7.3 Areas of Trapezoids and Kites

Learning Target: Find areas of trapezoids, kites, and composite figures.

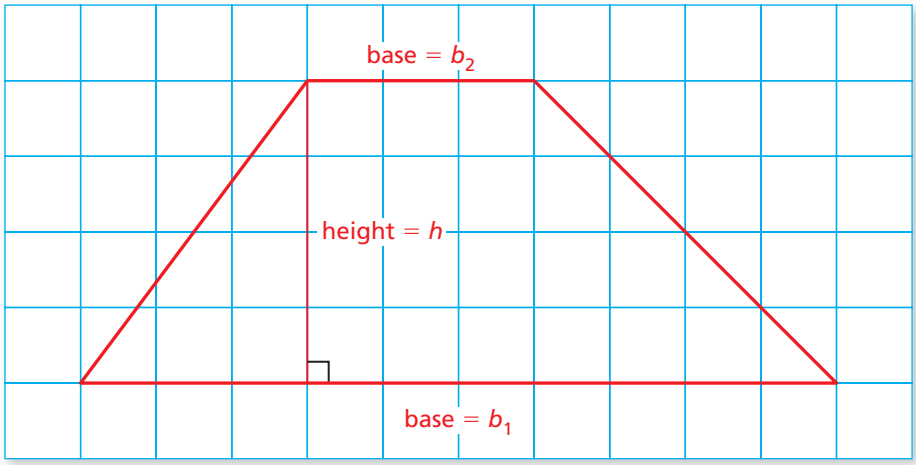
Success Criteria:
- I can explain how the area of a parallelogram is used to find the area of a trapezoid.
- I can decompose trapezoids and kites into smaller shapes.
- I can use decomposition to find the area of a figure.
- I can use the bases and the height of a trapezoid to find its area.

EXPLORATION 1 Deriving the Area Formula of a Trapezoid

Work with a partner.

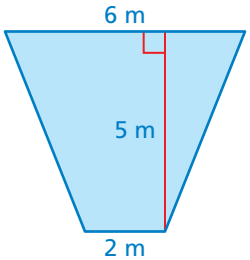
a. Draw *any* parallelogram on a piece of centimeter grid paper. Cut the parallelogram into two identical trapezoids. How can you use the area of the parallelogram to find the area of each trapezoid?

b. Copy the trapezoid below on a piece of centimeter grid paper. Find the area of the trapezoid. Explain how you found the area.



c. Draw *any* trapezoid on a piece of centimeter grid paper and find its area.

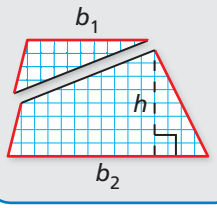
d. Use your results to write a formula for the area A of a trapezoid. Use the formula to find the area of the trapezoid shown.



Math Practice

Make a Plan

How can you use the diagram below to justify the formula you wrote in part (d)?



7.3 Lesson

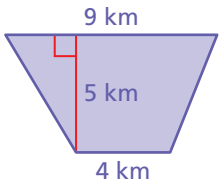
Key Vocabulary
kite, *p. 298*

You can use decomposition to find areas of trapezoids and *kites*. A **kite** is a quadrilateral that has two pairs of adjacent sides with the same length and opposite sides with different lengths.

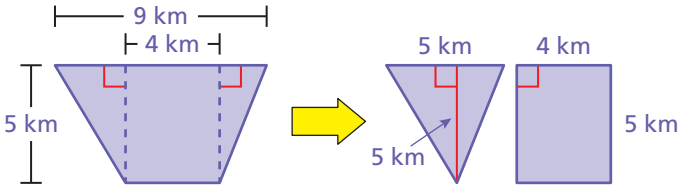
EXAMPLE 1 Finding Areas of Trapezoids and Kites

Find the area of each figure.

a.



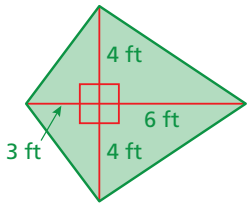
Decompose the trapezoid into a triangle and a rectangle. Find the sum of the areas of the figures.



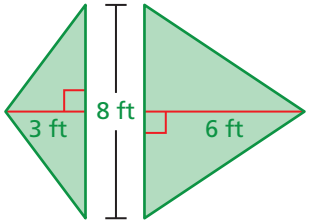
$$A = \frac{1}{2}(5)(5) + 5(4)$$
$$= 12\frac{1}{2} + 20$$
$$= 32\frac{1}{2}$$

The area of the trapezoid is $32\frac{1}{2}$ square kilometers.

b.



Decompose the kite into two triangles. Find the sum of the areas of the triangles.

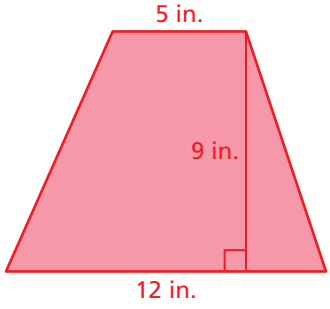


$$A = \frac{1}{2}(8)(3) + \frac{1}{2}(8)(6)$$
$$= 12 + 24$$
$$= 36$$

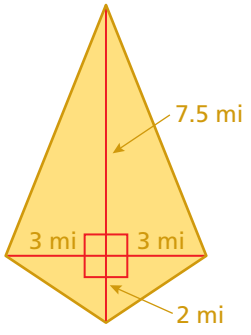
The area of the kite is 36 square feet.

Try It **Find the area of the figure.**

1.



2.



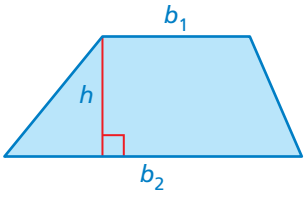
In Example 1(a), you could have used a copy of the trapezoid to form a parallelogram. As you may have discovered in the exploration, this leads to the following formula for the area of a trapezoid.

Key Idea

Area of a Trapezoid

Words The area A of a trapezoid is one-half the product of its height h and the sum of its bases b_1 and b_2.

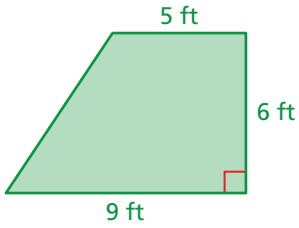
Algebra $A = \frac{1}{2}h(b_1 + b_2)$



EXAMPLE 2 Finding Areas of Trapezoids

Find the area of each trapezoid.

a.



$A = \frac{1}{2}h(b_1 + b_2)$ Write formula.

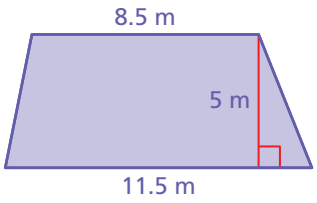
$= \frac{1}{2}(6)(5 + 9)$ Substitute.

$= \frac{1}{2}(6)(14)$ Add.

$= 42$ Multiply.

The area of the trapezoid is 42 square feet.

b.



$A = \frac{1}{2}h(b_1 + b_2)$

$= \frac{1}{2}(5)(8.5 + 11.5)$

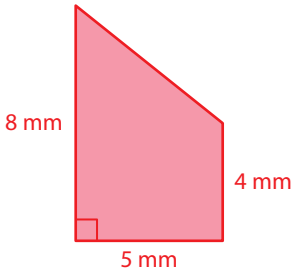
$= \frac{1}{2}(5)(20)$

$= 50$

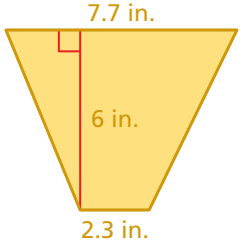
The area of the trapezoid is 50 square meters.

Try It **Find the area of the trapezoid.**

3.



4.

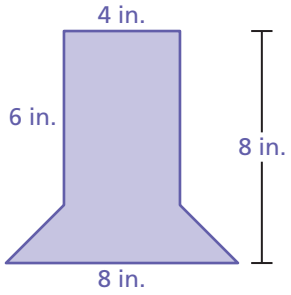


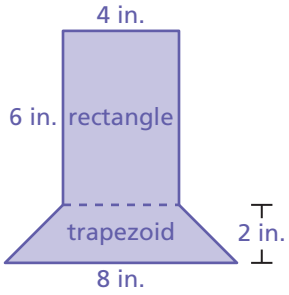
EXAMPLE 3 Finding the Area of a Composite Figure

There is often more than one way to separate composite figures. In Example 3, you can separate the figure into one rectangle and two triangles.

Find the area of the figure.

You can separate the figure into a rectangle and a trapezoid. Identify the height of the trapezoid. Then find the area of each shape.





Area of Rectangle

$$A = \ell w$$
$$= 6(4)$$
$$= 24$$

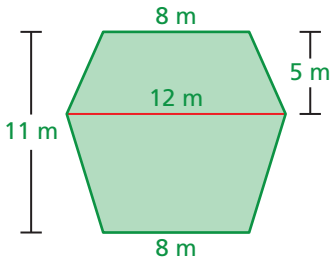
Area of Trapezoid

$$A = \frac{1}{2}h(b_1 + b_2)$$
$$= \frac{1}{2}(2)(4 + 8)$$
$$= 12$$

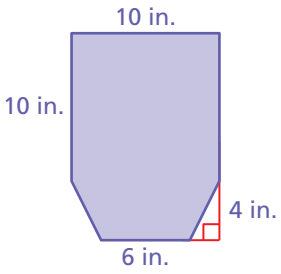
So, the area of the figure is $24 + 12 = 36$ square inches.

Try It **Find the area of the figure.**

5.



6.



Self-Assessment for Concepts & Skills

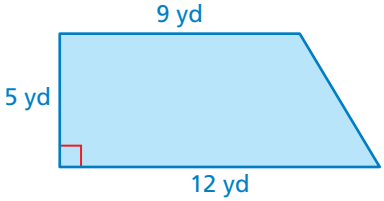
Solve each exercise. Then rate your understanding of the success criteria in your journal.

7. **WRITING** Explain how to use the area of a parallelogram to find the area of a trapezoid.

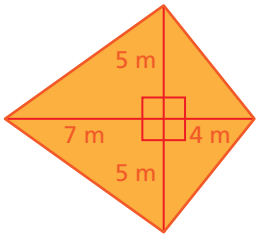
8. MP **REASONING** What measures do you need to find the area of a kite?

FINDING AREA **Find the area of the figure.**

9.

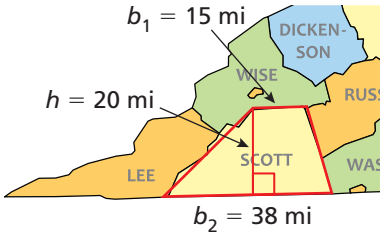


10.



EXAMPLE 4 Modeling Real Life

You can use a trapezoid to approximate the shape of Scott County, Virginia. The population is about 22,100. About how many people are there per square mile?



Understand the problem.

You are given the population and the dimensions of a county shaped like a trapezoid. You are asked to find the number of people per square mile.

Make a plan.

Use the formula for the area of a trapezoid to find the area of Scott County. Then divide the population by the area to find the number of people per square mile.

Solve and check.

$$A = \frac{1}{2}h(b_1 + b_2)$$ Write formula for area of a trapezoid.

$$= \frac{1}{2}(20)(15 + 38)$$ Substitute 20 for h, 15 for b_1, and 38 for b_2.

$$= \frac{1}{2}(20)(53)$$ Add.

$$= 530$$ Multiply.

The area of Scott County is about 530 square miles.

So, there are about $\frac{22{,}100 \text{ people}}{530 \text{ mi}^2} \approx 42$ people per square mile.

Check Reasonableness

Round the population to 20,000 and the area to 500 square miles to obtain an estimate that is simpler to calculate.

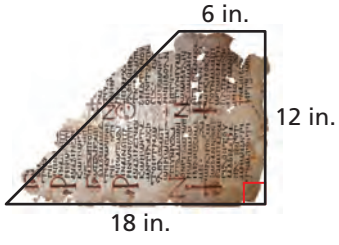
$20{,}000 \div 500 = 40$

The answer is reasonable because $40 \approx 42$. ✓

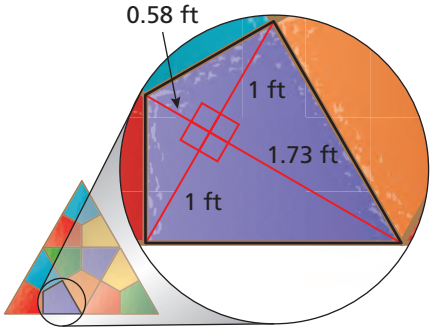
Self-Assessment for Problem Solving

Solve each exercise. Then rate your understanding of the success criteria in your journal.

11. DIG DEEPER! An archaeologist estimates that the manuscript shown was originally a rectangle with a length of 20 inches. Estimate the area of the fragment that is missing.



12. The stained-glass window is made of identical kite-shaped glass panes. The approximate dimensions of one pane are shown. The glass used to make the window costs $12.50 per square foot. Find the total cost of the glass used to make the window.



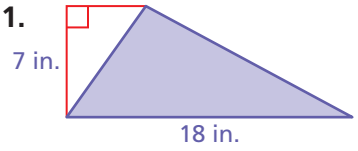
7.3 Practice

Go to **BigIdeasMath.com** to get HELP with solving the exercises.

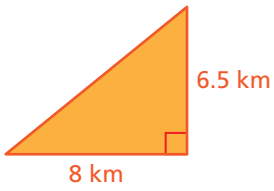
Review & Refresh

Find the area of the triangle.

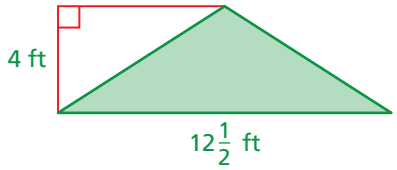
1.



2.



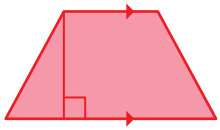
3.



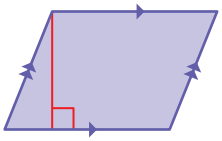
Classify the quadrilateral.

4.

5.



6.



7. On a normal day, 12 airplanes arrive at an airport every 15 minutes. Which rate does *not* represent this situation?

A. 24 airplanes every 30 minutes

B. 4 airplanes every 5 minutes

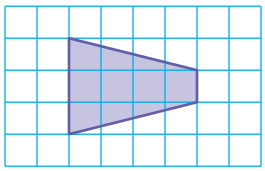
C. 6 airplanes every 5 minutes

D. 48 airplanes each hour

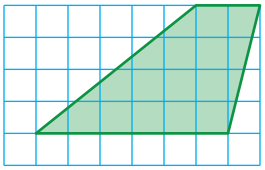
Concepts, Skills, & Problem Solving

USING TOOLS **Find the area of the trapezoid by forming a parallelogram.**
(See Exploration 1, p. 297.)

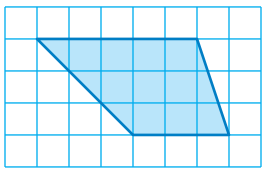
8.



9.

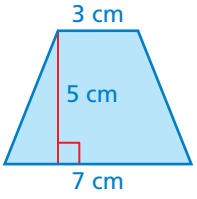


10.

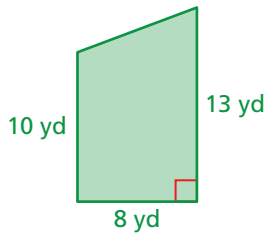


FINDING AREA **Use decomposition to find the area of the figure.**

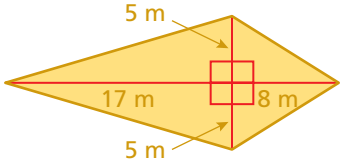
11.



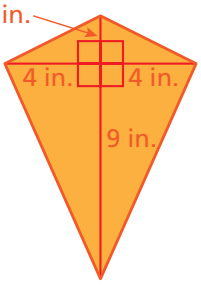
12.



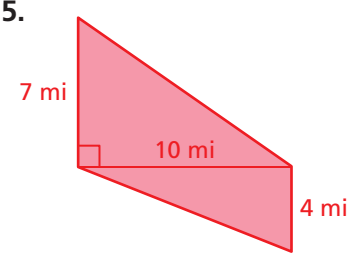
13.



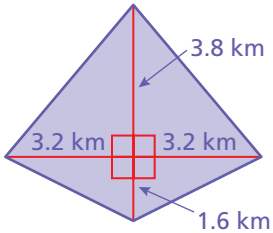
14.



15.

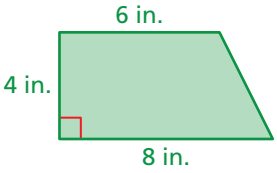


16.

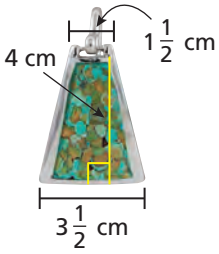


FINDING AREA **Find the area of the trapezoid.**

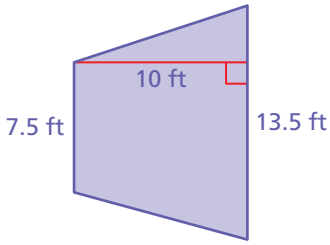
17.



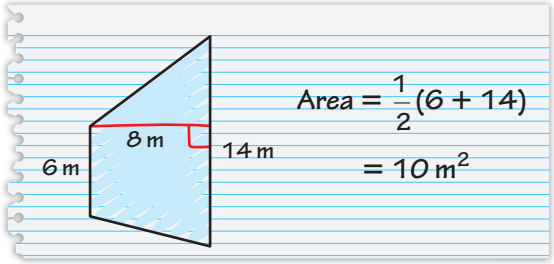
18.

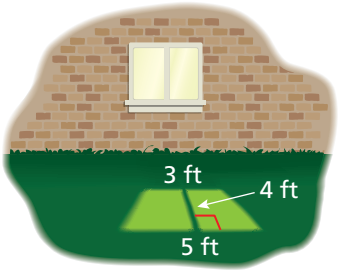


19.



20. **YOU BE THE TEACHER** Your friend finds the area of the trapezoid. Is your friend correct? Explain your reasoning.

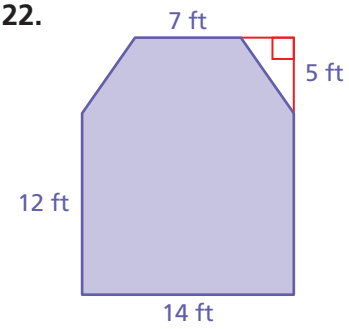




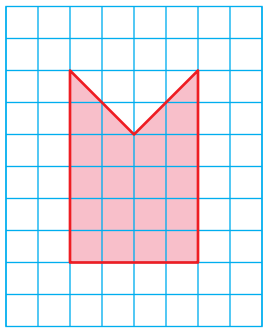
21. **MODELING REAL LIFE** Light shines through a window. What is the area of the trapezoid-shaped region created by the light?

COMPOSITE FIGURES **Find the area of the figure.**

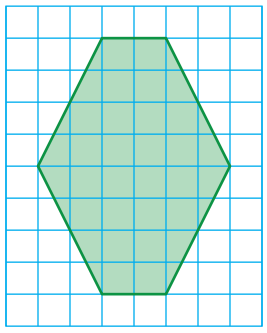
22.



23.

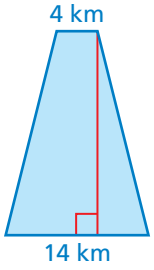


24.

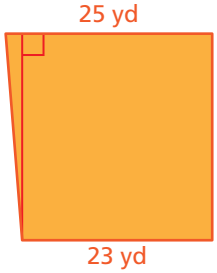


FINDING A MISSING DIMENSION **Find the height of the trapezoid.**

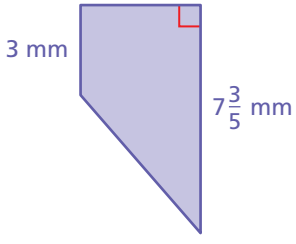
25. Area $= 180 \text{ km}^2$



26. Area $= 600 \text{ yd}^2$



27. Area $= 21\frac{1}{5} \text{ mm}^2$



FINDING AREA **Find the area (in square feet) of a trapezoid with height h and bases b_1 and b_2.**

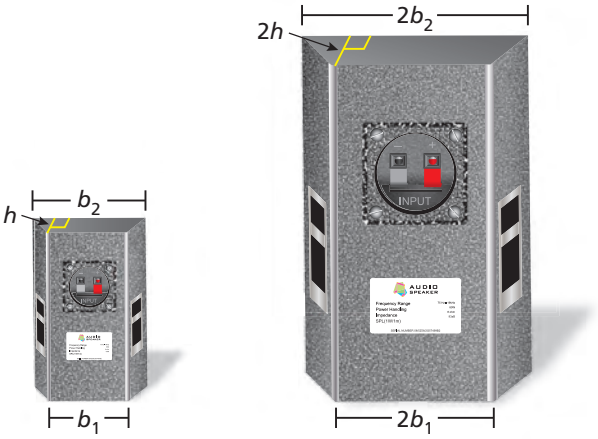
28. $h = 6$ in.
$b_1 = 9$ in.
$b_2 = 12$ in.

29. $h = 12$ yd
$b_1 = 5$ yd
$b_2 = 7$ yd

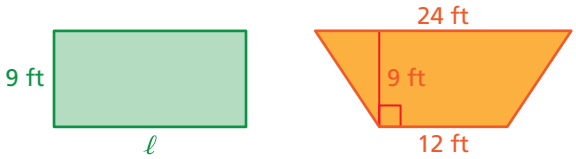
30. $h = 6$ m
$b_1 = 3$ m
$b_2 = 8$ m

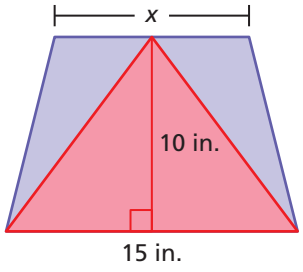
31. **OPEN-ENDED** The area of the trapezoidal student election sign is 5 square feet. Find two possible values for each base length.

32. **REASONING** How many times greater is the area of the floor covered by the larger speaker than by the smaller speaker?



33. **REASONING** The rectangle and the trapezoid have the same area. What is the length ℓ of the rectangle?





34. **CRITICAL THINKING** In the figure shown, the area of the trapezoid is less than twice the area of the triangle. Find the possible values of x. Can the trapezoid have the same area as the triangle? Explain your reasoning.

35. **STRUCTURE** In Section 7.1 Exercise 34 and Section 7.2 Exercise 29, you wrote a formula for the area of a rhombus in terms of its diagonals.

a. Use what you know about finding areas of figures to write a formula for the area of a kite in terms of its diagonals.

b. Are there any similarities between your formula in part (a) and the formula you found in Sections 7.1 and 7.2? Explain why or why not.