5 Algebraic Expressions and Properties

Chapter Learning Target:
Understand algebraic expressions.

Chapter Success Criteria:
- ■ I can identify parts of an algebraic expression.
- ■ I can write algebraic expressions.
- ■ I can solve a problem using algebraic expressions.
- ■ I can interpret algebraic expressions in real-life problems.

STEAM Video: "Shadow Drawings"

Shadow Drawings

Expressions can be used to represent the growth of living things over time. Can you think of any other real-life situations in which you would want to use an expression to represent a changing quantity?

Watch the STEAM Video "Shadow Drawings."
Then answer the following questions.

1. Tory traces the shadow of a plant each week on the same day of the week and at the same time of day. Why does she need to be so careful about the timing of the drawing?

2. The table shows the height of the plant each week for the first three weeks. About how tall was the plant after 1.5 weeks? Explain your reasoning.

Week	1	2	3
Height (inches)	7	14	22

3. Predict the height of the plant when Tory makes her next three weekly drawings.
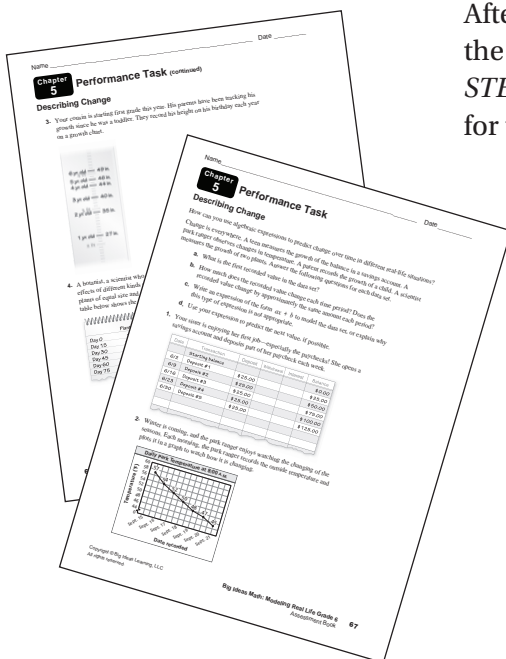
Describing Change

After completing this chapter, you will be able to use the concepts you learned to answer the questions in the *STEAM Video Performance Task*. You will be given data sets for the following real-life situations.

Savings account

Temperature

Human growth

Plant growth

You will be asked to use given data to write expressions and make predictions. Do the expressions provide accurate predictions far into the future?

Getting Ready for Chapter **5**

Chapter Exploration

1. Work with a partner.

   a. You babysit for 3 hours. You receive $24. What is your hourly wage?

   - Write the problem. Underline the important numbers and units you need to solve the problem.
   - Read the problem carefully a second time. Circle the key phrase for the question.

   > You babysit for 3 hours. You receive $24.
   >
   > What is your hourly wage?

   - Write each important number or phrase, with its units, on a piece of paper. Write $+, -, \times, \div,$ and $=$ on five other pieces of paper.

   hourly wage ($ per hour)

   - Arrange the pieces of paper to answer the question, "What is your hourly wage?"
   - Evaluate the expression that represents the hourly wage.

   hourly wage = ⬜ ÷ ⬜   Write.

   = ⬜   Evaluate.

   ▶ So, your hourly wage is $ ⬜ per hour.

   b. How can you use your hourly wage to find how much you will receive for any number of hours worked?

Vocabulary

The following vocabulary terms are defined in this chapter. Think about what each term might mean and record your thoughts.

algebraic expression   equivalent expressions
variable   factoring an expression
constant

5.1 Algebraic Expressions

Learning Target: Evaluate algebraic expressions given values of their variables.

Success Criteria:
- I can identify parts of an algebraic expression.
- I can evaluate algebraic expressions with one or more variables.
- I can evaluate algebraic expressions with one or more operations.

EXPLORATION 1

Evaluating Expressions

Math Practice

Make Sense of Quantities

What are the units in the problem? How does this help you write an expression?

Work with a partner. Identify any missing information that is needed to answer each question. Then choose a reasonable quantity and write an expression for each problem. After you have written the expression, evaluate it using mental math or some other method.

a. You receive $24 for washing cars. How much do you earn per hour?

b. You buy 5 silicone baking molds at a craft store. How much do you spend?

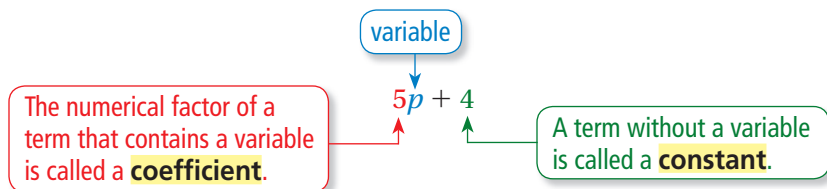c. You are running in a mud race. How much farther do you have to go after running 2000 feet?

d. A rattlesnake is 25 centimeters long when it hatches. The snake grows at a rate of about 1.6 centimeters per month for several months. What is the length of the rattlesnake?

An **algebraic expression** is an expression that may contain numbers, operations, and one or more *variables*. A **variable** is a symbol that represents one or more numbers. Each number or variable by itself, or product of numbers and variables in an algebraic expression, is called a **term**.

variable

$5p + 4$

The numerical factor of a term that contains a variable is called a **coefficient**.

A term without a variable is called a **constant**.

EXAMPLE 1 Identifying Parts of an Algebraic Expression

Identify the terms, coefficients, and constants in each expression.

a. $5x + 13$

b. $2z^2 + y + 3$

$5x + 13$

Terms:	$5x$,	13
Coefficient:	5	
Constant:		13

$2z^2 + y + 3$

Terms:	$2z^2$,	$1y$,	3
Coefficients:	2,	1	
Constant:			3

A variable by itself has a coefficient of 1. So, the term *y* in Example 1(b) has a coefficient of 1.

Try It Identify the terms, coefficients, and constants in the expression.

1. $12 + 10c$

2. $15 + 3w + \dfrac{1}{2}$

3. $z^2 + 9z$

EXAMPLE 2 Writing Algebraic Expressions Using Exponents

Write each expression using exponents.

a. $d \cdot d \cdot d \cdot d$

Because d is used as a factor 4 times, its exponent is 4.

▶ So, $d \cdot d \cdot d \cdot d = d^4$.

b. $1.5 \cdot h \cdot h \cdot h$

Because h is used as a factor 3 times, its exponent is 3.

▶ So, $1.5 \cdot h \cdot h \cdot h = 1.5h^3$.

Try It Write the expression using exponents.

4. $j \cdot j \cdot j \cdot j \cdot j \cdot j$

5. $9 \cdot k \cdot k \cdot k \cdot k$

To evaluate an algebraic expression, substitute a number for each variable. Then use the order of operations to find the value of the numerical expression.

EXAMPLE 3 **Evaluating Algebraic Expressions**

a. **Evaluate $k + 10$ when $k = 25$.**

$$k + 10 = 25 + 10 \qquad \text{Substitute 25 for } k.$$
$$= 35 \qquad \text{Add 25 and 10.}$$

b. **Evaluate $4 \cdot n$ when $n = 12$.**

$$4 \cdot n = 4 \cdot 12 \qquad \text{Substitute 12 for } n.$$
$$= 48 \qquad \text{Multiply 4 and 12.}$$

You can write the product of 4 and n in several ways.
$4 \cdot n$
$4n$
$4(n)$

Try It

6. Evaluate $24 + c$ when $c = 9$.

7. Evaluate $d - 17$ when $d = 30$.

8. Evaluate $18 \div q$ when $q = \dfrac{1}{2}$.

EXAMPLE 4 **Evaluating an Expression with Two Variables**

a. **Evaluate $n - m$ when $m = 12$ and $n = 30$.**

$$n - m = 30 - 12 \qquad \text{Substitute 30 for } n \text{ and 12 for } m.$$
$$= 18 \qquad \text{Subtract 12 from 30.}$$

b. **Evaluate $a \div b$ when $a = 16$ and $b = \dfrac{2}{3}$.**

$$a \div b = 16 \div \dfrac{2}{3} \qquad \text{Substitute 16 for } a \text{ and } \dfrac{2}{3} \text{ for } b.$$
$$= 16 \cdot \dfrac{3}{2} \qquad \text{Multiply by the reciprocal of } \dfrac{2}{3}, \text{ which is } \dfrac{3}{2}.$$
$$= 24 \qquad \text{Multiply.}$$

Try It Evaluate the expression when $p = 24$ and $q = 8$.

9. $p \div q$

10. $q + p$

11. $p - q$

12. $p \cdot q$

EXAMPLE 5 **Evaluating Expressions with Two Operations**

a. Evaluate $3x - 14$ when $x = 5$.

$3x - 14 = 3(5) - 14$ Substitute 5 for x.

$\quad\quad\quad = 15 - 14$ Using order of operations, multiply 3 and 5.

$\quad\quad\quad = 1$ Subtract 14 from 15.

b. Evaluate $n^2 + 8.5$ when $n = 2$.

$n^2 + 8.5 = 2^2 + 8.5$ Substitute 2 for n.

$\quad\quad\quad\quad = 4 + 8.5$ Using order of operations, evaluate 2^2.

$\quad\quad\quad\quad = 12.5$ Add 4 and 8.5.

Try It Evaluate the expression when $y = 6$.

13. $5y + 1$ **14.** $30 - 24 \div y$

15. $y^2 - 7$ **16.** $1.5 + y^2$

Self-Assessment *for Concepts & Skills*

Solve each exercise. Then rate your understanding of the success criteria in your journal.

17. WHICH ONE DOESN'T BELONG? Which expression does *not* belong with the other three? Explain your reasoning.

$2x + 1$ $5w \cdot c$ $3(4) + 5$ $2y \cdot z$

18. ALGEBRAIC EXPRESSIONS Identify the terms, coefficients, and constants in the expression $9h + 1$.

EVALUATING EXPRESSIONS Evaluate the expression when $m = 8$.

19. $m - 7$ **20.** $5m + 4$

21. MP NUMBER SENSE Does the value of the expression $20 - x$ increase, decrease, or stay the same as x increases? Explain.

22. OPEN-ENDED Write an algebraic expression using more than one operation. When you evaluate the expression, how do you know which operation to perform first?

23. MP STRUCTURE Is the expression $8.2 \div m \cdot m \cdot m \cdot m$ the same as the expression $8.2 \div m^4$? Explain your reasoning.

EXAMPLE 6 **Modeling Real Life**

You are saving to buy a meteorite fragment for $125. You begin with $45 and you save $3 each week. The expression $45 + 3w$ gives the amount of money you save after w weeks. Can you buy the meteorite after 20 weeks?

Understand the problem.

You are given an expression that represents your savings after w weeks. You are asked whether you have enough money to buy a $125 meteorite after 20 weeks.

Make a plan.

To find the amount of money you save after 20 weeks, evaluate the expression when $w = 20$. Then compare the value of the expression to the price of the meteorite.

Solve and check.

$$45 + 3w = 45 + 3(20) \qquad \text{Substitute 20 for } w.$$
$$= 45 + 60 \qquad \text{Multiply 3 and 20.}$$
$$= 105 \qquad \text{Add 45 and 60.}$$

▶ You cannot buy the $125 meteorite after 20 weeks because you only have $105.

Another Method You start with $45, so you need to save another $125 - 45 = \$80$. At $3 per week, it will take you $\frac{80}{3} \approx 27$ weeks of saving.

$$45 + 3(27) = 45 + 81 = \$126 \checkmark$$

Self-Assessment for Problem Solving

Solve each exercise. Then rate your understanding of the success criteria in your journal.

24. The expression $12.25m + 29.99$ gives the cost (in dollars) of a gym membership for m months. You have $180 to spend on a membership. Can you buy a one-year membership?

25. **DIG DEEPER!** The expression $p - 15$ gives the amount (in dollars) you pay after using the coupon when the original amount of a purchase is p dollars. The expression $30 + 6n$ gives the amount of money (in dollars) you save after n weeks. A jacket costs $78. Can you buy the jacket after 6 weeks? Explain.

Coupon

Good for $15 off any purchase of $75 or more

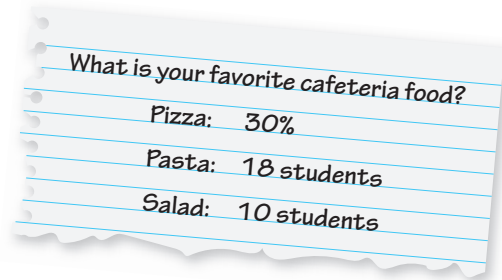## ▶ Review & Refresh

You ask 40 students which of three items from the cafeteria they like the best. You record the results on the piece of paper shown.

What is your favorite cafeteria food?
Pizza: 30%
Pasta: 18 students
Salad: 10 students

1. What percent of students answered salad?

2. How many students answered pizza?

3. What percent of students answered pasta?

Find the missing quantity in the double number line.

4. Fiber (grams)

5. Pastries
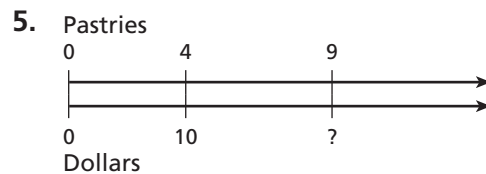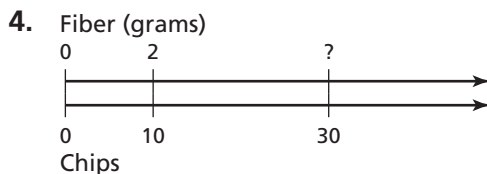
Divide. Write the answer in simplest form.

6. $1\frac{3}{8} \div \frac{3}{4}$

7. $2\frac{7}{9} \div 2$

8. $4 \div 4\frac{2}{5}$

9. $3\frac{2}{3} \div 1\frac{2}{7}$

▶ Concepts, Skills, & Problem Solving

EVALUATING EXPRESSIONS Write and evaluate an expression for the problem. (See Exploration 1, p. 201.)

10. The scores on your first two history tests are 82 and 95. By how many points did you improve on your second test?

11. You buy a hat for $12 and give the cashier a $20 bill. How much change do you receive?

12. You receive $8 for raking leaves for 2 hours. What is your hourly wage?

13. Music lessons cost $20 per week. How much do 6 weeks of lessons cost?

ALGEBRAIC EXPRESSIONS Identify the terms, coefficients, and constants in the expression.
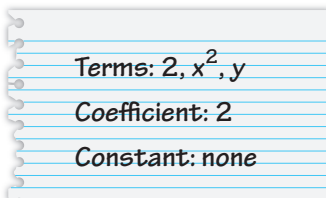
14. $7h + 3$

15. $g + 12 + 9g$

16. $5c^2 + 7d$

17. $2m^2 + 15 + 2p^2$

18. $6 + n^2 + \frac{1}{2}d$

19. $8x + \frac{x^2}{3}$

Terms: $2, x^2, y$
Coefficient: 2
Constant: none

20. **YOU BE THE TEACHER** Your friend finds the terms, coefficients, and constants in the algebraic expression $2x^2y$. Is your friend correct? Explain your reasoning.
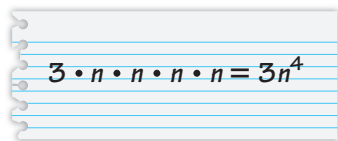
21. **PERIMETER** You can use the expression $2\ell + 2w$ to find the perimeter of a rectangle, where ℓ is the length and w is the width.

   a. Identify the terms, coefficients, and constants in the expression.

   b. Interpret the coefficients of the terms.

USING EXPONENTS Write the expression using exponents.

22. $b \cdot b \cdot b$

23. $g \cdot g \cdot g \cdot g \cdot g$

24. $8 \cdot w \cdot w \cdot w \cdot w$

25. $5.2 \cdot y \cdot y \cdot y$

26. $a \cdot a \cdot c \cdot c$

27. $2.1 \cdot x \cdot z \cdot z \cdot z \cdot z$

28. **YOU BE THE TEACHER** Your friend writes the product using exponents. Is your friend correct? Explain your reasoning.

$$3 \cdot n \cdot n \cdot n \cdot n = 3n^4$$

29. **AREA** Write an expression using exponents that represents the area of the square.

5d

As I was going to St. Ives
I met a man with seven wives
Each wife had seven sacks
Each sack had seven cats
Each cat had seven kits
Kits, cats, sacks, wives
How many were going to St. Ives?

30. **MP REASONING** Suppose the man in the St. Ives poem has x wives, each wife has x sacks, each sack has x cats, and each cat has x kits. Write an expression using exponents that represents the total number of kits, cats, sacks, and wives.

EVALUATING EXPRESSIONS Evaluate the expression when $a = 3$, $b = 2$, and $c = 12$.

31. $6 + a$

32. $b \cdot 5$

33. $c - 1$

34. $27 \div a$

35. $12 - b$

36. $c + 5$

37. $2a$

38. $c \div 6$

39. $a + b$

40. $c + a$

41. $c - a$

42. $a - b$

43. $\dfrac{c}{a}$

44. $\dfrac{c}{b}$

45. $b \cdot c$

46. $c(a)$

47. **MP PROBLEM SOLVING** You earn $15n$ dollars for mowing n lawns. How much do you earn for mowing 1 lawn? 7 lawns?

**EVALUATING EXPRESSIONS** Copy and complete the table.

48.

x	3	6	9
x · 8			

49.

x	2	4	8
64 ÷ x			

50. MODELING REAL LIFE Due to gravity, an object falls $16t^2$ feet in t seconds. You drop a rock from a bridge that is 75 feet above the water. Will the rock hit the water in 2 seconds? Explain.

EVALUATING EXPRESSIONS Evaluate the expression when $a = 10$, $b = 9$, and $c = 4$.

51. $2a + 3$

52. $4c - 7.8$

53. $\dfrac{a}{4} + \dfrac{1}{3}$

54. $\dfrac{24}{b} + 8$

55. $c^2 + 6$

56. $a^2 - 18$

57. $a + 9c$

58. $bc + 12.3$

59. $3a + 2b - 6c$

$$5m + 3 = 5 \cdot 8 + 3$$
$$= 5 \cdot 11$$
$$= 55$$
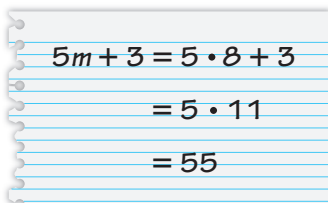
60. YOU BE THE TEACHER Your friend evaluates the expression when $m = 8$. Is your friend correct? Explain your reasoning.

61. MP PROBLEM SOLVING After m months, the height of a plant is $(10 + 3m)$ millimeters. How tall is the plant after 8 months? 3 years?

62. MP STRUCTURE You use a video streaming service to rent x new releases and y standard rentals. Which expression tells you how much money you will need?

$3x + 4y$ $4x + 3y$ $7(x + y)$

Standard Rentals
$3

63. OPEN-ENDED You float 2000 feet along a lazy river water ride. The ride takes less than 10 minutes. Give two examples of possible times and speeds.

New Releases
$4

64. DIG DEEPER! The expression $20a + 13c$ is the cost (in dollars) for a adults and c students to enter a science center.

   a. How much does it cost for an adult? a student? Explain your reasoning.

   b. Find the total cost for 4 adults and 24 students.

   c. You find the cost for a group. Then the numbers of adults and students in the group both double. Does the cost double? Explain your answer using an example.

   d. In part (b), the number of adults is cut in half, but the number of students doubles. Is the cost the same? Explain your answer.

65. MP REASONING The volume of the cube (in cubic inches) is equal to four times the area of one of its faces (in square inches). What is the volume of the cube?
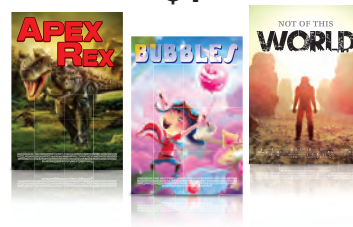
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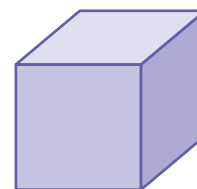
5.2 Writing Expressions

Learning Target: Write algebraic expressions and solve problems involving algebraic expressions.

Success Criteria:
• I can write numerical expressions.
• I can write algebraic expressions.
• I can write and evaluate algebraic expressions that represent real-life problems.

EXPLORATION 1

Writing Expressions

Work with a partner. You use a $20 bill to buy lunch at a café. You order a sandwich from the menu board shown.

CHEF
RECOMMENDS
Sandwiches

Chicken Salad
$4.95
* * * * * * *
Reuben
$6.45

Egg Salad
$4.65
* * * * * * *
Roast Beef
$6.75

Grilled Cheese
$4.65
* * * * * * *
BLT
$5.25

Prices include tax.

Math Practice

Use Expressions
How do the numerical expressions help you generalize the situation and write an algebraic expression?

a. Complete the table. In the last column, write a numerical expression for the amount of change you receive.

Sandwich	Price (dollars)	Change Received (dollars)
Reuben		
BLT		
Egg salad		
Roast beef		

b. **MP REPEATED REASONING** Write an algebraic expression that represents the amount of change you receive when you order any sandwich from the menu board.

c. The expression $20 - 4.65s$ represents the amount of change one customer receives after ordering from the menu board. Explain what each part of the expression represents. Do you know what the customer ordered? Explain your reasoning.

5.2 Lesson

Some words can imply math operations.

Operation	Addition	Subtraction	Multiplication	Division
Key Words and Phrases	added to plus sum of more than increased by total of and	subtracted from minus difference of less than decreased by fewer than take away	multiplied by times product of twice	divided by quotient of

EXAMPLE 1 Writing Numerical Expressions

Write each phrase as an expression.

 a. 8 fewer than 21

   $21 - 8$ The phrase *fewer than* means *subtraction*.

 b. the product of 30 and 9

   30×9, or $30 \cdot 9$ The phrase *product of* means *multiplication*.

Try It **Write the phrase as an expression.**

 1. the sum of 18 and 35   **2.** 6 times 50

EXAMPLE 2 Writing Algebraic Expressions

Write each phrase as an expression.

 a. 14 more than a number x

   $x + 14$ The phrase *more than* means *addition*.

 b. a number y minus 75

   $y - 75$ The word *minus* means *subtraction*.

 c. the quotient of 3 and a number z

   $3 \div z$, or $\dfrac{3}{z}$ The phrase *quotient of* means *division*.

Common Error

When writing expressions involving subtraction or division, order is important. For example, the quotient of a number x and 2 means

   $x \div 2$, not $2 \div x$.

Try It **Write the phrase as an expression.**

 3. 25 less than a number b **4.** a number x divided by 4

 5. the total of a number t and 11   **6.** 100 decreased by a number k

Writing an Algebraic Expression

The length of Interstate 90 from the West Coast to the East Coast is
153.5 miles more than 2 times the length of Interstate 15 from southern
California to northern Montana. Let *m* be the length of Interstate 15.
Which expression can you use to represent the length of Interstate 90?

A. $2m + 153.5$ **B.** $2m - 153.5$

C. $153.5 - 2m$ **D.** $153.5m + 2$

> Variables can be lowercase or uppercase. Make sure you consistently use the same case for a variable when solving a problem.

> The word *times* means *multiplication.* So, multiply 2 and *m*.

> The phrase *more than* means *addition.* So, add 2*m* and 153.5.

$$2m + 153.5$$

▶ The correct answer is **A.**

Try It

7. Your friend has 5 more than twice as many game tokens as you. Let *t* be the number of game tokens you have. Write an expression for the number of game tokens your friend has.

Self-Assessment for Concepts & Skills

Solve each exercise. Then rate your understanding of the success criteria in your journal.

WRITING EXPRESSIONS Write the phrase as an expression.

8. the sum of 7 and 11 9. 5 subtracted from 9

10. **DIFFERENT WORDS, SAME QUESTION** Which is different?
Write "both" expressions.

12 more than x	x increased by 12
x take away 12	the sum of x and 12

11. **MP PRECISION** Your friend says that the phrases below have the same meaning. Is your friend correct? Explain your reasoning.

the difference of a number x and 12

the difference of 12 and a number x

EXAMPLE 4 **Modeling Real Life**

You plant a cypress tree that is 10 inches tall. Each year, its height increases by 15 inches. Write an expression that represents the height (in inches) after t years. What is the height after 9 years?

Make a table showing the height of the tree each year for the first several years. Use the results to write an expression and evaluate the expression when $t = 9$.

10 in.

The height is *increasing*, so *add* 15 each year, as shown in the table.

Year, t	Height (inches)
0	10
1	$10 + 15(1) = 25$
2	$10 + 15(2) = 40$
3	$10 + 15(3) = 55$
4	$10 + 15(4) = 70$

When t is 0, the height is 10 inches.

You can see that an expression is $10 + 15t$.

Sometimes, as in Example 3, a variable represents a single value. Other times, as in Example 4, a variable can represent more than one value.

Evaluate $10 + 15t$ when $t = 9$.

$$10 + 15t = 10 + 15(9) = 145$$

▶ So, the height (in inches) after t years is $10 + 15t$. After 9 years, the height of the tree is 145 inches.

Self-Assessment for Problem Solving

Solve each exercise. Then rate your understanding of the success criteria in your journal.

Leisure Lake Outfitters

Paddleboard Rentals
Rental Fee: $15
Hourly Rate: $12 per hour

12. A company rents paddleboards by charging a rental fee plus an hourly rate. Write an expression that represents the cost (in dollars) of renting a paddleboard for h hours. How much does an eight-hour rental cost?

13. **DIG DEEPER!** A county fair charges an entry fee of $7 and $0.75 for each ride token. You have $15. Write an expression that represents the amount (in dollars) you have left after entering the fair and purchasing n tokens. How many tokens can you purchase? How much money do you have left after purchasing 6 tokens?

NO CASH VALUE

▶ Review & Refresh

Identify the terms, coefficients, and constants in the expression.

1. $4f + 8$

2. $\dfrac{4}{5} + 3s + 2$

3. $9h^2 + \dfrac{8}{9}p + 1$

Copy and complete the statement.

4. $\dfrac{2\text{ c}}{\text{min}} = \dfrac{\boxed{}\text{ gal}}{\text{h}}$

5. $\dfrac{12\text{ m}}{\text{sec}} \approx \dfrac{\boxed{}\text{ ft}}{\text{min}}$

6. $\dfrac{3\text{ lb}}{\text{sec}} \approx \dfrac{\boxed{}\text{ kg}}{\text{h}}$

Divide. Write the answer in simplest form.

7. $\dfrac{1}{2} \div \dfrac{5}{8}$

8. $\dfrac{1}{3} \div \dfrac{3}{4}$

9. $\dfrac{2}{5} \div 3$

10. $3 \div \dfrac{6}{7}$

▶ Concepts, Skills, & Problem Solving

MP STRUCTURE The expression represents the amount of change you receive after buying n sandwiches. Explain what each part of the expression represents. (See Exploration 1, p. 209.)

11. $10 - 5.25n$

12. $20 - 4.95n$

13. $100 - 6.75n$

WRITING EXPRESSIONS Write the phrase as an expression.

14. 5 less than 8

15. the product of 3 and 12

16. 28 divided by 7

17. the total of 6 and 10

18. 3 fewer than 18

19. 17 added to 15

20. 13 subtracted from a number x

21. 5 times a number d

22. the quotient of 18 and a number a

23. the difference of a number s and 6

24. 7 increased by a number w

25. a number t cubed

YOU BE THE TEACHER Your friend writes the phrase as an expression. Is your friend correct? Explain your reasoning.

26.
> The quotient of 8 and a number y is $\dfrac{y}{8}$.

27.
> 16 decreased by a number x is $16 - x$.

28. **MP NUMBER SENSE** Five friends share the cost of a dinner equally.

 a. Write an expression that represents the cost (in dollars) per person.

 b. Make up a reasonable total cost and test your expression.

29. **MODELING REAL LIFE** A biologist analyzes 15 bacteria samples each day.

 a. Copy and complete the table.

 b. Write an expression that represents the total number of samples analyzed after n days.

| Days	1	2	3	4	5
Total Samples					

30. **MP** **PROBLEM SOLVING** To rent a moving truck for the day, it costs $33 plus $1 for each mile driven.

   a. Write an expression that represents the cost (in dollars) to rent the truck.

   b. You drive the truck 300 miles. How much do you pay?

WRITING PHRASES Give two ways to write the expression as a phrase.

31. $n + 6$ **32.** $4w$ **33.** $15 - b$ **34.** $14 - 3z$

EVALUATING EXPRESSIONS Write the phrase as an expression. Then evaluate the expression when $x = 5$ and $y = 20$.

35. 3 less than the quotient of a number y and 4

36. the sum of a number x and 4, all divided by 3

37. 6 more than the product of 8 and a number x

38. the quotient of 40 and the difference of a number y and 16
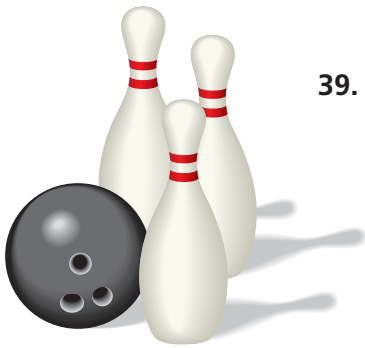
39. **MODELING REAL LIFE** It costs $3 to bowl a game and $2 for shoe rental.

   a. Write an expression that represents the total cost (in dollars) of g games.

   b. Use your expression to find the total cost of 8 games.

40. **MODELING REAL LIFE** Florida has 8 less than 5 times the number of counties in Arizona. Georgia has 25 more than twice the number of counties in Florida.

   a. Write an expression that represents the number of counties in Florida.

   b. Write an expression that represents the number of counties in Georgia.

   c. Arizona has 15 counties. How many do Florida and Georgia have?

41. **MP** **PATTERNS** There are 140 people in a singing competition. The graph shows the results for the first five rounds.

   a. Write an expression that represents the number of people after each round.

   b. Assuming this pattern continues, how many people compete in the ninth round? Explain your reasoning.

Singing Competition
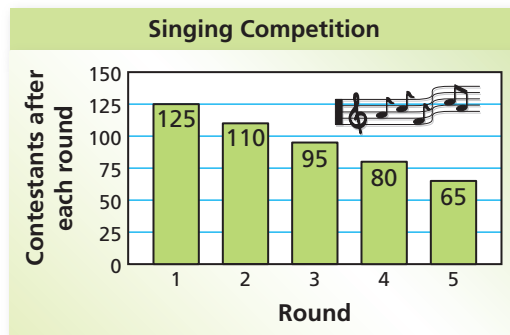
42. **MP** **NUMBER SENSE** The difference between two numbers is 8. The lesser number is a. Write an expression that represents the greater number.

43. **MP** **NUMBER SENSE** One number is four times another. The greater number is x. Write an expression that represents the lesser number.

5.3 Properties of Addition and Multiplication

Learning Target: Identify equivalent expressions and apply properties to generate equivalent expressions.

Success Criteria:
- I can explain the meaning of equivalent expressions.
- I can use properties of addition to generate equivalent expressions.
- I can use properties of multiplication to generate equivalent expressions.

EXPLORATION 1

Identifying Equivalent Expressions

Work with a partner.

a. Choose four values for a variable x. Then evaluate each expression for each value of x. Are any of the expressions *equivalent*? Explain your reasoning.

x				
$4 + x + 4$				

x				
$16x$				

x				
$4 \cdot (x \cdot 4)$				

x				
$x + 4 + 4$				

x				
$x + 8$				

x				
$(4 \cdot x) \cdot 4$				

b. You have used the following properties in a previous course. Use the examples to explain the meaning of each property.

Commutative Property of Addition: $3 + 5 = 5 + 3$

Commutative Property of Multiplication: $9 \cdot 3 = 3 \cdot 9$

Associative Property of Addition: $8 + (3 + 1) = (8 + 3) + 1$

Associative Property of Multiplication: $12 \cdot (6 \cdot 2) = (12 \cdot 6) \cdot 2$

Are these properties true for algebraic expressions? Explain your reasoning.

Math Practice

Use Counterexamples
Use a counterexample to show that the Commutative Property is not true for division.

Expressions that result in the same number for any value of each variable are **equivalent expressions**. You can use the Commutative and Associative Properties to write equivalent expressions.

Key Ideas

Commutative Properties

Words Changing the order of addends or factors does not change the sum or product.

Numbers $5 + 8 = 8 + 5$

$5 \cdot 8 = 8 \cdot 5$

Algebra $a + b = b + a$

$a \cdot b = b \cdot a$

Associative Properties

Words Changing the grouping of addends or factors does not change the sum or product.

Numbers $(7 + 4) + 2 = 7 + (4 + 2)$

$(7 \cdot 4) \cdot 2 = 7 \cdot (4 \cdot 2)$

Algebra $(a + b) + c = a + (b + c)$

$(a \cdot b) \cdot c = a \cdot (b \cdot c)$

EXAMPLE 1 **Using Properties to Write Equivalent Expressions**

One way to check whether expressions are equivalent is to evaluate each expression for any value of the variable. In Example 1(a), use $x = 2$.

$7 + (12 + x) \stackrel{?}{=} 19 + x$

$7 + (12 + 2) \stackrel{?}{=} 19 + 2$

$21 = 21$ ✓

a. **Simplify the expression $7 + (12 + x)$.**

$7 + (12 + x) = (7 + 12) + x$ Associative Property of Addition

$= 19 + x$ Add 7 and 12.

b. **Simplify the expression $(6.1 + x) + 8.4$.**

$(6.1 + x) + 8.4 = (x + 6.1) + 8.4$ Commutative Property of Addition

$= x + (6.1 + 8.4)$ Associative Property of Addition

$= x + 14.5$ Add 6.1 and 8.4.

c. **Simplify the expression $5(11y)$.**

$5(11y) = (5 \cdot 11)y$ Associative Property of Multiplication

$= 55y$ Multiply 5 and 11.

Try It **Simplify the expression. Explain each step.**

1. $10 + (a + 9)$ **2.** $\left(c + \dfrac{2}{3}\right) + \dfrac{1}{2}$ **3.** $5(4n)$

Key Ideas

Addition Property of Zero

Words The sum of any number and 0 is that number.

Numbers $7 + 0 = 7$ **Algebra** $a + 0 = a$

Multiplication Properties of Zero and One

Words The product of any number and 0 is 0.

The product of any number and 1 is that number.

Numbers $9 \cdot 0 = 0$ **Algebra** $a \cdot 0 = 0$

$4 \cdot 1 = 4$ $a \cdot 1 = a$

EXAMPLE 2 ## Using Properties to Write Equivalent Expressions

a. **Simplify the expression $9 \cdot 0 \cdot p$.**

$9 \cdot 0 \cdot p = (9 \cdot 0) \cdot p$ Associative Property of Multiplication

$= 0 \cdot p$ Multiplication Property of Zero

$= 0$ Multiplication Property of Zero

b. **Simplify the expression $4.5 \cdot r \cdot 1$.**

$4.5 \cdot r \cdot 1 = 4.5 \cdot (r \cdot 1)$ Associative Property of Multiplication

$= 4.5 \cdot r$ Multiplication Property of One

$= 4.5r$ Rewrite.

Try It **Simplify the expression. Explain each step.**

4. $12 \cdot b \cdot 0$ **5.** $1 \cdot m \cdot 24$ **6.** $(t + 15) + 0$

Self-Assessment for Concepts & Skills

Solve each exercise. Then rate your understanding of the success criteria in your journal.

USING PROPERTIES **Simplify the expression. Explain each step.**

7. $(7 + c) + 4$ **8.** $4(b \cdot 6)$ **9.** $0 \cdot b \cdot 9$

10. **WRITING** Explain what it means for expressions to be equivalent. Then give an example of equivalent expressions.

11. **OPEN-ENDED** Write an algebraic expression that can be simplified using the Associative Property of Multiplication and the Multiplication Property of One.

EXAMPLE 3 **Modeling Real Life**

You and six friends play on a basketball team. A sponsor paid $100 for the league fee, x dollars for each player's T-shirt, and $68.25 for basketballs. Write an expression that represents the total amount (in dollars) the sponsor paid. Then find the total amount paid when each T-shirt costs $14.50.

Use a verbal model to write an expression that represents the sum of the league fee, the cost of the T-shirts, and the cost of the basketballs. Then evaluate the expression when $x = 14.5$.

Common Error

You **and** six friends are on the team, so use 7, not 6, to represent the number of T-shirts.

League fee (dollars)		Number of T-shirts	$\cdot$	Cost per T-shirt (dollars)		Cost of basketballs (dollars)
$100	+	7		x	+	$68.25

$$100 + 7x + 68.25 = 7x + 100 + 68.25 \qquad \text{Commutative Property of Addition}$$
$$= 7x + (100 + 68.25) \qquad \text{Associative Property of Addition}$$
$$= 7x + 168.25 \qquad \text{Add 100 and 68.25.}$$

Evaluate $7x + 168.25$ when $x = 14.5$.

$$7x + 168.25 = 7(14.5) + 168.25 = 101.5 + 168.25 = 269.75$$

▶ An expression that represents the total amount (in dollars) is $7x + 168.25$. When each T-shirt costs $14.50, the sponsor pays $269.75.

Self-Assessment for Problem Solving

Solve each exercise. Then rate your understanding of the success criteria in your journal.

12. You and five friends form a team for an outdoor adventure race. Your team needs to raise money to pay for $130 of travel fees, x dollars for each team member's entry fee, and $85.50 for food. Use an algebraic expression to find the total amount your team needs to raise when the entry fee is $25.50 per person.

13. You have $50 and a $15 gift card to spend online. You purchase a pair of headphones for $34.99 and 8 songs for x dollars each. Use an algebraic expression to find the amount you have left when each song costs $1.10.

5.3 Practice

? Go to *BigIdeasMath.com* to get HELP with solving the exercises.

▶ ## Review & Refresh

Write the phrase as an expression.

1. 10 added to a number p

2. the product of 6 and a number m

3. the quotient of a number b and 15

4. 7 fewer than a number s

Write the prime factorization of the number.

5. 36

6. 144

7. 147

8. 205

Evaluate the expression.

9. $8.092 + 3.5$

10. $16.78 - 12.237$

11. $9.17 + 1.83 + 2.641$

12. $8.43 - 6.218 + 4.2$

Represent the ratio relationship using a graph.

13.

Oil (teaspoons)	8	16	24
Flour (cups)	1	2	3

14.

Atoms	4	8	12
Protons	64	128	192

▶▶ ## Concepts, Skills, & Problem Solving

MATCHING Match the expression with an equivalent expression.
(See Exploration 1, p. 215.)

15. $3 + 3 + y$

16. $(y \cdot y) \cdot 3$

17. $3 \cdot 1 \cdot y$

18. $(3 + 0) + (y + y)$

A. $y \cdot 3$

B. $y + 3 + 3$

C. $y(3 \cdot y)$

D. $(3 + y) + y$

IDENTIFYING PROPERTIES Tell which property the statement illustrates.

19. $5 \cdot p = p \cdot 5$

20. $2 + (12 + r) = (2 + 12) + r$

21. $4 \cdot (x \cdot 10) = (4 \cdot x) \cdot 10$

22. $x + 7.5 = 7.5 + x$

23. $(c + 2) + 0 = c + 2$

24. $a \cdot 1 = a$

25. **YOU BE THE TEACHER** Your friend states the property that the statement illustrates. Is your friend correct? Explain your reasoning.

$(7 + x) + 3 = (x + 7) + 3$

Associative Property of Addition

USING PROPERTIES Simplify the expression. Explain each step.

26. $6 + (5 + x)$

27. $(14 + y) + 3$

28. $6(2b)$

29. $7(9w)$

30. $3.2 + (x + 5.1)$

31. $(0 + a) + 8$

32. $9 \cdot c \cdot 4$

33. $(18.6 \cdot d) \cdot 1$

34. $\left(3k + 4\frac{1}{5}\right) + 8\frac{3}{5}$

35. $(2.4 + 4n) + 9$

36. $(3s) \cdot 8$

37. $z \cdot 0 \cdot 12$

38. GEOMETRY The expression $12 + x + 4$ represents the perimeter of a triangle. Simplify the expression.

39. MP PRECISION A case of scout cookies has 10 cartons. A carton has 12 boxes. The amount you earn on a whole case is $10(12x)$ dollars.

    a. What does x represent?

    b. Simplify the expression.

40. MODELING REAL LIFE A government estimates the cost to design new radar technology over a period of m months. The government estimates $840,000 for equipment, $15,000 for software, and $40,000 per month for wages. Use an algebraic expression to find the total cost the government estimates when the project takes 16 months to complete.

**WRITING EXPRESSIONS** Write the phrase as an expression. Then simplify the expression.

41. 7 plus the sum of a number x and 5

42. the product of 8 and a number y, multiplied by 9

USING PROPERTIES Copy and complete the statement using the specified property.

	Property	Statement
43.	Associative Property of Multiplication	$7(2y) = $
44.	Commutative Property of Multiplication	$13.2 \cdot (x \cdot 1) = $
45.	Associative Property of Addition	$17 + (6 + 2x) = $
46.	Addition Property of Zero	$2 + (c + 0) = $
47.	Multiplication Property of One	$1 \cdot w \cdot 16 = $

48. GEOMETRY Five identical triangles form the trapezoid shown.

    a. What is the perimeter of the trapezoid?

    b. How can you use some or all of the triangles to form a new trapezoid with a perimeter of $3x + 14$? Explain your reasoning.
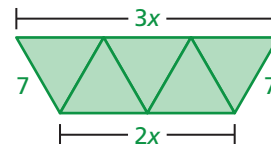
49. DIG DEEPER! You and a friend sell hats at a fair booth. You sell 16 hats on the first shift and 21 hats on the third shift. Your friend sells x hats on the second shift.

    a. The expression $37(14) + 10x$ represents the amount (in dollars) that you both earn. How can you tell that your friend is selling the hats for a lower price?

    b. You earn more money than your friend. What can you say about the value of x?

5.4 The Distributive Property

**Learning Target:** Apply the Distributive Property to generate equivalent expressions.

Success Criteria:
- I can explain how to apply the Distributive Property.
- I can use the Distributive Property to simplify algebraic expressions.
- I can use the Distributive Property to combine like terms.

EXPLORATION 1   Using Models to Simplify Expressions

Work with a partner.

a. Use the models to simplify the expressions. Explain your reasoning.

$$3(x + 4) = \boxed{}$$

$$8(6 + y) = \boxed{}$$

$$5n + 4n = \boxed{}$$

$$5n - 4n = \boxed{}$$
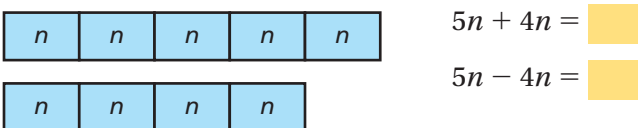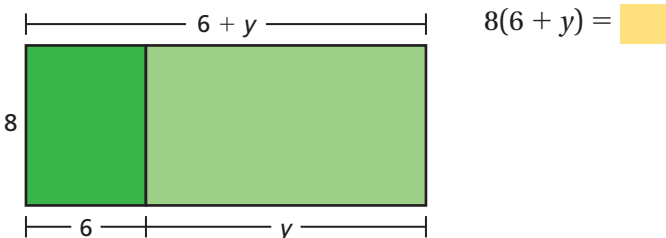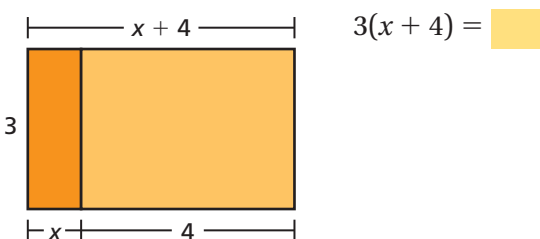
b. In part (a), check that the original expressions are equivalent to the simplified expressions.

c. You used the Distributive Property in a previous course. Use the example to explain the meaning of the property.

Distributive Property: $6(20 + 3) = 6(20) + 6(3)$

Is this property true for algebraic expressions? Explain your reasoning.

🔑 *Key Idea*

Distributive Property

Words To multiply a sum or difference by a number, multiply each term in the sum or difference by the number outside the parentheses. Then simplify.

Numbers $3(7 + 2) = 3 \times 7 + 3 \times 2$ **Algebra** $a(b + c) = ab + ac$

$3(7 - 2) = 3 \times 7 - 3 \times 2$ $a(b - c) = ab - ac$

EXAMPLE 1 ## Simplifying Algebraic Expressions

Use the Distributive Property to simplify each expression.

a. $4(n + 5)$

$4(n + 5) = 4(n) + 4(5)$ Distributive Property

$= 4n + 20$ Multiply.

b. $12(2y - 3)$

$12(2y - 3) = 12(2y) - 12(3)$ Distributive Property

$= 24y - 36$ Multiply.

c. $\dfrac{1}{2}(6y - 2z)$

$\dfrac{1}{2}(6y - 2z) = \dfrac{1}{2}(6y) - \dfrac{1}{2}(2z)$ Distributive Property

$= 3y - z$ Multiply.

d. $9(6 + x + 2)$

$9(6 + x + 2) = 9(6) + 9(x) + 9(2)$ Distributive Property

$= 54 + 9x + 18$ Multiply.

$= 9x + 54 + 18$ Commutative Property of Addition

$= 9x + (54 + 18)$ Associative Property of Addition

$= 9x + 72$ Add 54 and 18.

> You can use the Distributive Property when there are more than two terms in the sum or difference.

Try It Use the Distributive Property to simplify the expression.
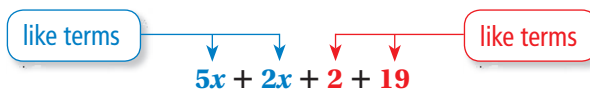
1. $7(a + 2)$ **2.** $3(d - 11)$

3. $12\left(a + \dfrac{2}{3}b\right)$ **4.** $7(2 + 6 - 4d)$

In an algebraic expression, **like terms** are terms that have the same variables raised to the same exponents. Constant terms are also like terms.

like terms ──────┐   ┌────── like terms

$$5x + 2x + 2 + 19$$

You can use the Distributive Property to *combine* like terms.

EXAMPLE 2 **Combining Like Terms**

Simplify each expression.

a. $3x + 9 + 2x - 5$

$3x + 9 + 2x - 5 = 3x + 2x + 9 - 5$	Commutative Property of Addition
$= (3 + 2)x + 9 - 5$	Distributive Property
$= 5x + 4$	Simplify.

b. $y + y + y$

$y + y + y = 1y + 1y + 1y$	Multiplication Property of One
$= (1 + 1 + 1)y$	Distributive Property
$= 3y$	Add coefficients.

c. $7z + 2(z - 5y)$

$7z + 2(z - 5y) = 7z + 2(z) - 2(5y)$	Distributive Property
$= 7z + 2z - 10y$	Multiply.
$= (7 + 2)z - 10y$	Distributive Property
$= 9z - 10y$	Add coefficients.

> When you combine like terms, you are using the Distributive Property. You are applying the rules $ab + ac = a(b + c)$ and $ab - ac = a(b - c)$.

Try It Simplify the expression.

5. $8 + 3z - z$

6. $3(b + 5) + b + 2$

Self-Assessment for Concepts & Skills

Solve each exercise. Then rate your understanding of the success criteria in your journal.

7. **WRITING** One meaning of the word *distribute* is *to give something to each member of a group*. How can this help you remember the Distributive Property?

SIMPLIFYING EXPRESSIONS Use the Distributive Property to simplify the expression.

8. $3(x + 10)$

9. $15(4n - 2)$

10. $2w + 4 + 13w + 1$

EXAMPLE 3 **Modeling Real Life**

José is x years old. His brother, Felipe, is 2 years older than José. Their aunt, Maria, is three times as old as Felipe. Write and simplify an expression that represents Maria's age in years.

Use a table to organize the given information and write an expression that represents each person's age in years.

Name	Description	Expression
José	He is x years old.	x
Felipe	He is 2 years *older* than José. So, *add* 2 to x.	$x + 2$
Maria	She is three *times* as old as Felipe. So, *multiply* 3 and $(x + 2)$.	$3(x + 2)$

Simplify the expression that represents Maria's age.

$$3(x + 2) = 3(x) + 3(2) \qquad \text{Distributive Property}$$
$$= 3x + 6 \qquad \text{Multiply.}$$

▶ Maria's age in years is represented by the expression $3x + 6$.

Look Back

If José is 10 years old, then Felipe is $10 + 2 = 12$ years old and Maria is $3(12) = 36$ years old. So, you should obtain 36 when you evaluate $3x + 6$ for $x = 10$.

$$3x + 6 = 3(10) + 6$$
$$= 36 \checkmark$$

Self-Assessment *for Problem Solving*

Solve each exercise. Then rate your understanding of the success criteria in your journal.

11. You purchase a remote-controlled drone for d dollars. Your friend purchases a drone that costs $35 more than your drone. Your brother purchases a drone that costs three times as much as your friend's drone. Write and simplify an expression that represents the cost (in dollars) of your brother's drone.
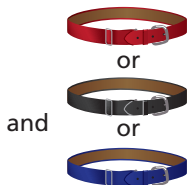
12. Write and simplify an expression that represents the total cost (in dollars) of buying the items shown for each member of a baseball team.

and

or

or

Pants: $10   Belt: $$x$

13. **DIG DEEPER!** One molecule of caffeine contains x oxygen atoms, twice as many nitrogen atoms as oxygen atoms, 4 more carbon atoms than nitrogen atoms, and 1.25 times as many hydrogen atoms as carbon atoms. Write and simplify an expression that represents the number of hydrogen atoms in one molecule of caffeine.

5.4 Practice

▷ Review & Refresh

Simplify the expression. Explain each step.

1. $(s + 4) + 8$ **2.** $(12 + x) + 2$ **3.** $3(4n)$

You are given the difference of the numbers of boys and girls in a class and the ratio of boys to girls. How many boys and how many girls are in the class?

4. 3 more boys; 5 for every 4   **5.** 8 more girls; 3 for every 5

6. 4 more girls; 9 for every 13   **7.** 6 more boys; 7 for every 4

Divide.

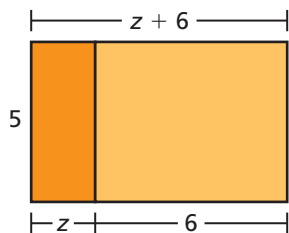**8.** $301 \div 7$ **9.** $1722 \div 14$ **10.** $629 \div 12$ **11.** $8068 \div 31$

▶▶ Concepts, Skills, & Problem Solving

USING MODELS Use the model to simplify the expression. Explain your reasoning. (See Exploration 1, p. 221.)

12. $5(z + 6) = \boxed{}$

13. $4s + 2s = \boxed{}$

SIMPLIFYING EXPRESSIONS Use the Distributive Property to simplify the expression.

14. $3(x + 4)$ **15.** $10(b - 6)$ **16.** $6(s - 9)$ **17.** $7(8 + y)$

18. $8(12 + a)$ **19.** $9(2n + 1)$ **20.** $12(6 - k)$ **21.** $18(5 - 3w)$

22. $9(3 + c + 4)$ **23.** $\frac{1}{4}(8 + x + 4)$ **24.** $8(5g + 5 - 2)$ **25.** $6(10 + z + 3)$

26. $4(x + y)$ **27.** $25(x - y)$ **28.** $7(p + q + 9)$ **29.** $\frac{1}{2}(2n + 4 + 6m)$

MATCHING Match the expression with an equivalent expression.

30. $6(n + 4)$ **31.** $2(3n + 9)$ **32.** $6(n + 2)$ **33.** $3(2n + 3)$

   A. $3(2n + 6)$ **B.** $6n + 9$ **C.** $3(2n + 8)$ **D.** $6n + 12$

34. **MP STRUCTURE** Each day, you run on a treadmill for r minutes and lift weights for 15 minutes. Which expressions can you use to find how many minutes of exercise you do in 5 days? Explain your reasoning.

$5(r + 15)$ $5r + 5 \cdot 15$ $5r + 15$ $r(5 + 15)$

35. **MODELING REAL LIFE** A cheetah can run 103 feet per second. A zebra can run x feet per second. Write and simplify an expression that represents how many feet farther the cheetah can run in 10 seconds.

COMBINING LIKE TERMS Simplify the expression.

36. $6(x + 4) + 1$

37. $5 + 8(3 + x)$

38. $x + 3 + 5x$

39. $7y + 6 - 1 + 12y$

40. $4d + 9 - d - 8$

41. $n + 3(n - 1)$

42. $2v + 8v - 5v$

43. $5(z + 4) + 5(2 - z)$

44. $2.7(w - 5.2)$

45. $\frac{2}{3}y + \frac{1}{6}y + y$

46. $\frac{3}{4}\left(z + \frac{2}{5}\right) + 2z$

47. $7(x + y) - 7x$

48. $4x + 9y + 3(x + y)$

49. **YOU BE THE TEACHER** Your friend simplifies the expression. Is your friend correct? Explain your reasoning.

$$8x - 2x + 5x = 8x - 7x$$
$$= (8 - 7)x$$
$$= x$$

50. **MP REASONING** Evaluate each expression by (1) using the Distributive Property and (2) evaluating inside the parentheses first. Which method do you prefer? Is your preference the same for both expressions? Explain your reasoning.

   a. $2(3.22 - 0.12)$
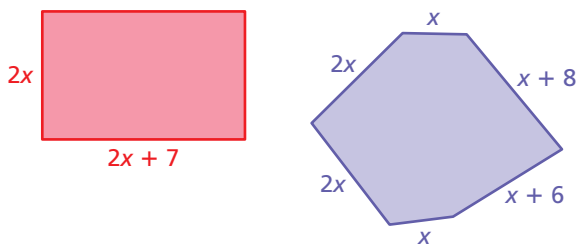
   b. $12\left(\frac{1}{2} + \frac{2}{3}\right)$

51. **DIG DEEPER!** An art club sells 42 large candles and 56 small candles.

   a. Write and simplify an expression that represents the profit.

   b. A large candle costs $5, and a small candle costs $3. What is the club's profit?

   Price: $10 Cost: $x
   Price: $5 Cost: $y

   Profit = Price − Cost

52. **MP REASONING** Find the difference between the perimeters of the rectangle and the hexagon. Interpret your answer.

   2x
   2x + 7

   x
   2x
   x + 8
   2x
   x + 6
   x

53. **PUZZLE** Add one set of parentheses to the expression $7 \cdot x + 3 + 8 \cdot x + 3 \cdot x + 8 - 9$ so that it is equivalent to $2(9x + 10)$.

5.5 Factoring Expressions

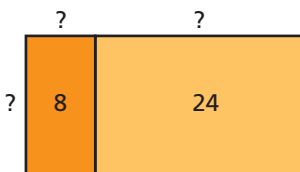**Learning Target:** Factor numerical and algebraic expressions.

Success Criteria:
- I can use the Distributive Property to factor numerical expressions.
- I can identify the greatest common factor of terms including variables.
- I can use the Distributive Property to factor algebraic expressions.
- I can interpret factored expressions in real-life problems.

EXPLORATION 1 Finding Dimensions

Work with a partner.

a. The models show the area (in square units) of each part of a rectangle. Use the models to find missing values that complete the expressions. Explain your reasoning.
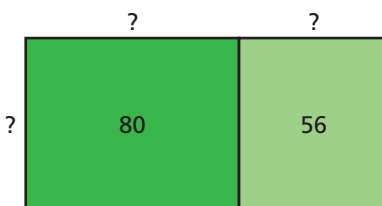
?	?
8	24

$$8 + 24 = \boxed{} \left(\boxed{} + \boxed{} \right)$$

Math Practice

Evaluate Results

Do your answers in the first two models seem reasonable? How can you check your answers?

?	?
80	56

$$80 + 56 = \boxed{} \left(\boxed{} + \boxed{} \right)$$

?	?
$3x$	18

$$3x + 18 = \boxed{} \left(\boxed{} + \boxed{} \right)$$

b. In part (a), check that the original expressions are equivalent to the expressions you wrote. Explain your reasoning.

c. Explain how you can use the Distributive Property to rewrite a sum of two whole numbers with a common factor.

5.5 Lesson

🔑 *Key Idea*

Factoring an Expression

Words   Writing a numerical expression or algebraic expression as a product of factors is called **factoring the expression**. You can use the Distributive Property to factor expressions.

Numbers $3 \cdot 7 + 3 \cdot 2 = 3(7 + 2)$ **Algebra** $ab + ac = a(b + c)$

$3 \cdot 7 - 3 \cdot 2 = 3(7 - 2)$ $ab - ac = a(b - c)$

EXAMPLE 1   **Factoring Numerical Expressions**

When you factor an expression, you can factor out any common factor.

a.   Factor $18 + 30$ using the GCF.

One way to find the GCF of 18 and 30 is to list their factors.

   Factors of 18: ①,②,③,⑥, 9, 18

   Factors of 30: ①,②,③, 5,⑥, 10, 15, 30   *Circle the common factors.*

The GCF of 18 and 30 is 6.

Write each term of the expression as a product of the GCF and the remaining factor. Then use the Distributive Property to factor the expression.

   $18 + 30 = 6(3) + 6(5)$ *Rewrite using GCF.*

   $ = 6(3 + 5)$ *Distributive Property*

b.   Factor $20 - 12$ using the GCF.

One way to find the GCF of 20 and 12 is to list their factors.

   Factors of 20: ①,②,④, 5, 10, 20

   Factors of 12: ①,②, 3,④, 6, 12   *Circle the common factors.*

The GCF of 20 and 12 is 4.

Write each term of the expression as a product of the GCF and the remaining factor. Then use the Distributive Property to factor the expression.

   $20 - 12 = 4(5) - 4(3)$ *Rewrite using GCF.*

   $ = 4(5 - 3)$ *Distributive Property*

Try It   **Factor the expression using the GCF.**

   1. $9 + 15$ **2.** $60 + 45$ **3.** $30 - 20$

EXAMPLE 2 **Factoring Algebraic Expressions**

a. **Factor $3x + 42$ using the GCF.**

You can find the GCF of $3x$ and 42 by writing their prime factorizations.

$3x = 3 \cdot x$

$42 = 2 \cdot 3 \cdot 7$

Circle the common prime factor.

The GCF of $3x$ and 42 is 3. Use the GCF to factor the expression.

$3x + 42 = 3(x) + 3(14)$ Rewrite using GCF.

$\qquad = 3(x + 14)$ Distributive Property

b. **Factor $63z - 27y$ using the GCF.**

You can find the GCF of $63z$ and $27y$ by writing their prime factorizations.

$63z = 3 \cdot 3 \cdot 7 \cdot z$

$27y = 3 \cdot 3 \cdot 3 \cdot y$

Circle the common prime factors.

The GCF of $63z$ and $27y$ is $3 \cdot 3 = 9$. Use the GCF to factor the expression.

$63z - 27y = 9(7z) - 9(3y)$ Rewrite using GCF.

$\qquad = 9(7z - 3y)$ Distributive Property

Try It **Factor the expression using the GCF.**

4. $7x + 49$ **5.** $8y - 44$ **6.** $25a + 10b$

Self-Assessment *for Concepts & Skills*

Solve each exercise. Then rate your understanding of the success criteria in your journal.

FACTORING EXPRESSIONS **Factor the expression using the GCF.**

7. $16 + 24$ **8.** $49 - 28$ **9.** $8y + 14$

10. **WHICH ONE DOESN'T BELONG?** Which expression does *not* belong with the other three? Explain your reasoning.

$3(8n + 12)$ $\qquad$ $4(6n + 9)$ $\qquad$ $6(4n + 3)$ $\qquad$ $12(2n + 3)$

11. **MP** **REASONING** Use what you know about factoring to explain how you can factor the expression $18x + 30y + 9z$. Then factor the expression.

12. **CRITICAL THINKING** Identify the GCF of the terms $(x \cdot x)$ and $(4 \cdot x)$. Explain your reasoning. Then use the GCF to factor the expression $x^2 + 4x$.

EXAMPLE 3 **Modeling Real Life**

You receive a discount on each book you buy for your electronic reader. The original price of each book is x dollars. You buy 5 books for a total of $(5x - 15)$ dollars. Factor the expression. What can you conclude about the discount?

To factor $5x - 15$, you can find the GCF of $5x$ and 15 by writing their prime factorizations.

$$5x = \boxed{5} \cdot x$$
$$15 = \boxed{5} \cdot 3$$

Circle the common prime factor.

So, the GCF of $5x$ and 15 is 5. Use the GCF to factor the expression.

$$5x - 15 = 5(x) - 5(3) \quad \text{Rewrite using GCF.}$$
$$= 5(x - 3) \quad \text{Distributive Property}$$

The factor 5 represents the number of books purchased. The factor $(x - 3)$ represents the discounted price of each book. This factor is a difference of two terms, showing that the original price, x, of each book is decreased by \$3.

▶ So, the factored expression shows a \$3 discount for every book you buy. The original expression shows a total savings of \$15.

Check Suppose that the original price of each book is \$6. Verify that each expression has the same value when $x = 6$.

$$5x - 15 = 5(6) - 15 = 15$$
$$5(x - 3) = 5(6 - 3) = 15 \checkmark$$

Self-Assessment for Problem Solving

Solve each exercise. Then rate your understanding of the success criteria in your journal.

13. A youth club receives a discount on each pizza purchased for a party. The original price of each pizza is x dollars. The club leader purchases 8 pizzas for a total of $(8x - 32)$ dollars. Factor the expression. What can you conclude about the discount?

14. Three crates of food are packed on a shuttle departing for the Moon. Each crate weighs x pounds. On the Moon, the combined weight of the crates is $(3x - 81)$ pounds. What can you conclude about the weight of each crate on the Moon?

▶ *Review & Refresh*

Use the Distributive Property to simplify the expression.

1. $2(n + 8)$ **2.** $3(4 + m)$ **3.** $7(b - 3)$ **4.** $10(4 - w)$

Write the phrase as an expression.

5. 5 plus a number p

6. 18 less than a number r

7. 11 times a number d

8. a number c divided by 25

Decide whether the rates are equivalent.

9. 84 feet in 12 seconds
217 feet in 31 seconds

10. 12 cups of soda for every 54 cups of juice
8 cups of soda for every 36 cups of juice
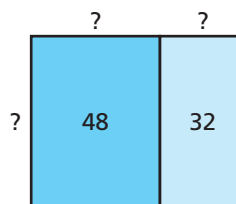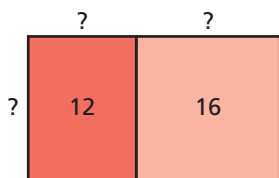
Match the decimal with its equivalent percent.

11. 0.36   **12.** 3.6   **13.** 0.0036   **14.** 0.036

A. 0.36%   **B.** 360%   **C.** 36%   **D.** 3.6%

▶▶ *Concepts, Skills, & Problem Solving*

FINDING DIMENSIONS The model shows the area (in square units) of each part of a rectangle. Use the model to find missing values that complete the expression. Explain your reasoning. (See Exploration 1, p. 227.)

15. $12 + 16 = \boxed{}\left(\boxed{} + \boxed{}\right)$

16. $48 + 32 = \boxed{}\left(\boxed{} + \boxed{}\right)$

FACTORING NUMERICAL EXPRESSIONS Factor the expression using the GCF.

17. $7 + 14$ **18.** $12 + 42$ **19.** $22 + 11$ **20.** $70 + 95$

21. $60 - 36$ **22.** $100 - 80$ **23.** $84 + 28$ **24.** $48 + 80$

25. $19 + 95$ **26.** $44 - 11$ **27.** $18 - 12$ **28.** $48 + 16$

29. $98 - 70$ **30.** $58 + 28$ **31.** $72 - 39$ **32.** $69 + 84$

33. **MP REASONING** The whole numbers a and b are divisible by c, where b is greater than a. Is $a + b$ divisible by c? Is $b - a$ divisible by c? Explain your reasoning.

34. **MULTIPLE CHOICE** Which expression is *not* equivalent to $81x + 54$?

A. $27(3x + 2)$ **B.** $3(27x + 18)$ **C.** $9(9x + 6)$ **D.** $6(13x + 9)$

FACTORING ALGEBRAIC EXPRESSIONS Factor the expression using the GCF.

35. $2x + 10$

36. $15x + 6$

37. $26x - 13$

38. $50x - 60$

39. $36x + 9$

40. $14x - 98$

41. $18p + 26$

42. $16m + 40$

43. $24 + 72n$

44. $50 + 65h$

45. $76d - 24$

46. $27 - 45c$

47. $18t + 38x$

48. $90y + 65z$

49. $10x - 25y$

50. $24y + 88x$

51. OPEN-ENDED Use the Distributive Property to write two expressions that are equivalent to $8x + 16$.

MATCHING Match the expression with an equivalent expression.

52. $8x + 16y$

53. $4x + 8y$

54. $16x + 8y$

55. $8x + 4y$

A. $4(2x + y)$

B. $2(4y + 2x)$

C. $4(2x + 4y)$

D. $8(y + 2x)$

56. YOU BE THE TEACHER Your friend factors the expression $24x + 56$. Is your friend correct? Explain your reasoning.

$$24x + 56 = 8(3x) + 8(7)$$
$$= (8 + 8) \cdot (3x + 7)$$
$$= 16(3x + 7)$$

57. MODELING REAL LIFE You sell soup mixes for a fundraiser. For each soup mix you sell, the company that makes the soup receives x dollars, and you receive the remaining amount. You sell 16 soup mixes for a total of $(16x + 96)$ dollars. How much money do you receive for each soup mix that you sell?

58. MP PROBLEM SOLVING A clothing store is having a sale on holiday socks. Each pair of socks costs x dollars. You leave the store with 6 pairs of socks and spend a total of $(6x - 14)$ dollars. You pay with $40. How much change do you receive? Explain your reasoning.

59. MP STRUCTURE You buy 37 concert tickets for $8 each, and then sell all 37 tickets for $11 each. The work below shows two ways you can determine your profit. Describe each solution method. Which do you prefer? Explain your reasoning.

$$\text{Profit} = 37(11) - 37(8)$$
$$= 407 - 296$$
$$= \$111$$

$$\text{Profit} = 37(11) - 37(8)$$
$$= 37(11 - 8)$$
$$= 37(3)$$
$$= \$111$$

60. MP NUMBER SENSE The prime factorizations of two numbers are shown, where a and b represent prime numbers. Write the sum of the two numbers as an expression of the form $14\left(\boxed{} + \boxed{}\right)$. Explain your reasoning.

Number 1: $2 \cdot 11 \cdot 5 \cdot a$

Number 2: $7 \cdot b \cdot 3 \cdot 3$

5 Connecting Concepts

▶ Using the Problem-Solving Plan

1. A store sells 18 pairs of the wireless earbuds shown. Customers saved a total of $882 on the earbuds. Find the original price of the earbuds.

SUPER SALE

35% off wireless earbuds

Understand the problem.
You know the percent discount on a pair of wireless earbuds, the number of pairs of earbuds sold, and the total amount of money that customers saved. You are asked to find the original price of the earbuds.

Make a plan.
First, write an expression that represents the total amount of money that customers pay for the earbuds. Then factor the expression to find the discount (in dollars) on each pair of earbuds. Finally, solve a percent problem to find the original price.

Solve and check.
Use the plan to solve the problem. Then check your solution.

2. All of the weight plates in a gym are labeled in kilograms. You want to convert the weights to pounds. Write an expression to find the number of pounds in z kilograms. Then find the weight in pounds of a plate that weighs 20.4 kilograms.

3. You buy apple chips and banana chips in the ratio of 2 : 7.

   a. How many ounces of banana chips do you buy when you buy n ounces of apple chips? Explain.

   b. You buy 12 ounces of apple chips. How many ounces of banana chips do you buy?

Performance Task

Describing Change

At the beginning of this chapter, you watched a STEAM video called "Shadow Drawings." You are now ready to complete the performance task related to this video, available at **BigIdeasMath.com**. Be sure to use the problem-solving plan as you work through the performance task.

▶ Review Vocabulary

Write the definition and give an example of each vocabulary term.

algebraic expression, *p. 202*
variable, *p. 202*
term, *p. 202*

coefficient, *p. 202*
constant, *p. 202*
equivalent expressions, *p. 216*

like terms, *p. 223*
factoring an expression, *p. 228*

▶ Graphic Organizers

You can use an **Example and Non-Example Chart** to list examples and non-examples of a concept. Here is an Example and Non-Example Chart for the ***Commutative Property of Addition***.

Commutative Property of Addition

Examples	Non-Examples
$a + b = b + a$	$a \cdot b = b \cdot a$
$2.1 + 9 = 9 + 2.1$	$(7 + 4) + 2 = 7 + (4 + 2)$
$17 + 34 = 34 + 17$	$b \cdot 0 = 0$
$(6 + x) + 8 = (x + 6) + 8$	$46 \cdot 1 = 46$
$(3 + y) + 1 = 1 + (3 + y)$	$2(12 + x) = 2(12) + 2(x)$

Choose and complete a graphic organizer to help you study the concept.

1. algebraic expressions

2. variable

3. Commutative Property of Multiplication

4. Associative Property of Addition

5. Associative Property of Multiplication

6. Addition Property of Zero

7. Multiplication Property of Zero

8. Multiplication Property of One

9. Distributive Property

"I finished my Example and Non-Example Chart about things we need on the moon."

▶ Chapter Self-Assessment

As you complete the exercises, use the scale below to rate your understanding of the success criteria in your journal.

1	2	3	4
I do not understand.	I can do it with help.	I can do it on my own.	I can teach someone else.

5.1 Algebraic Expressions *(pp. 201–208)*

Learning Target: Evaluate algebraic expressions given values of their variables.

Identify the terms, coefficients, and constants in the expression.

1. $9x + 2 + 8y$

2. $3x^2 + x + 7$

3. $1 + \dfrac{q}{4} + 7q$

Evaluate the expression when $x = 20$, $y = 4$, and $z = 7$.

4. $x \div 5$

5. $12 - z$

6. $4y$

7. $y + x$

8. $x \cdot z$

9. $x - y$

10. $3z + 8$

11. $8y - x$

12. $\dfrac{x^2}{y}$

13. The amount earned (in dollars) for recycling p pounds of copper is $2p$. How much do you earn for recycling 28 pounds of copper?

14. While playing a video game, you score p game points and b triple bonus points. An expression for your score is $p + 3b$. What is your score when you earn 245 game points and 20 triple bonus points?

15. Tickets for a baseball game cost a dollars for adults and c dollars for children. The expression $2a + 3c$ represents the cost (in dollars) for a family to go to the game. What is the cost for the family when an adult ticket is $17 and a child ticket is $12?

16. Add one set of parentheses to the expression $2x^2 + 4 - 5$ so that the value of the expression is 75 when $x = 6$.

5.2 Writing Expressions *(pp. 209–214)*

Learning Target: Write algebraic expressions and solve problems involving algebraic expressions.

Write the phrase as an expression.

17. 9 fewer than 23

18. 6 more than the quotient of 15 and 3

19. the product of a number d and 32

20. a number t decreased by 17

21. Your basketball team scored 4 fewer than twice as many points as the other team.

    a. Write an expression that represents the number of points your team scored.

    b. The other team scored 24 points. How many points did your team score?

22. The boiling temperature (in degrees Celsius) of platinum is 199 more than four times the boiling temperature (in degrees Celsius) of zinc.

    a. Write an expression that represents the boiling temperature (in degrees Celsius) of platinum.

    b. The boiling temperature of zinc is 907 degrees Celsius. What is the boiling temperature of platinum?

23. Write an algebraic expression with two variables, x and y, that has a value of 50 when $x = 3$ and $y = 5$.

5.3 Properties of Addition and Multiplication *(pp. 215–220)*

Learning Target: Identify equivalent expressions and apply properties to generate equivalent expressions.

Simplify the expression. Explain each step.

24. $10 + (2 + y)$

25. $(21 + b) + 1$

26. $3(7x)$

27. $1(3.2w)$

28. $5.3 + (w + 1.2)$

29. $(0 + t) + 9$

30. The expression $7 + 3x + 4$ represents the perimeter of the triangle. Simplify the expression.

31. Write an algebraic expression that can be simplified using the Associative Property of Addition.

7

3x

4

5.4 The Distributive Property (pp. 221–226)

Learning Target: Apply the Distributive Property to generate equivalent expressions.

Use the Distributive Property to simplify the expression.

32. $2(x + 12)$ **33.** $11(4b - 3)$ **34.** $8(s - 1)$ **35.** $6(6 + y)$

Simplify the expression.

36. $5(n + 3) + 4n$ **37.** $t + 2 + 6t$ **38.** $3z + 14 + 5z - 9$

39. A family of three goes to a salon. Each person gets a haircut and highlights. The cost of each haircut is \$15, and the cost per person for highlights is x dollars. Write and simplify an expression that represents the total cost (in dollars) for the family at the salon.

40. Each day, you take vocal lessons for v minutes and trumpet lessons for 30 minutes. Write and simplify an expression to find how many minutes of lessons you take in 4 days.

5.5 Factoring Expressions (pp. 227–232)

Learning Target: Factor numerical and algebraic expressions.

Factor the expression using the GCF.

41. $42 - 12$ **42.** $15 + 35$ **43.** $36x - 28$

44. $24 + 64x$ **45.** $60 - 150x$ **46.** $16x + 56y$

47. A soccer team receives a discount on each jersey purchased. The original price of each jersey is x dollars. The team buys 18 jerseys for a total of $(18x - 36)$ dollars. What can you conclude about the discount?

48. You sell apple cider for a fundraiser. For each gallon of cider you sell, the company that makes the cider receives x dollars, and you receive the remaining amount. You sell 15 gallons of cider for $(15x + 45)$ dollars. How much money do you receive for each gallon of cider that you sell?

1. Identify the terms, coefficients, and constants of $\frac{q}{3} + 6 + 9q$.

2. Evaluate $4b - a$ when $a = 12$ and $b = 7$.

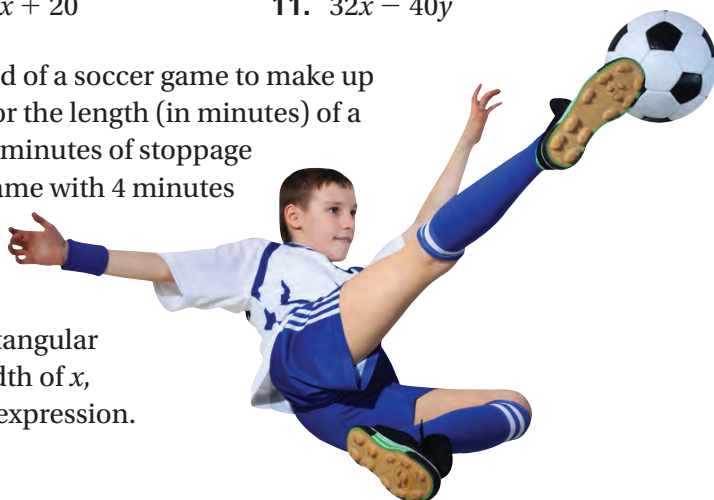**Write the phrase as an expression.**

3. 25 more than 50

4. 6 less than the quotient of 32 and a number y

Simplify the expression. Explain each step.

5. $3.1 + (8.6 + m)$

6. $\left(\frac{2}{3} \cdot t\right) \cdot 1\frac{1}{2}$

7. $4(x + 8)$

8. $4t + 7 + 2t - 2$

Factor the expression using the GCF.

9. $18 + 24$

10. $15x + 20$

11. $32x - 40y$

12. Playing time is added at the end of a soccer game to make up for stoppages. An expression for the length (in minutes) of a 90-minute soccer game with x minutes of stoppage time is $90 + x$. How long is a game with 4 minutes of stoppage time?

13. The expression $15 \cdot x \cdot 6$ represents the volume of a rectangular prism with a length of 15, a width of x, and a height of 6. Simplify the expression.

14. The Coiling Dragon Cliff Skywalk in China is 128 feet longer than the length x (in feet) of the Tianmen Skywalk in China. The world's longest glass-bottom bridge, located in China's Zhangjiajie National Park, is about 4.3 times longer than the Coiling Dragon Cliff Skywalk. Write and simplify an expression that represents the length (in feet) of the world's longest glass-bottom bridge.

15. A youth group is making and selling sandwiches to raise money. The cost to make each sandwich is h dollars. The group sells 150 sandwiches for a total of $(150h + 450)$ dollars. How much profit does the group earn for each sandwich sold?

16. You make party favors for an event. You tie 9 inches of ribbon around each party favor. Write an expression for the number of inches of ribbon needed for n party favors. The ribbon costs $3 for each *yard*. Write an expression for the total cost (in dollars) of the ribbon.
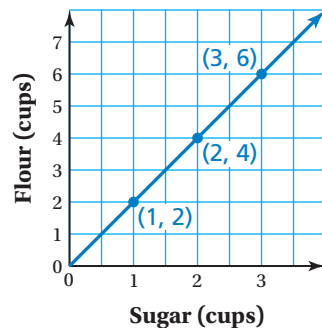
1. The student council is organizing a school fair. Council members are making signs to show the prices for admission and for each game a person can play.

> **SCHOOL FAIR**
> Admission $2.00
> Price per game $0.25

Let x represent the number of games. Which expression can you use to determine the total amount (in dollars) a person pays for admission and playing x games?

A. 2.25

B. $2.25x$

C. $2 + 0.25x$

D. $2x + 0.25$

2. Which ratio relationship is represented in the graph?

F. 2 cups of flour for every $\frac{1}{2}$ cup of sugar

G. 6 cups of flour for every 3 cups of sugar

H. 1 cup of flour for every 4 cups of sugar

I. $\frac{1}{2}$ cup of flour for every 1 cup of sugar

3. At a used bookstore, you can purchase two types of books.

You can use the expression $3h + 2p$ to find the total cost (in dollars) for h hardcover books and p paperback books. What is the total cost (in dollars) for 6 hardcover books and 4 paperback books?

Hardcover Books - $3   Paperback Books - $2

4. Your friend divided two decimal numbers. Her work is shown in the box below. What should your friend change in order to divide the two decimal numbers correctly?

$$2.08 \\ 0.07\overline{)14.56} \rightarrow 7\overline{)14.56}$$

A. Rewrite the problem as $0.07\overline{)0.1456}$.

B. Rewrite the problem as $0.07\overline{)1456}$.

C. Rewrite the problem as $7\overline{)0.1456}$.

D. Rewrite the problem as $7\overline{)1456}$.

5. What is the value of $4.391 + 5.954$?

F. 9.12145   **G.** 9.245

H. 9.345   **I.** 10.345

6. The circle graph shows the eye color of students in a sixth-grade class. Nine students in the class have brown eyes. How many students are in the class?

A. 4 students

B. 18 students

C. 20 students

D. 405 students

Eye Color

Blue 20%

Green 35%

Brown 45%

7. Properties of Addition and Multiplication are used to simplify an expression.

$$36 \cdot 23 + 33 \cdot 64 = 36 \cdot 23 + 64 \cdot 33$$
$$= 36 \cdot 23 + 64 \cdot (23 + 10)$$
$$= 36 \cdot 23 + 64 \cdot 23 + 64 \cdot 10$$
$$= x \cdot 23 + 64 \cdot 10$$

What number belongs in place of the x?

8. What is the prime factorization of 1350?

    F. $10 \cdot 135$ **G.** $2 \cdot 3 \cdot 5$

    H. $6 \cdot 225$ **I.** $2 \cdot 3^3 \cdot 5^2$

9. A horse gallops at a speed of 44 feet per second. What is the speed of the horse in miles per hour?

    A. $\dfrac{1}{2}$ mile per hour   **B.** 30 miles per hour

    C. $64 \dfrac{8}{15}$ miles per hour   **D.** 158,400 miles per hour

10. Which equation correctly demonstrates the Distributive Property?

    F. $a(b + c) = ab + c$

    G. $a(b + c) = ab + ac$

    H. $a + (b + c) = (a + b) + (a + c)$

    I. $a + (b + c) = (a + b) \cdot (a + c)$

11. Which number is equivalent to $2 \dfrac{4}{5} \cdot 1 \dfrac{2}{7}$?

    A. $2 \dfrac{8}{45}$ **B.** $2 \dfrac{8}{35}$

    C. $3 \dfrac{3}{5}$ **D.** $4 \dfrac{3}{35}$

12. Which pair of numbers does *not* have a least common multiple of 24?

    F. 2, 12   **G.** 3, 8

    H. 6, 8   **I.** 12, 24

13. Use the Properties of Multiplication to simplify the expression in an efficient way. Show your work and explain how you used the Properties of Multiplication.

Think
Solve
Explain

$$(25 \times 18) \times 4$$
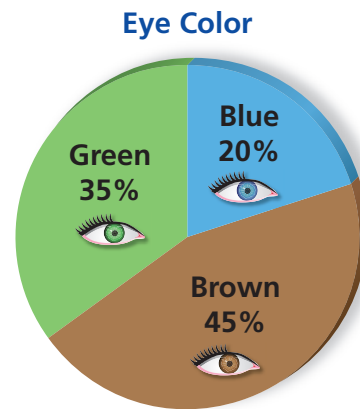
14. Which number is *not* a perfect square?

    A. 64   **B.** 81

    C. 96   **D.** 100