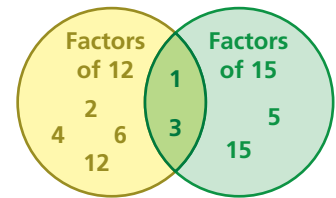


1.4 Greatest Common Factor

Learning Target: Find the greatest common factor of two numbers.

- Success Criteria:**
- I can explain the meaning of factors of a number.
 - I can use lists of factors to identify the greatest common factor of numbers.
 - I can use prime factors to identify the greatest common factor of numbers.

A **Venn diagram** uses circles to describe relationships between two or more sets. The Venn diagram shows the factors of 12 and 15. Numbers that are factors of both 12 and 15 are represented by the overlap of the two circles.



EXPLORATION 1

Identifying Common Factors

Work with a partner. In parts (a)–(d), create a Venn diagram that represents the factors of each number and identify any *common factors*.

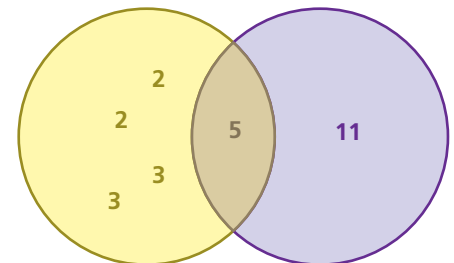
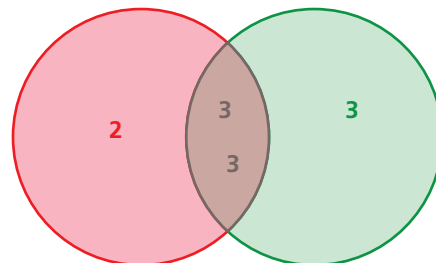
- 36 and 48
 - 16 and 56
 - 30 and 75
 - 54 and 90
- e. Look at the Venn diagrams in parts (a)–(d). Explain how to identify the *greatest common factor* of each pair of numbers. Then circle it in each diagram.

EXPLORATION 2

Using Prime Factors

Work with a partner.

- Each Venn diagram represents the prime factorizations of two numbers. Identify each pair of numbers. Explain your reasoning.



- Create a Venn diagram that represents the prime factorizations of 36 and 48.
- Repeat part (b) for the remaining number pairs in Exploration 1.
- MP STRUCTURE** Make a conjecture about the relationship between the greatest common factors you found in Exploration 1 and the numbers in the overlaps of the Venn diagrams you just created.

Math Practice

Interpret a Solution

What does the diagram representing the prime factorizations mean?

1.4 Lesson

Factors that are shared by two or more numbers are called **common factors**. The greatest of the common factors is called the **greatest common factor** (GCF). One way to find the GCF of two or more numbers is by listing factors.

EXAMPLE 1 Finding the GCF Using Lists of Factors

Find the GCF of 24 and 40.

List the factors of each number.

Factors of 24: ①, ②, 3, ④, 6, ⑧, 12, 24

Circle the common factors.

Factors of 40: ①, ②, ④, 5, ⑧, 10, 20, 40

The common factors of 24 and 40 are 1, 2, 4, and 8. The greatest of these common factors is 8.

▶ So, the GCF of 24 and 40 is 8.

Try It Find the GCF of the numbers using lists of factors.

1. 8, 36

2. 18, 72

3. 14, 28, 49

Key Vocabulary

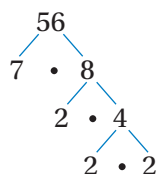
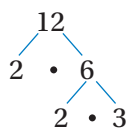
Venn diagram, p. 21
common factors, p. 22
greatest common factor, p. 22

Another way to find the GCF of two or more numbers is by using prime factors. The GCF is the product of the common prime factors of the numbers.

EXAMPLE 2 Finding the GCF Using Prime Factorizations

Find the GCF of 12 and 56.

Make a factor tree for each number.



Write the prime factorization of each number.

$$12 = 2 \cdot 2 \cdot 3$$

Circle the common prime factors.

$$56 = 2 \cdot 2 \cdot 2 \cdot 7$$

$$\begin{array}{c} \uparrow \quad \uparrow \\ 2 \cdot 2 = 4 \end{array}$$

Find the product of the common prime factors.

▶ So, the GCF of 12 and 56 is 4.

Try It Find the GCF of the numbers using prime factorizations.

4. 20, 45

5. 32, 90

6. 45, 75, 120

Examples 1 and 2 show two different methods for finding the GCF. After solving with one method, you can use the other method to check your answer.

EXAMPLE 3**Finding Two Numbers with a Given GCF**

Which pair of numbers has a GCF of 15?

- A. 10, 15 B. 30, 60 C. 21, 45 D. 45, 75

The number 15 cannot be a factor of the lesser number 10. So, you can eliminate Choice A.

The number 15 cannot be a factor of a number that does not have a 0 or 5 in the ones place. So, you can eliminate Choice C.

List the factors for Choices B and D. Then identify the GCF for each.

Choice B: Factors of 30: ①, ②, ③, ⑤, ⑥, ⑩, ⑮, ③①

Factors of 60: ①, ②, ③, 4, ⑤, ⑥, ⑩, 12, ⑮, 20, ③①, 60

The GCF of 30 and 60 is 30.

Choice D: Factors of 45: ①, ③, ⑤, 9, ⑮, 45

Factors of 75: ①, ③, ⑤, ⑮, 25, 75

The GCF of 45 and 75 is 15.

▶ The correct answer is **D**.

Try It

7. Write a pair of numbers whose greatest common factor is 10.

**Self-Assessment for Concepts & Skills**

Solve each exercise. Then rate your understanding of the success criteria in your journal.

FINDING THE GCF Find the GCF of the numbers.

8. 16, 40 9. 35, 63 10. 18, 72, 144

11. **MULTIPLE CHOICE** Which number is *not* a factor of 10? Explain.

- A. 1 B. 2 C. 4 D. 5

12. **DIFFERENT WORDS, SAME QUESTION** Which is different? Find “both” answers.

What is the greatest common factor of 24 and 32?

What is the greatest common divisor of 24 and 32?

What is the greatest common prime factor of 24 and 32?

What is the product of the common prime factors of 24 and 32?

EXAMPLE 4

Modeling Real Life

You are filling piñatas for your friend's birthday party. The list shows the gifts you are putting into the piñatas. You want identical groups of gifts in each piñata with no gifts left over. What is the greatest number of piñatas you can make?

* 18 kazoos

* 24 mints

* 42 lollipops

The GCF of the numbers of gifts represents the greatest number of identical groups of gifts you can make with no gifts left over. So, to find the number of piñatas, find the GCF.

Write the prime factorization of each number.

$$18 = 2 \cdot 3 \cdot 3$$

$$24 = 2 \cdot 3 \cdot 2 \cdot 2$$

$$42 = 2 \cdot 3 \cdot 7$$

Circle the common prime factors.

$$2 \cdot 3 = 6$$

Find the product of the common prime factors.

The GCF of 18, 24, and 42 is 6.

► So, you can make at most 6 piñatas.

Check Verify that 6 identical piñatas will use all of the gifts.

$$18 \text{ kazoos} \div 6 \text{ piñatas} = 3 \text{ kazoos per piñata}$$

$$24 \text{ mints} \div 6 \text{ piñatas} = 4 \text{ mints per piñata}$$

$$42 \text{ lollipops} \div 6 \text{ piñatas} = 7 \text{ lollipops per piñata} \quad \checkmark$$



Self-Assessment for Problem Solving

Solve each exercise. Then rate your understanding of the success criteria in your journal.

13. You use 30 sandwiches and 42 granola bars to make identical picnic baskets. You make the greatest number of picnic baskets with no food left over. How many sandwiches and how many granola bars are in each basket?
14. You fill bags with cookies to give to your friends. You bake 45 chocolate chip cookies, 30 peanut butter cookies, and 15 oatmeal cookies. You want identical groups of cookies in each bag with no cookies left over. What is the greatest number of bags you can make?



1.4 Practice



Go to BigIdeasMath.com to get HELP with solving the exercises.

► Review & Refresh

List the factor pairs of the number.

1. 20 2. 16 3. 56 4. 87

Tell whether the statement is *always*, *sometimes*, or *never* true.

5. A rectangle is a rhombus. 6. A rhombus is a square.
7. A square is a rectangle. 8. A trapezoid is a parallelogram.

► Concepts, Skills, & Problem Solving

USING A VENN DIAGRAM Use a Venn diagram to find the greatest common factor of the numbers. (See Exploration 1, p. 21.)

9. 12, 30 10. 32, 54 11. 24, 108

FINDING THE GCF Find the GCF of the numbers using lists of factors.

12. 6, 15 13. 14, 84 14. 45, 76
15. 39, 65 16. 51, 85 17. 40, 63
18. 12, 48 19. 24, 52 20. 30, 58

FINDING THE GCF Find the GCF of the numbers using prime factorizations.

21. 45, 60 22. 27, 63 23. 36, 81
24. 72, 84 25. 61, 73 26. 38, 95
27. 60, 75 28. 42, 60 29. 42, 63
30. 24, 96 31. 189, 200 32. 90, 108

OPEN-ENDED Write a pair of numbers with the indicated GCF.

33. 5 34. 12 35. 37

36. **MODELING REAL LIFE** A teacher is making identical activity packets using 92 crayons and 23 sheets of paper. What is the greatest number of packets the teacher can make with no items left over?

37. **MODELING REAL LIFE** You are making balloon arrangements for a birthday party. There are 16 white balloons and 24 red balloons. Each arrangement must be identical. What is the greatest number of arrangements you can make using every balloon?



YOU BE THE TEACHER Your friend finds the GCF of the two numbers. Is your friend correct? Explain your reasoning.

38.

$42 = 2 \cdot 3 \cdot 7$
 $154 = 2 \cdot 7 \cdot 11$
 The GCF is 7.

39.

$36 = 2^2 \cdot 3^2$
 $60 = 2^2 \cdot 3 \cdot 5$
 The GCF is $2^2 \cdot 3 = 12$.

FINDING THE GCF Find the GCF of the numbers.

40. 35, 56, 63

41. 30, 60, 78

42. 42, 70, 84

43. 40, 55, 72

44. 18, 54, 90

45. 16, 48, 88

46. 52, 78, 104

47. 96, 120, 156

48. 280, 300, 380

49. **OPEN-ENDED** Write three numbers that have a GCF of 16. What method did you use to find your answer?

CRITICAL THINKING Tell whether the statement is *always*, *sometimes*, or *never* true. Explain your reasoning.

50. The GCF of two even numbers is 2. 51. The GCF of two prime numbers is 1.

52. When one number is a multiple of another, the GCF of the numbers is the greater of the numbers.

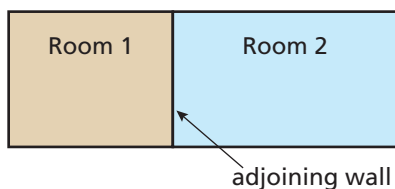
53. **MP PROBLEM SOLVING** A science museum makes gift bags for students using 168 magnets, 48 robot figurines, and 24 packs of freeze-dried ice cream. What is the greatest number of gift bags that can be made using all of the items? How many of each item are in each gift bag?

54. **VENN DIAGRAM** Consider the numbers 252, 270, and 300.

- Create a Venn diagram using the prime factors of the numbers.
- Use the Venn diagram to find the GCF of 252, 270, and 300.
- What is the GCF of 252 and 270? 252 and 300? 270 and 300? Explain how you found your answers.

55. **MP REASONING** You are making fruit baskets using 54 apples, 36 oranges, and 73 bananas.

- Explain why you cannot make identical fruit baskets without leftover fruit.
- What is the greatest number of identical fruit baskets you can make with the least amount of fruit left over? Explain how you found your answer.



56. **DIG DEEPER!** Two rectangular, adjacent rooms share a wall. One-foot-by-one-foot tiles cover the floor of each room. Describe how the greatest possible length of the adjoining wall is related to the total number of tiles in each room. Draw a diagram that represents one possibility.