y, that make the statement $x - y = 0.6$ true.

MP **REASONING** **Tell whether the difference of the two numbers is *always*, *sometimes*, or *never* positive. Explain your reasoning.**

55. two negative fractions

56. a positive decimal and a negative decimal

57. MP **STRUCTURE** Fill in the blanks to complete the decimals.

$$5.\square 4 - \square.\square = -3.61$$

1 Connecting Concepts

Problem-Solving Strategies

Using an appropriate strategy will help you make sense of problems as you study the mathematics in this course. You can use the following strategies to solve problems that you encounter.

- Use a verbal model.
- Draw a diagram.
- Write an equation.
- Solve a simpler problem.
- Sketch a graph or number line.
- Make a table.
- Make a list.
- Break the problem into parts.

Using the Problem-Solving Plan

1. A land surveyor uses a coordinate plane to draw a map of a park, where each unit represents 1 mile. The park is in the shape of a parallelogram with vertices $(-2.5, 1.5)$, $(-1.5, -2.25)$, $(2.75, -2.25)$, and $(1.75, 1.5)$. Find the area of the park.

Understand the problem.

You know the vertices of the parallelogram-shaped park and that each unit represents 1 mile. You are asked to find the area of the park.

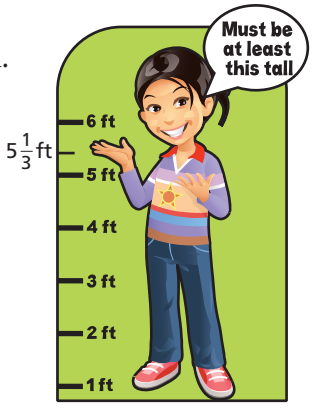
Make a plan.

Use a coordinate plane to draw a map of the park. Then find the height and base length of the park. Find the area by using the formula for the area of a parallelogram.

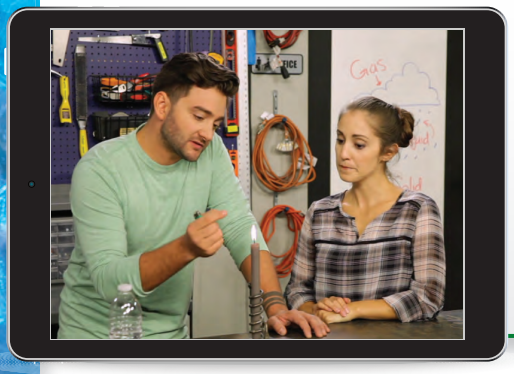
Solve and check.

Use the plan to solve the problem. Then check your solution.

2. The diagram shows the height requirement for driving a go-cart. You are $5\frac{1}{4}$ feet tall. Write and solve an inequality to represent how much taller you must be to drive a go-cart.



Performance Task



Melting Matters

At the beginning of this chapter, you watched a STEAM Video called "Freezing Solid." You are now ready to complete the performance task related to this video, available at ***BigIdeasMath.com***. Be sure to use the problem-solving plan as you work through the performance task.

1 Chapter Review

Go to *BigIdeasMath.com* to download blank graphic organizers.

Review Vocabulary

Write the definition and give an example of each vocabulary term.

integers, *p. 3*
rational number, *p. 3*
absolute value, *p. 4*
additive inverse, *p. 11*

Graphic Organizers

You can use a **Definition and Example Chart** to organize information about a concept. Here is an example of a Definition and Example Chart for the vocabulary term ***absolute value***.

Absolute value: the distance between a number and 0 on a number line

Example
$|3| = 3$

Example
$|-5| = 5$

Example
$|0| = 0$

Choose and complete a graphic organizer to help you study the concept.

1. integers
2. rational numbers
3. adding integers
4. Additive Inverse Property
5. adding rational numbers
6. subtracting integers
7. subtracting rational numbers

"I made a **Definition and Example Chart** to give my owner ideas for my birthday next week."

Chapter Self-Assessment

As you complete the exercises, use the scale below to rate your understanding of the success criteria in your journal.

1	2	3	4
I do not understand.	I can do it with help.	I can do it on my own.	I can teach someone else.

1.1 Rational Numbers *(pp. 3–8)*

Learning Target: Understand absolute values and ordering of rational numbers.

Find the absolute value.

1. $|3|$
2. $|-9|$
3. $\left|\frac{3}{4}\right|$
4. $|-5.2|$
5. $\left|-\frac{6}{7}\right|$
6. $|4.15|$

Copy and complete the statement using <, >, or =.

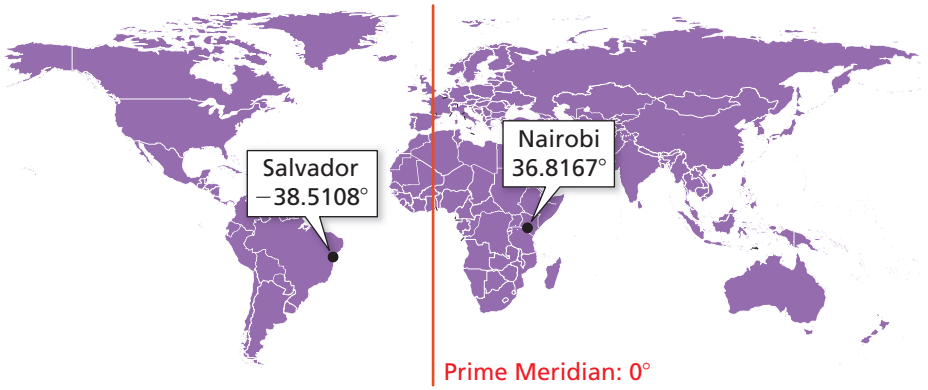
7. $|-2|$ ☐ -2
8. $\left|-\frac{1}{3}\right|$ ☐ $\left|-\frac{5}{6}\right|$
9. $-|1.7|$ ☐ -1.7

10. Order $|2.25|$, $|-1.5|$, $1\frac{1}{4}$, $\left|2\frac{1}{2}\right|$, and -2 from least to greatest.

11. Your friend is in Death Valley, California, at an elevation of –282 feet. You are near the Mississippi River in Illinois at an elevation of 279 feet. Who is closer to sea level?

12. Give values for a and b so that $a < b$ and $|a| > |b|$.

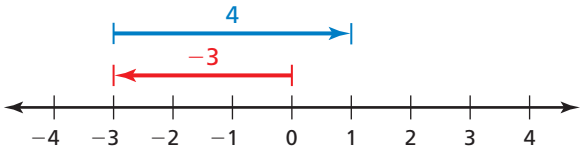
13. The map shows the longitudes (in degrees) for Salvador, Brazil, and Nairobi, Kenya. Which city is closer to the Prime Meridian?



1.2 Adding Integers *(pp. 9–16)*

Learning Target: Find sums of integers.

14. Write an addition expression represented by the number line. Then find the sum.



Find the sum. Use a number line to verify your answer.

15. $-16 + (-11)$

16. $-15 + 5$

17. $100 + (-75)$

18. $-32 + (-2)$

19. $-2 + (-7) + 15$

20. $9 + (-14) + 3$

21. During the first play of a football game, you lose 3 yards. You gain 7 yards during the second play. What is your total gain of yards for these two plays?

22. Write an addition expression using integers that equals -2. Use a number line to justify your answer.

23. Describe a real-life situation that uses the sum of the integers -8 and 12.

1.3 Adding Rational Numbers *(pp. 17–22)*

Learning Target: Find sums of rational numbers.

Find the sum. Write fractions in simplest form.

24. $\frac{9}{10} + \left(-\frac{4}{5}\right)$

25. $-4\frac{5}{9} + \frac{8}{9}$

26. $-1.6 + (-2.4)$

27. Find the sum of $-4 + 6\frac{2}{5} + (-2.7)$. Explain each step.

28. You open a new bank account. The table shows the activity of your account for the first month. Positive numbers represent deposits and negative numbers represent withdrawals. What is your balance (in dollars) in the account at the end of the first month?

Date	Amount (dollars)
3/5	100
3/12	−12.25
3/16	25.82
3/21	14.95
3/29	−18.56

1.4 Subtracting Integers *(pp. 23–28)*

Learning Target: Find differences of integers.

Find the difference. Use a number line to verify your answer.

29. $8 - 18$ **30.** $-16 - (-5)$ **31.** $-18 - 7$ **32.** $-12 - (-27)$

33. Your score on a game show is -300. You answer the final question incorrectly, so you lose 400 points. What is your final score?

34. Oxygen has a boiling point of $-183°C$ and a melting point of $-219°C$. What is the temperature difference of the melting point and the boiling point?

35. In one month, you earn \$16 for mowing the lawn, \$15 for babysitting, and \$20 for allowance. You spend \$12 at the movie theater. How much more money do you need to buy a \$45 video game?

36. Write a subtraction expression using integers that equals -6.

37. Write two negative integers whose difference is positive.

1.5 Subtracting Rational Numbers *(pp. 29–36)*

Learning Target: Find differences of rational numbers and find distances between numbers on a number line.

Find the difference. Write fractions in simplest form.

38. $-\frac{5}{12} - \frac{3}{10}$ **39.** $3\frac{3}{4} - \frac{7}{8}$ **40.** $3.8 - (-7.45)$

41. Find the distance between -3.71 and -2.59 on a number line.

42. A turtle is $20\frac{5}{6}$ inches below the surface of a pond. It dives to a depth of $32\frac{1}{4}$ inches. What is the change in the turtle's position?

43. The lowest temperature ever recorded on Earth was $-89.2°C$ at Soviet Vostok Station in Antarctica. The highest temperature ever recorded was $56.7°C$ at Greenland Ranch in California. What is the difference between the highest and lowest recorded temperatures?

1 Practice Test

Find the absolute value.

1. $\left|-\frac{4}{5}\right|$ **2.** $|6.43|$ **3.** $|-22|$

Copy and complete the statement using <, >, or =.

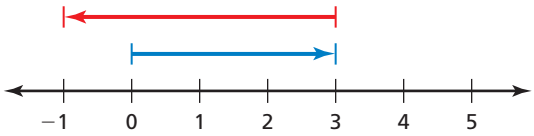
4. $4 \square |-8|$ **5.** $|-7| \square -12$ **6.** $-7 \square |3|$

Add or subtract. Write fractions in simplest form.

7. $-6 + (-11)$ **8.** $2 - (-9)$ **9.** $-\frac{4}{9} + \left(-\frac{23}{18}\right)$

10. $\frac{17}{12} - \left(-\frac{1}{8}\right)$ **11.** $9.2 + (-2.8)$ **12.** $2.86 - 12.1$

13. Write an addition expression and write a subtraction expression represented by the number line. Then evaluate the expressions.



14. The table shows your scores, relative to *par*, for nine holes of golf. What is your total score for the nine holes?

Hole	1	2	3	4	5	6	7	8	9
Score	+1	−2	−1	0	−1	+3	−1	−3	+1

15. The elevation of a fish is −27 feet. The fish descends 32 feet, and then rises 14 feet. What is its new elevation?

16. The table shows the rainfall (in inches) for three months compared to the yearly average. Is the total rainfall for the three-month period greater than or less than the yearly average? Explain.

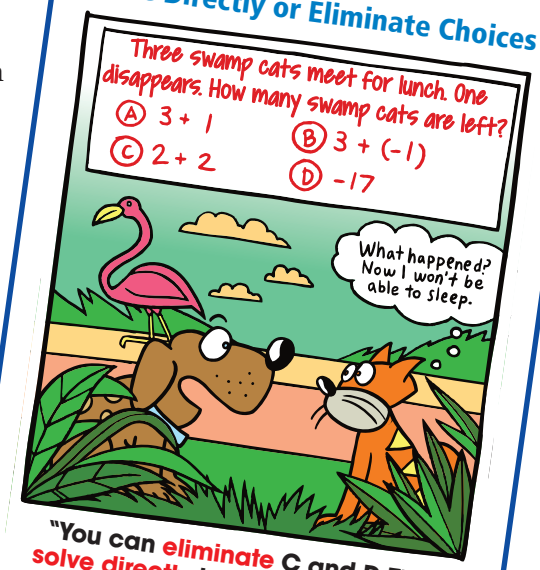
October	November	December
−0.86	2.56	−1.24

17. Bank Account A has $750.92, and Bank Account B has $675.44. Account A changes by –$216.38, and Account B changes by −$168.49. Which account has the greater balance? Explain.

18. On January 1, you recorded the lowest temperature as 23°F and the highest temperature as 6°C. A formula for converting from degrees Fahrenheit F to degrees Celsius C is $C = \frac{5}{9}F - \frac{160}{9}$. What is the temperature range (in degrees Celsius) for January 1?

1 Cumulative Practice

Test-Taking Strategy
Solve Directly or Eliminate Choices



"You can eliminate C and D. Then solve directly to determine that the correct answer is B."

1. A football team gains 2 yards on the first play, loses 5 yards on the second play, loses 3 yards on the third play, and gains 4 yards on the fourth play. What is the team's total gain or loss?

A. a gain of 14 yards **B.** a gain of 2 yards

C. a loss of 2 yards **D.** a loss of 14 yards

2. Which expression is *not* equal to 0?

F. $5 - 5$ **G.** $-7 + 7$

H. $6 - (-6)$ **I.** $-8 - (-8)$

3. What is the value of the expression?

$$|-2 - (-2.5)|$$

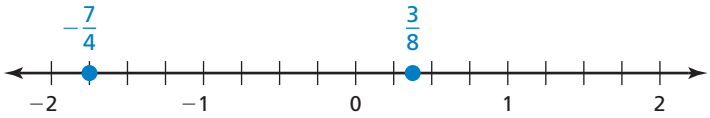
A. -4.5 **B.** -0.5

C. 0.5 **D.** 4.5

4. What is the value of the expression?

$$17 - (-8)$$

5. What is the distance between the two numbers on the number line?



F. $-2\frac{1}{8}$ **G.** $-1\frac{3}{8}$

H. $1\frac{3}{8}$ **I.** $2\frac{1}{8}$

6. What is the value of the expression when $a = 8$, $b = 3$, and $c = 6$?

$$\left| a^2 - 2ac + 5b \right|$$

A. -65

B. -17

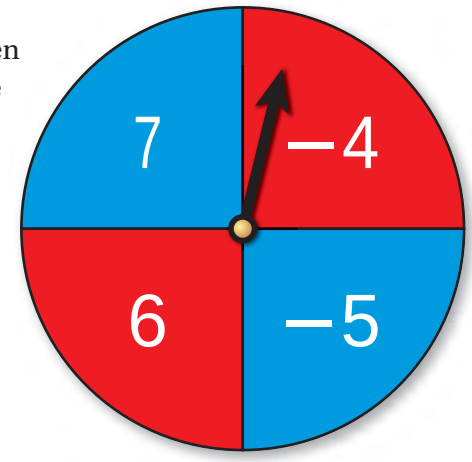
C. 17

D. 65

7. What is the value of the expression?

$$-9.74 + (-2.23)$$

8. Four friends are playing a game using the spinner shown. Each friend starts with a score of 0 and then spins four times. When you spin blue, you add the number to your score. When you spin red, you subtract the number from your score. The highest score after four spins wins. Each friend's spins are shown. Which spins belong to the winner?



F. 6, 7, 7, 6

G. $-4, -4, 7, -5$

H. $6, -5, -4, 7$

I. $-5, 6, -5, 6$

9. What number belongs in the box to make the equation true?

$$3\frac{1}{2} \div 5\frac{2}{3} = \frac{7}{2} \times \square$$

A. $\frac{3}{17}$

B. $\frac{3}{2}$

C. $\frac{17}{3}$

D. $\frac{13}{2}$

10. What is the value of the expression?

$$\frac{5.2 - 2.25}{0.05}$$

F. -346

G. 0.59

H. 5.9

I. 59

11. You leave school and walk 1.237 miles west. Your friend leaves school and walks 0.56 mile east. How far apart are you and your friend?

A. 0.677 mile

B. 0.69272 mile

C. 1.293 miles

D. 1.797 miles

12. Which property does the equation represent?

$$-80 + 30 + (-30) = -80 + [30 + (-30)]$$

F. Commutative Property of Addition

G. Associative Property of Addition

H. Additive Inverse Property

I. Addition Property of Zero

13. The values of which two points have the greatest sum?

A. R and S

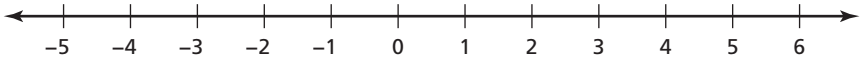
B. R and U

C. S and T

D. T and U

14. Consider the number line shown.

Think
Solve
Explain



Part A Use the number line to explain how to add -2 and -3.

Part B Use the number line to explain how to subtract 5 from 2.

15. Which expression represents a *negative* value?

F. $2 - |-7 + 3|$

G. $|-12 + 9|$

H. $|5| + |11|$

I. $|8 - 14|$