1.3 Prime Factorization

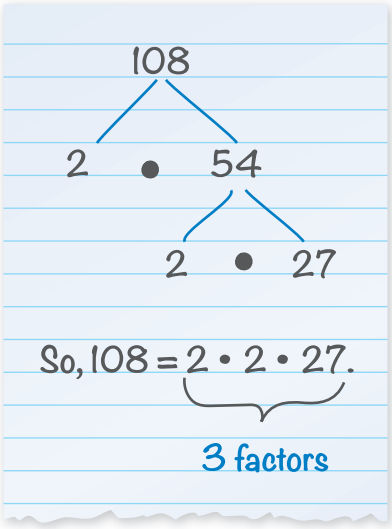
Learning Target: Write a number as a product of prime factors and represent the product using exponents.

Success Criteria:
- I can find factor pairs of a number.
- I can explain the meanings of prime and composite numbers.
- I can create a factor tree to find the prime factors of a number.
- I can write the prime factorization of a number.

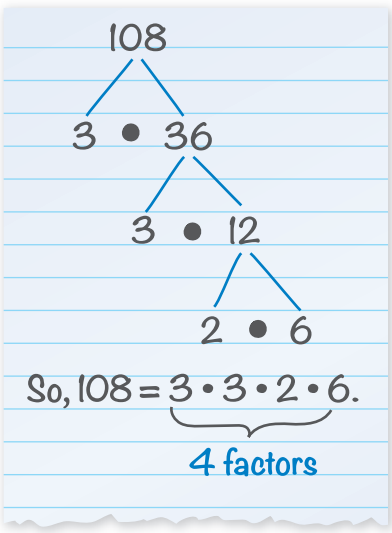
EXPLORATION 1 Rewriting Numbers as Products of Factors

Work with a partner. Two students use *factor trees* to write 108 as a product of factors, as shown below.

Student A



Student B



a. Without using 1 as a factor, can you write 108 as a product with more factors than each student used? Justify your answer.

Math Practice

Interpret Results
How do you know your answer makes sense?

b. Use factor trees to write 80, 162, and 300 as products of as many factors as possible. Do not use 1 as a factor.

c. Compare your results in parts (a) and (b) with other groups. For each number, identify the product with the greatest number of factors. What do these factors have in common?

1.3 Lesson

Because 2 is a factor of 10 and $2 \cdot 5 = 10$, 5 is also a factor of 10. The pair 2, 5 is called a **factor pair** of 10.

Key Vocabulary

factor pair, *p. 16*
prime factorization, *p. 16*
factor tree, *p. 16*

EXAMPLE 1 Finding Factor Pairs

The brass section of a marching band has 30 members. The band director arranges the brass section in rows. Each row has the same number of members. How many possible arrangements are there?

Use the factor pairs of 30 to find the number of arrangements.

$30 = 1 \cdot 30$ There could be 1 row of 30 or 30 rows of 1.

$30 = 2 \cdot 15$ There could be 2 rows of 15 or 15 rows of 2.

$30 = 3 \cdot 10$ There could be 3 rows of 10 or 10 rows of 3.

$30 = 5 \cdot 6$ There could be 5 rows of 6 or 6 rows of 5.

$30 = 6 \cdot 5$ The factors 5 and 6 are already listed.

When making an organized list of factor pairs, stop finding pairs when the factors begin to repeat.

There are 8 possible arrangements: 1 row of 30, 30 rows of 1, 2 rows of 15, 15 rows of 2, 3 rows of 10, 10 rows of 3, 5 rows of 6, or 6 rows of 5.

Try It

List the factor pairs of the number.

1. 18 **2.** 24 **3.** 51

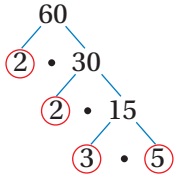
4. WHAT IF? The woodwinds section of the marching band has 38 members. Which has more possible arrangements, the brass section or the woodwinds section? Explain.

Key Idea

Prime Factorization

The **prime factorization** of a composite number is the number written as a product of its prime factors.

You can use factor pairs and a **factor tree** to help find the prime factorization of a number. The factor tree is complete when only prime factors appear in the product. A factor tree for 60 is shown.



$60 = 2 \cdot 2 \cdot 3 \cdot 5$, or $2^2 \cdot 3 \cdot 5$

Remember

A *prime number* is a whole number greater than 1 with exactly two factors, 1 and itself. A *composite number* is a whole number greater than 1 with factors in addition to 1 and itself.

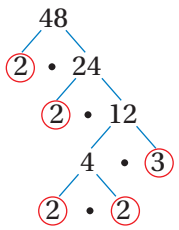
EXAMPLE 2 Writing a Prime Factorization

Write the prime factorization of 48.

Choose any factor pair of 48 to begin the factor tree.

Notice that beginning with different factor pairs results in the same prime factorization. Every composite number has only one prime factorization.

Tree 1



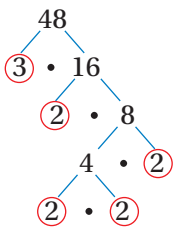
$48 = 2 \cdot 2 \cdot 3 \cdot 2 \cdot 2$

Find a factor pair and draw "branches."

Circle the prime factors as you find them.

Find factors until each branch ends at a prime factor.

Tree 2



$48 = 3 \cdot 2 \cdot 2 \cdot 2 \cdot 2$

▶ The prime factorization of 48 is $2 \cdot 2 \cdot 2 \cdot 2 \cdot 3$, or $2^4 \cdot 3$.

Try It **Write the prime factorization of the number.**

5. 20 **6.** 88 **7.** 90 **8.** 462

Self-Assessment for Concepts & Skills

Solve each exercise. Then rate your understanding of the success criteria in your journal.

WRITING A PRIME FACTORIZATION **Write the prime factorization of the number.**

9. 14 **10.** 86 **11.** 40 **12.** 516

13. **WRITING** Explain the difference between prime numbers and composite numbers.

14. MP **STRUCTURE** Your friend lists the following factor pairs and concludes that there are 6 factor pairs of 12. Explain why your friend is incorrect.

1, 12	2, 6	3, 4
12, 1	6, 2	4, 3

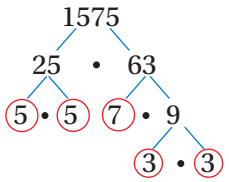
15. **WHICH ONE DOESN'T BELONG?** Which factor pair does *not* belong with the other three? Explain your reasoning.

2, 28	4, 14	6, 9	7, 8

EXAMPLE 3 Using a Prime Factorization

What is the greatest perfect square that is a factor of 1575?

Because 1575 has many factors, it is not efficient to list all of its factors and check for perfect squares. Use a factor tree to write the prime factorization of 1575. Then analyze the prime factors to find perfect square factors.



$1575 = 3 \cdot 3 \cdot 5 \cdot 5 \cdot 7$

The prime factorization shows that 1575 has three factors other than 1 that are perfect squares.

$3 \cdot 3 = 9$

$5 \cdot 5 = 25$

$(3 \cdot 5) \cdot (3 \cdot 5) = 15 \cdot 15 = 225$

So, the greatest perfect square that is a factor of 1575 is 225.

Self-Assessment for Problem Solving

Solve each exercise. Then rate your understanding of the success criteria in your journal.

16. A group of 20 friends plays a card game. The game can be played with 2 or more teams of equal size. Each team must have at least 2 members. List the possible numbers and sizes of teams.

17. You arrange 150 chairs in rows for a school play. You want each row to have the same number of chairs. How many possible arrangements are there? Are all of the possible arrangements appropriate for the play? Explain.

18. What is the least perfect square that is a factor of 4536? What is the greatest perfect square that is a factor of 4536?

19. DIG DEEPER! The prime factorization of a number is $2^4 \times 3^4 \times 5^4 \times 7^2$. Is the number a perfect square? Explain your reasoning.

1.3 Practice

Go to **BigIdeasMath.com** to get HELP with solving the exercises.

Review & Refresh

Evaluate the expression.

1. $2 + 4^2(5 - 3)$ **2.** $2^3 + 4 \times 3^2$ **3.** $9 \times 5 - 2^4\left(\frac{5}{2} - \frac{1}{2}\right)$

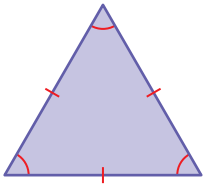
Plot the points in a coordinate plane. Draw a line segment connecting the points.

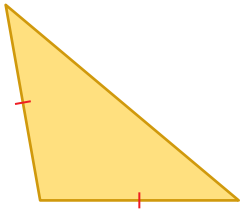
4. (1, 1) and (4, 3) **5.** (2, 3) and (5, 9) **6.** (2, 5) and (4, 8)

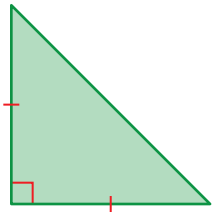
Use the Distributive Property to find the quotient. Justify your answer.

7. $408 \div 4$ **8.** $628 \div 2$ **9.** $969 \div 3$

Classify the triangle in as many ways as possible.

10. 

11. 

12. 

Concepts, Skills, & Problem Solving

REWRITING A NUMBER **Write the number as a product of as many factors as possible.** (See Exploration 1, p. 15.)

13. 60 **14.** 63 **15.** 120 **16.** 150

FINDING FACTOR PAIRS **List the factor pairs of the number.**

17. 15 **18.** 22 **19.** 34 **20.** 39

21. 45 **22.** 54 **23.** 59 **24.** 61

25. 100 **26.** 58 **27.** 25 **28.** 76

29. 52 **30.** 88 **31.** 71 **32.** 91

WRITING A PRIME FACTORIZATION **Write the prime factorization of the number.**

33. 16 **34.** 25 **35.** 30 **36.** 26

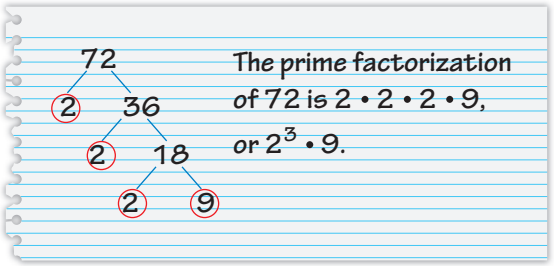
37. 84 **38.** 54 **39.** 65 **40.** 77

41. 46 **42.** 39 **43.** 99 **44.** 24

45. 315 **46.** 490 **47.** 140 **48.** 640

USING A PRIME FACTORIZATION **Find the number represented by the prime factorization.**

49. $2^2 \cdot 3^2 \cdot 5$ **50.** $3^2 \cdot 5^2 \cdot 7$ **51.** $2^3 \cdot 11^2 \cdot 13$



52. **YOU BE THE TEACHER** Your friend finds the prime factorization of 72. Is your friend correct? Explain your reasoning.

USING A PRIME FACTORIZATION **Find the greatest perfect square that is a factor of the number.**

53. 250	**54.** 275	**55.** 392	**56.** 338
57. 244	**58.** 650	**59.** 756	**60.** 1290
61. 2205	**62.** 1890	**63.** 495	**64.** 4725

65. **VOCABULARY** A botanist separates plants into equal groups of 5 for an experiment. Is the total number of plants in the experiment *prime* or *composite*? Explain.

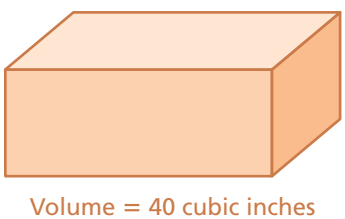
66. MP **REASONING** A teacher divides 36 students into equal groups for a scavenger hunt. Each group should have at least 4 students but no more than 8 students. What are the possible group sizes?

67. **CRITICAL THINKING** Is 2 the only even prime number? Explain.

68. MP **LOGIC** One table at a bake sale has 75 cookies. Another table has 60 cupcakes. Which table allows for more rectangular arrangements? Explain.

69. **PERFECT NUMBERS** A *perfect number* is a number that equals the sum of its factors, not including itself. For example, the factors of 28 are 1, 2, 4, 7, 14, and 28. Because $1 + 2 + 4 + 7 + 14 = 28$, 28 is a perfect number. What are the perfect numbers between 1 and 27?

70. MP **REPEATED REASONING** Choose any two perfect squares and find their product. Then multiply your answer by another perfect square. Continue this process. Are any of the products perfect squares? What can you conclude?



71. MP **PROBLEM SOLVING** The stage manager of a school play creates a rectangular stage that has whole number dimensions and an area of 42 square yards. String lights will outline the stage. What is the least number of yards of string lights needed to enclose the stage?

72. **DIG DEEPER!** Consider the rectangular prism shown. Using only whole number dimensions, how many different prisms are possible? Explain.