6.4 Writing Equations in Two Variables

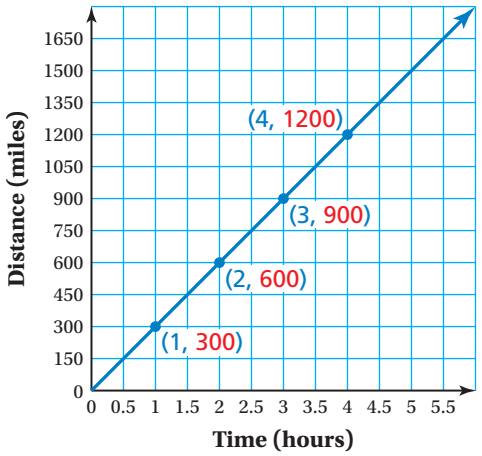
Learning Target: Write equations in two variables and analyze the relationship between the two quantities.

Success Criteria:
- I can determine whether an ordered pair is a solution of an equation in two variables.
- I can distinguish between independent and dependent variables.
- I can write and graph an equation in two variables.
- I can create equations in two variables to solve real-life problems.

EXPLORATION 1 Writing Equations in Two Variables

Work with a partner. In Section 3.4 Exploration 1, you used a ratio table to create a graph for an airplane traveling 300 miles per hour. Below is one possible ratio table and graph.

Time (hours)	1	2	3	4
Distance (miles)	300	600	900	1200



Math Practice

Look for Patterns
How can you use the patterns in the table to help you write an equation?

a. Describe the relationship between the two quantities. Which quantity *depends* on the other quantity?

b. Use variables to write an equation that represents the relationship between the time and the distance. What can you do with this equation? Provide an example.

c. Suppose the airplane is 1500 miles away from its destination. Write an equation that represents the relationship between time and distance from the destination. How can you represent this relationship using a graph?

6.4 Lesson

An **equation in two variables** represents two quantities that change in relationship to one another. A **solution of an equation in two variables** is an ordered pair that makes the equation true.

Key Vocabulary

equation in two variables, *p. 266*

solution of an equation in two variables, *p. 266*

independent variable, *p. 266*

dependent variable, *p. 266*

EXAMPLE 1 Identifying Solutions of Equations in Two Variables

Tell whether the ordered pair is a solution of the equation.

a. $y = 2x$; $(3, 6)$

$6 \stackrel{?}{=} 2(3)$ Substitute.

$6 = 6$ ✓ Compare.

So, $(3, 6)$ is a solution.

b. $y = 4x - 3$; $(4, 12)$

$12 \stackrel{?}{=} 4(4) - 3$

$12 \neq 13$ ✗

So, $(4, 12)$ is *not* a solution.

Try It **Tell whether the ordered pair is a solution of the equation.**

1. $y = 7x$; $(2, 21)$

2. $y = 5x + 1$; $(3, 16)$

Equations in two variables have an *independent variable* and a *dependent variable*. The variable representing the quantity that can change freely is the **independent variable**. The other variable is called the **dependent variable** because its value *depends* on the independent variable.

EXAMPLE 2 Using an Equation in Two Variables

The equation $y = 64 - 8x$ represents the amount y (in fluid ounces) of chemical remaining in a flask after you pour x cups. Identify the independent and dependent variables. How much of the chemical remains in the flask after you pour 5 cups?

Because the amount y of fluid ounces remaining depends on the number x of cups you pour, y is the dependent variable and x is the independent variable.

Use the equation to find the value of y when $x = 5$.

$y = 64 - 8x$ Write the equation.

$= 64 - 8(5)$ Substitute 5 for x.

$= 24$ Simplify.

There are 24 fluid ounces remaining.

Try It

3. The equation $y = 10x + 25$ represents the amount y (in dollars) in your savings account after x weeks. Identify the independent and dependent variables. How much is in your savings account after 8 weeks?

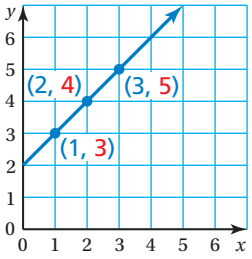
Key Idea

Tables, Graphs, and Equations

You can use tables and graphs to represent equations in two variables. The independent variable is graphed on the horizontal axis, and the dependent variable is graphed on the vertical axis. The table and graph below represent the equation $y = x + 2$.

When you draw a line through the points, you graph *all* the solutions of the equation.

Independent Variable, x	Dependent Variable, y	Ordered Pair, (x, y)
1	3	(1, 3)
2	4	(2, 4)
3	5	(3, 5)



EXAMPLE 3 Graphing an Equation in Two Variables

Graph $y = 2x + 1$.

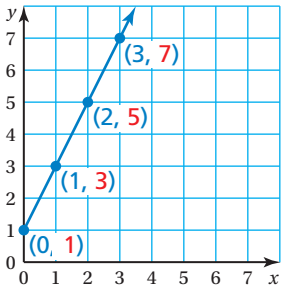
To graph the equation, first make a table.

Independent Variable, x	$y = 2x + 1$	Dependent Variable, y	Ordered Pair, (x, y)
0	$y = 2(0) + 1$	1	(0, 1)
1	$y = 2(1) + 1$	3	(1, 3)
2	$y = 2(2) + 1$	5	(2, 5)
3	$y = 2(3) + 1$	7	(3, 7)

Remember

In a coordinate plane, the horizontal axis is often called the *x-axis*. The vertical axis is often called the *y-axis*. In real-life problems, other variables can be used.

Then plot the ordered pairs and draw a line through the points.



Try It **Graph the equation.**

4. $y = 3x$ **5.** $y = 4x + 1$ **6.** $y = \frac{1}{2}x + 2$

EXAMPLE 4 Writing and Graphing an Equation in Two Variables

An athlete burns 200 calories weight lifting. The athlete then works out on an elliptical trainer and burns 10 calories for every minute. Write and graph an equation that represents the total number of calories burned during the workout.

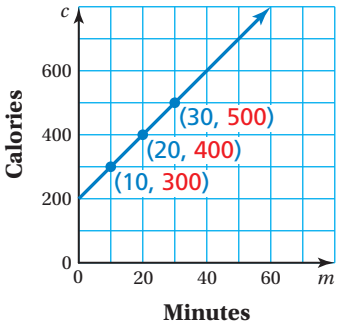
Use a verbal model to write an equation.

Verbal Model

Total number of calories burned	=	Calories burned weight lifting	+	Calories burned per minute	•	Number of minutes

Variables Let c be the total number of calories burned, and let m be the number of minutes on the elliptical trainer.

Equation $c = 200 + 10 \cdot m$



To graph the equation, first notice that the total number of calories burned depends on the number of minutes. So, create a table and plot the ordered pairs with minutes m on the horizontal axis and calories c on the vertical axis. Then draw a line through the points.

Minutes, m	Calories, c
10	300
20	400
30	500

Try It

7. It costs \$25 to rent a kayak plus \$8 for each hour. Write and graph an equation that represents the total cost (in dollars) of renting the kayak.

Self-Assessment for Concepts & Skills

Solve each exercise. Then rate your understanding of the success criteria in your journal.

8. WRITING Describe the difference between independent variables and dependent variables.

IDENTIFYING SOLUTIONS **Tell whether the ordered pair is a solution of the equation.**

9. $y = 3x + 8$; $(4, 20)$

10. $y = 6x - 14$; $(7, 29)$

11. MP PRECISION Explain how to graph an equation in two variables.

12. WHICH ONE DOESN'T BELONG? Which one does *not* belong with the other three? Explain your reasoning.

$y = 12x + 25$ $c = 10t - 5$ $a = 7b + 11$ $n = 4n - 6$

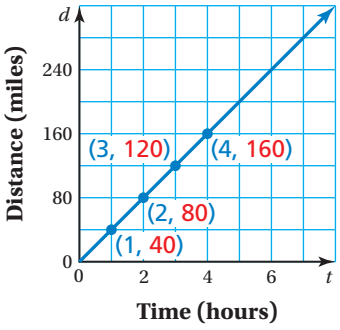
You can model many rate problems by using the *distance formula*, $d = rt$, where d is the distance traveled, r is the speed, and t is the time.

EXAMPLE 5 Modeling Real Life

A train averages 40 miles per hour between two cities. Write and graph an equation that represents the relationship between the time and the distance traveled. How long does it take the train to travel 220 miles?

The rate r is 40 miles per hour. Using the distance formula, an equation that represents the relationship between time and distance traveled is $d = 40t$.

Make a table and graph the equation.



Time (hours), t	1	2	3	4
Distance (miles), d	40	80	120	160

Use the equation to find the value of t when $d = 220$.

$d = 40t$ Write the equation.

$220 = 40t$ Substitute 220 for d.

$5.5 = t$ Divide each side by 40.

The train travels 220 miles in 5.5 hours.

You can also tell from the graph that the distance is about 220 miles after 5.5 hours.

Another Method Use a ratio table.

Time (hours), t	1	0.5	5.5 ✓
Distance (miles), d	40	20	220

÷2 ×11 ÷2 ×11

Self-Assessment for Problem Solving

Solve each exercise. Then rate your understanding of the success criteria in your journal.

13. A sky lantern rises at an average speed of 8 feet per second. Write and graph an equation that represents the relationship between the time and the distance risen. How long does it take the lantern to rise 100 feet?

14. You and a friend start biking in opposite directions from the same point. You travel 108 feet every 8 seconds. Your friend travels 63 feet every 6 seconds. How far apart are you and your friend after 15 minutes?

6.4 Practice

Go to **BigIdeasMath.com** to get HELP with solving the exercises.

Review & Refresh

Solve the equation.

1. $4x = 36$ **2.** $\frac{x}{8} = 5$ **3.** $\frac{4x}{3} = 8$ **4.** $\frac{2}{5}x = 6$

Divide. Write the answer in simplest form.

5. $3\frac{1}{2} \div \frac{4}{5}$ **6.** $7 \div 5\frac{1}{4}$ **7.** $\frac{3}{11} \div 1\frac{1}{8}$ **8.** $7\frac{1}{2} \div 1\frac{1}{3}$

9. Find the area of the carpet tile. Then find the area covered by 120 carpet tiles.

Copy and complete the statement. Round to the nearest hundredth if necessary.

10. 8 m = ____ cm **11.** 88 oz = ____ lb

12. 3 c ≈ ____ mL **13.** 15 km ≈ ____ mi

Divide.

14. $6\overline{)34.8}$ **15.** $4\overline{)12.8}$ **16.** $45.92 \div 2.8$ **17.** $39.525 \div 4.25$

Concepts, Skills, & Problem Solving

WRITING EQUATIONS **Use variables to write an equation that represents the relationship between the time and the distance.** (See Exploration 1, p. 265.)

18. An eagle flies 40 miles per hour.

19. A person runs 175 yards per minute.

IDENTIFYING SOLUTIONS **Tell whether the ordered pair is a solution of the equation.**

20. $y = 4x$; (0, 4) **21.** $y = 3x$; (2, 6) **22.** $y = 5x - 10$; (3, 5)

23. $y = x + 7$; (1, 6) **24.** $y = x + 4$; (2, 4) **25.** $y = x - 5$; (6, 11)

26. $y = 6x + 1$; (2, 13) **27.** $y = 7x + 2$; (2, 0) **28.** $y = 2x - 3$; (4, 5)

29. $y = 3x - 3$; (1, 0) **30.** $7 = y - 5x$; (4, 28) **31.** $y + 3 = 6x$; (3, 15)

32. **YOU BE THE TEACHER** Your friend determines whether (5, 1) is a solution of $y = 3x + 2$. Is your friend correct? Explain your reasoning.

$y = 3x + 2$; (5, 1)

$1 \stackrel{?}{=} 3(5) + 2$

$1 \neq 17$

So, (5, 1) is not a solution.

IDENTIFYING VARIABLES **Identify the independent and dependent variables.**

33. The equation $A = 25w$ represents the area A (in square feet) of a rectangular dance floor with a width of w feet.

34. The equation $c = 0.09s$ represents the amount c (in dollars) of commission a salesperson receives for making a sale of s dollars.

35. The equation $t = 12p + 12$ represents the total cost t (in dollars) of a meal with a tip of p percent (in decimal form).

36. The equation $h = 60 - 4m$ represents the height h (in inches) of the water in a tank m minutes after it starts to drain.

OPEN-ENDED **Complete the table by describing possible independent or dependent variables.**

	Independent Variable	Dependent Variable
37.	The number of hours you study for a test	
38.	The speed you are pedaling a bike	
39.		Your monthly cell phone bill
40.		The amount of money you earn

GRAPHING EQUATIONS **Graph the equation.**

41. $y = 2x$

42. $y = 5x$

43. $y = 6x$

44. $y = x + 2$

45. $y = x + 0.5$

46. $y = x + 4$

47. $y = x + 10$

48. $y = 3x + 2$

49. $y = 2x + 4$

50. $y = \frac{2}{3}x + 8$

51. $y = \frac{1}{4}x + 6$

52. $y = 2.5x + 12$

53. **MODELING REAL LIFE** A cheese pizza costs \$5. Additional toppings cost \$1.50 each. Write and graph an equation that represents the total cost (in dollars) of a pizza.

54. **MODELING REAL LIFE** It costs \$35 for a membership at a wholesale store. The monthly fee is \$15. Write and graph an equation that represents the total cost (in dollars) of a membership.

55. **MP PROBLEM SOLVING** The maximum size of a text message is 160 characters. A space counts as one character.

a. Write an equation that represents the number of remaining (unused) characters in a text message as you type.

b. Identify the independent and dependent variables.

c. How many characters remain in the message shown?

56. **CHOOSE TOOLS** A car averages 60 miles per hour on a road trip. Use a graph to represent the relationship between the time and the distance traveled.

PRECISION **Write and graph an equation that represents the relationship between the time and the distance traveled.**

57.

Moves 2 meters every 3 hours

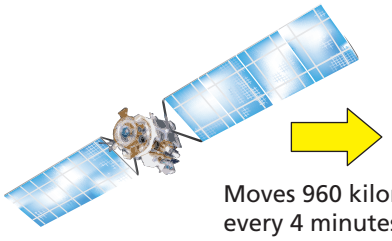
58.

Rises 5 stories every 6 seconds

59.

Moves 660 feet every 10 seconds

60.



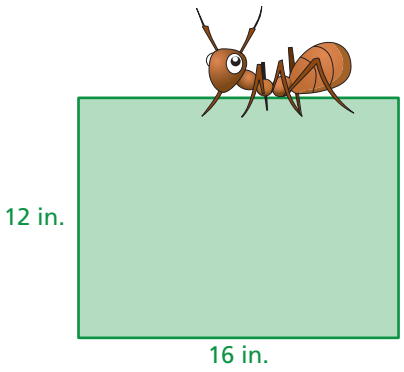
Moves 960 kilometers every 4 minutes

IDENTIFYING SOLUTIONS **Fill in the blank so that the ordered pair is a solution of the equation.**

61. $y = 8x + 3$; (1, ____)

62. $y = 12x + 2$; (____, 14)

63. $y = 9x + 4$; (____, 22)



64. **DIG DEEPER!** Can the dependent variable cause a change in the independent variable? Explain.

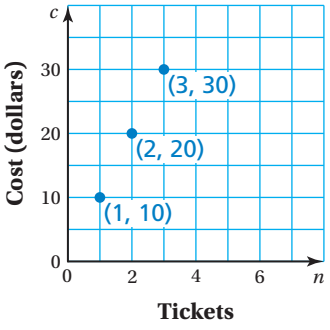
65. **OPEN-ENDED** Write an equation that has (3, 4) as a solution.

66. **MODELING REAL LIFE** You walk 5 city blocks in 12 minutes. How many city blocks can you walk in 2 hours?

67. **GEOMETRY** How fast should the ant walk to go around the rectangle in 4 minutes?

68. **MODELING REAL LIFE** To estimate how far you are from lightning (in miles), count the number of seconds between a lightning flash and the thunder that follows. Then divide the number of seconds by 5. Use two different methods to find the number of seconds between a lightning flash and the thunder that follows when a storm is 2.4 miles away.

69. **REASONING** The graph represents the cost c (in dollars) of buying n tickets to a baseball game.



a. Should the points be connected with a line to show all the solutions? Explain your reasoning.

b. Write an equation that represents the graph.

6 Connecting Concepts

Using the Problem-Solving Plan

1. A tornado forms 12.25 miles from a weather station. It travels away from the station at an average speed of 440 yards per minute. How far from the station is the tornado after 30 minutes?

Understand the problem.

You know the initial distance between the tornado and the station, and the average speed the tornado is traveling away from the station. You are asked to determine how far the tornado is from the station after 30 minutes.

Make a plan.

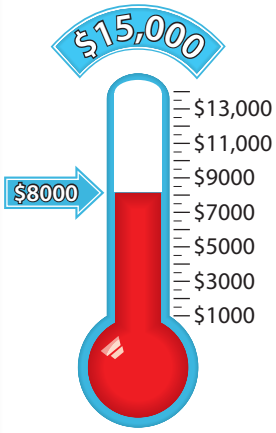
First, convert the average speed to miles per minute. Then write an equation that represents the distance d (in miles) between the tornado and the station after t minutes. Use the equation to find the value of d when $t = 30$.

Solve and check.

Use the plan to solve the problem. Then check your solution.

2. You buy 96 cans of soup to donate to a food bank. The store manager discounts the cost of each case for a total discount of \$40. Use an equation in two variables to find the discount for each case of soup. What is the total cost when each can of soup originally costs \$1.20?

3. The diagram shows the initial amount raised by an organization for cancer research. A business agrees to donate \$2 for every \$5 donated by the community during an additional fundraising event. Write an equation that represents the total amount raised (in dollars). How much money does the community need to donate for the organization to reach its fundraising goal?



Performance Task

Planning the Climb

At the beginning of this chapter, you watched a STEAM video called "Rock Climbing." You are now ready to complete the performance task related to this video, available at ***BigIdeasMath.com***. Be sure to use the problem-solving plan as you work through the performance task.

6 Chapter Review

Go to *BigIdeasMath.com* to download blank graphic organizers.

Review Vocabulary

Write the definition and give an example of each vocabulary term.

equation, *p. 246*
solution, *p. 252*
inverse operations, *p. 253*
equation in two variables, *p. 266*
solution of an equation in two variables, *p. 266*
independent variable, *p. 266*
dependent variable, *p. 266*

Graphic Organizers

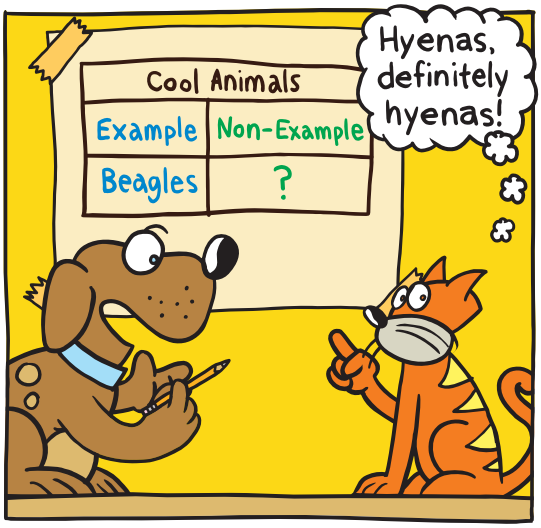
You can use an **Example and Non-Example Chart** to list examples and non-examples of a concept. Here is an Example and Non-Example Chart for the vocabulary term ***equation***.

Equation

Examples	Non-Examples
$x = 5$	5
$2a = 16$	$2a$
$x + 4 = 19$	$x + 4$
$5 = x + 3$	$x + 3$
$12 - 7 = 5$	$12 - 7$
$\frac{3}{4}y = 6$	$\frac{3}{4}$

Choose and complete a graphic organizer to help you study the concept.

1. inverse operations
2. solving equations using addition or subtraction
3. solving equations using multiplication or division
4. equations in two variables
5. independent variables
6. dependent variables



"I need a good non-example of a cool animal for my Example and Non-Example Chart."

Chapter Self-Assessment

As you complete the exercises, use the scale below to rate your understanding of the success criteria in your journal.

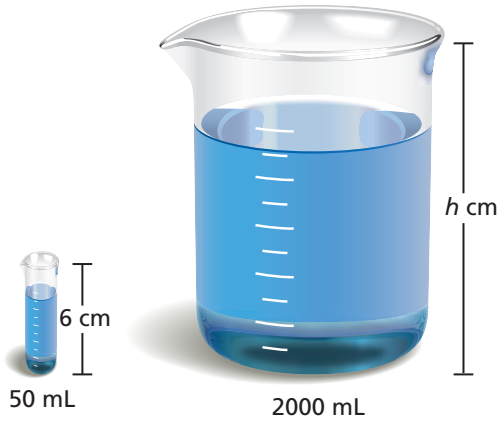
1	2	3	4
I do not understand.	I can do it with help.	I can do it on my own.	I can teach someone else.

6.1 Writing Equations in One Variable *(pp. 245–250)*

Learning Target: Write equations in one variable and write equations that represent real-life problems.

Write the word sentence as an equation.

1. The product of a number m and 2 is 8.
2. 6 less than a number t is 7.
3. A number m increased by 5 is 7.
4. 8 is the quotient of a number g and 3.
5. The height of the 50-milliliter beaker is one-third the height of the 2000-milliliter beaker. Write an equation you can use to find the height (in centimeters) of the 2000-milliliter beaker.



6. There are 16 teams in a basketball tournament. After two rounds, 12 teams are eliminated. Write and solve an equation to find the number of teams remaining after two rounds.
7. Write an equation that has a solution of $x = 8$.
8. Write a word sentence for the equation $y + 3 = 5$.

6.2 Solving Equations Using Addition or Subtraction *(pp. 251–258)*

Learning Target: Write and solve equations using addition or subtraction.

9. Tell whether $x = 7$ is a solution of $x + 9 = 16$.

Solve the equation. Check your solution.

10. $x - 1 = 8$ **11.** $m + 7 = 11$ **12.** $21 = p - 12$

Write the word sentence as an equation. Then solve the equation.

13. 5 more than a number x is 9.

14. 82 is the difference of a number b and 24.

15. A stuntman is running on the roof of a train. His combined speed is the sum of the speed of the train and his running speed. The combined speed is 73 miles per hour, and his running speed is 15 miles per hour. Find the speed of the train.

16. Before swallowing a large rodent, a python weighs 152 pounds. After swallowing the rodent, the python weighs 164 pounds. Find the weight of the rodent.

6.3 Solving Equations Using Multiplication or Division *(pp. 259–264)*

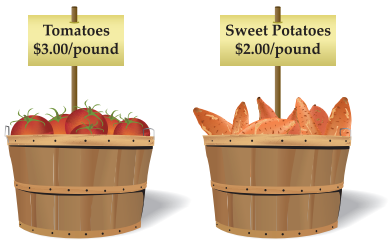
Learning Target: Write and solve equations using multiplication or division.

Solve the equation. Check your solution.

17. $6 \cdot q = 54$ **18.** $k \div 3 = 21$ **19.** $\frac{5}{7}a = 25$

20. The weight of an object on the Moon is about 16.5% of its weight on Earth. The weight of an astronaut on the Moon is 24.75 pounds. How much does the astronaut weigh on Earth?

21. Write an equation that can be solved using multiplication and has a solution of $x = 12$.



22. At a farmers' market, you buy 4 pounds of tomatoes and 2 pounds of sweet potatoes. You spend 80% of the money in your wallet. How much money is in your wallet before you pay?

6.4 Writing Equations in Two Variables *(pp. 265–272)*

Learning Target: Write equations in two variables and analyze the relationship between the two quantities.

Tell whether the ordered pair is a solution of the equation.

23. $y = 3x + 1$; $(2, 7)$

24. $y = 7x - 4$; $(4, 22)$

25. The equation $E = 360m$ represents the kinetic energy E (in joules) of a roller-coaster car with a mass of m kilograms. Identify the independent and dependent variables.

Graph the equation.

26. $y = x + 1$

27. $y = 7x$

28. $y = 4x + 3$

29. $y = \frac{1}{2}x + 5$

30. A taxi ride costs \$3 plus \$2.50 per mile. Write and graph an equation that represents the total cost (in dollars) of a taxi ride. What is the total cost of a five-mile taxi ride?

31. Write and graph an equation that represents the total cost (in dollars) of renting the bounce house. How much does it cost to rent the bounce house for 6 hours?

32. A car averages 50 miles per hour on a trip. Write and graph an equation that represents the relationship between the time and the distance traveled. How long does it take the car to travel 525 miles?

6 Practice Test

1. Write "7 times a number s is 84" as an equation.

Solve the equation. Check your solution.

2. $15 = 7 + b$

3. $v - 6 = 16$

4. $5x = 70$

5. $\frac{6m}{7} = 30$

6. Tell whether (3, 27) is a solution of $y = 9x$.

7. Tell whether (8, 36) is a solution of $y = 4x + 2$.

8. The drawbridge shown consists of two identical sections that open to allow boats to pass. Write an equation you can use to find the length s (in feet) of each section of the drawbridge.

9. Each ticket to a school dance is \$4. The total amount collected in ticket sales is \$332. Find the number of students attending the dance.

10. A soccer team sells T-shirts for a fundraiser. The company that makes the T-shirts charges \$10 per shirt plus a \$20 shipping fee per order.

a. Write and graph an equation that represents the total cost (in dollars) of ordering the shirts.

b. Choose an ordered pair that lies on your graph in part (a). Interpret it in the context of the problem.

11. You hand in 2 homework pages to your teacher. Your teacher now has 32 homework pages to grade. Find the number of homework pages that your teacher originally had to grade.

GUEST CHECK

Order No. 240796

1	Cheeseburger	7	49
2	Onion rings	7	00
1	Chx finger basket	9	50
1	12" Combo sub	6	50
2	Sodas	3	00
	Tax	2	51
	Total	36	00

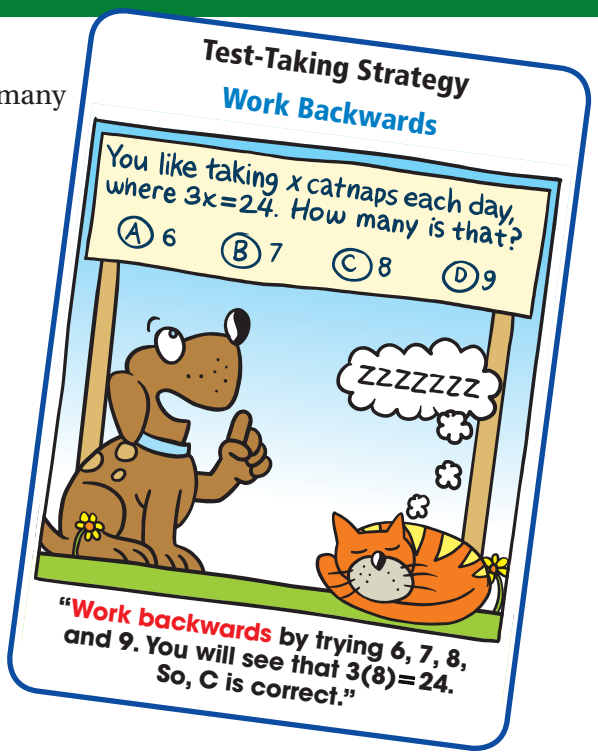
12. Write an equation that represents the total cost (in dollars) of the meal shown with a tip that is a percent of the check total. What is the total cost of the meal when the tip is 15%?

6 Cumulative Practice

1. You buy roses at a flower shop for $3 each. How many roses can you buy with $27?

A. 9 **B.** 10

C. 24 **D.** 81



2. You are making identical fruit baskets using 16 apples, 24 pears, and 32 bananas. What is the greatest number of baskets you can make using all of the fruit?

F. 2 **G.** 4

H. 8 **I.** 16

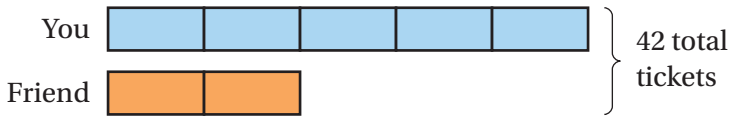
3. Which equation represents the word sentence?

The sum of 18 and 5 is equal to 9 less than a number y.

A. $18 - 5 = 9 - y$ **B.** $18 + 5 = 9 - y$

C. $18 + 5 = y - 9$ **D.** $18 - 5 = y - 9$

4. The tape diagram shows the ratio of tickets sold by you and your friend. How many more tickets did you sell than your friend?



F. 6 **G.** 12

H. 18 **I.** 30

5. What is the value of x that makes the equation true?

$$59 + x = 112$$

6. The steps your friend took to divide two mixed numbers are shown.

$$3\frac{3}{5} \div 1\frac{1}{2} = \frac{18}{5} \times \frac{3}{2}$$
$$= \frac{27}{5}$$
$$= 5\frac{2}{5}$$

What should your friend change in order to divide the two mixed numbers correctly?

A. Find a common denominator of 5 and 2.

B. Multiply by the reciprocal of $\frac{18}{5}$.

C. Multiply by the reciprocal of $\frac{3}{2}$.

D. Rename $3\frac{3}{5}$ as $2\frac{8}{5}$.

7. A company ordering parts receives a charge of \$25 for shipping and handling plus \$20 per part. Which equation represents the cost c (in dollars) of ordering p parts?

F. $c = 25 + 20p$

G. $c = 20 + 25p$

H. $p = 25 + 20c$

I. $p = 20 + 25c$

8. Which property is illustrated by the statement?

$$5(a + 6) = 5(a) + 5(6)$$

A. Associative Property of Multiplication

B. Commutative Property of Multiplication

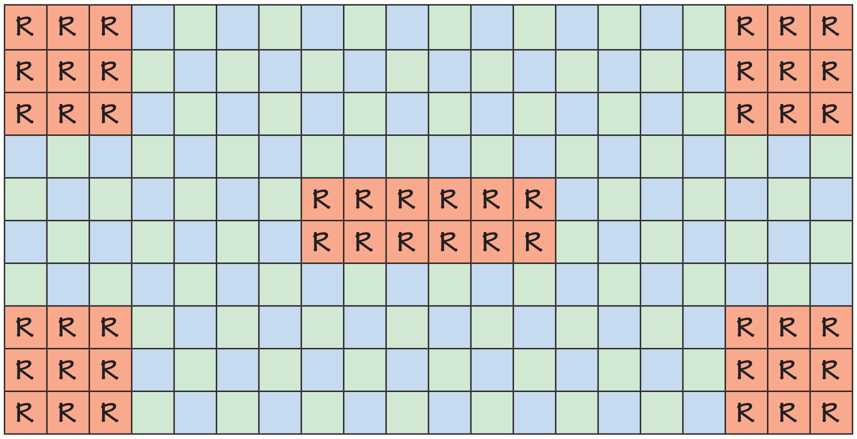
C. Commutative Property of Addition

D. Distributive Property

9. What is the value of the expression?

$$46.8 \div 0.156$$

10. In the mural below, the squares that are painted red are marked with the letter R.



What percent of the mural is painted red?

F. 24% **G.** 25%

H. 48% **I.** 50%

11. Which expression is equivalent to $28x + 70$?

A. $14(2x + 5)$ **B.** $14(5x + 2)$

C. $2(14x + 5)$ **D.** $14(7x)$

12. What is the first step in evaluating the expression?

$$3 \cdot (5 + 2)^2 \div 7$$

F. Multiply 3 and 5. **G.** Add 5 and 2.

H. Evaluate 5^2. **I.** Evaluate 2^2.

13. Jeff wants to save \$4000 to buy a used car. He has already saved \$850. He plans to save an additional \$150 each week.

Part A Write and solve an equation to represent the number of weeks remaining until he can afford the car.

Jeff saves \$150 per week by saving $\frac{3}{4}$ of what he earns at his job each week.

He works 20 hours per week.

Part B Write an equation to represent the amount per hour that Jeff must earn to save \$150 per week. Explain your reasoning.

Part C What is the amount per hour that Jeff must earn? Show your work and explain your reasoning.