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6.1 Perpendicular and Angle Bisectors

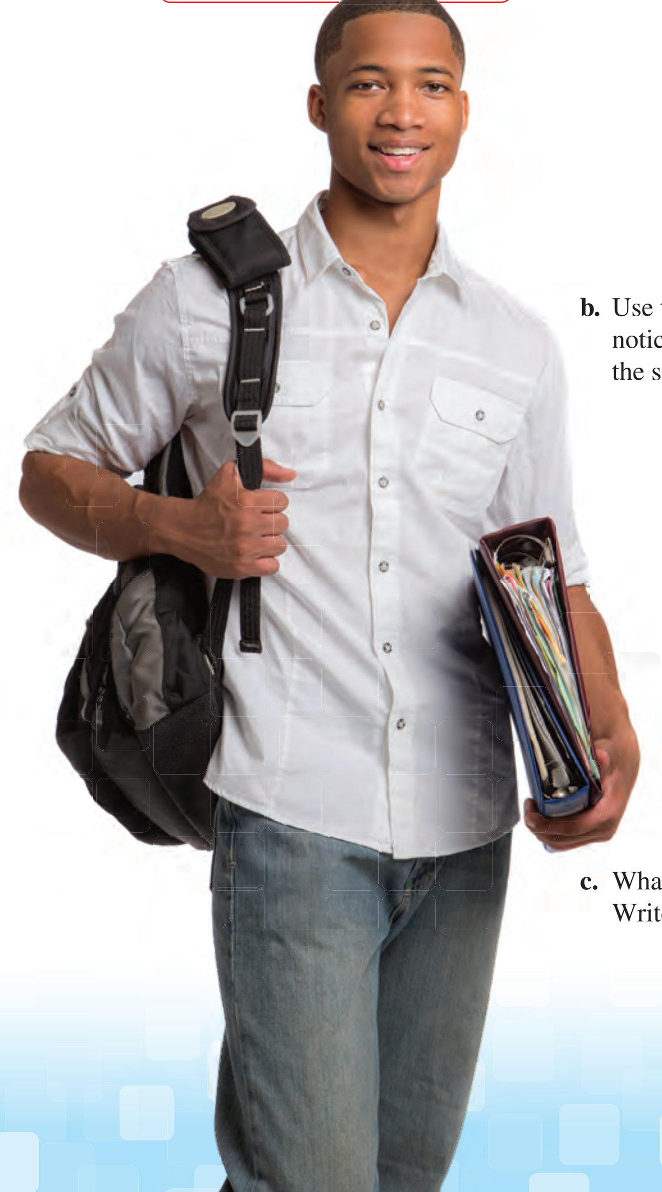
Learning Target Use theorems about perpendicular and angle bisectors.

Success Criteria
- I can identify a perpendicular bisector and an angle bisector.
- I can use theorems about bisectors to find measures in figures.
- I can write equations of perpendicular bisectors.

EXPLORE IT! Drawing Perpendicular and Angle Bisectors

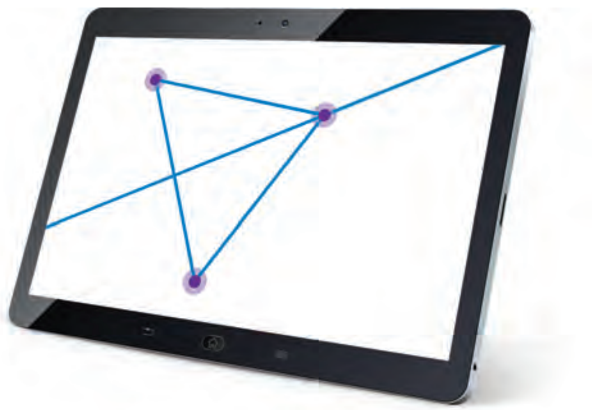
Math Practice

Use Technology to Explore
What advantages does technology have over a compass and straightedge when making these conjectures?

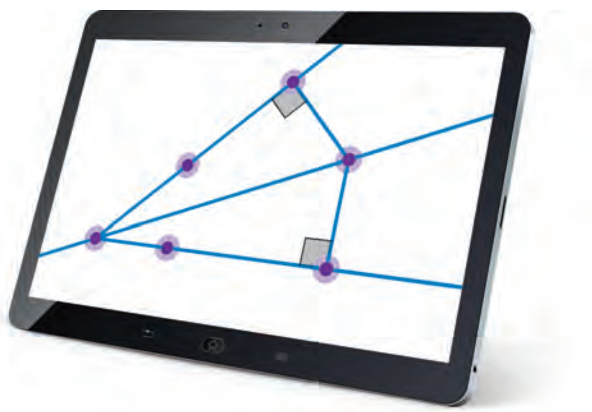


Work with a partner.

a. Use technology to draw any segment and its perpendicular bisector. What do you notice about the distances between any point on the perpendicular bisector and the endpoints of the line segment? Explain why this is true.



b. Use technology to draw any angle and its angle bisector. What do you notice about the distances between any point on the angle bisector and the sides of the angle? Explain why this is true.



c. What conjectures can you make using your results in parts (a) and (b)? Write your conjectures as conditional statements written in if-then form.

Using Perpendicular Bisectors

Vocabulary

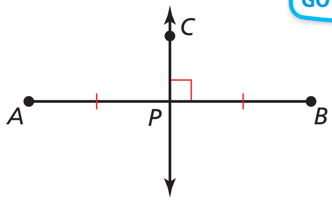
equidistant, *p. 292*

STUDY TIP

A perpendicular bisector can be a segment, a ray, a line, or a plane.

Recall that a *perpendicular bisector* of a line segment is the line that is perpendicular to the segment at its midpoint.

A point is **equidistant** from two figures when the point is the *same distance* from each figure.



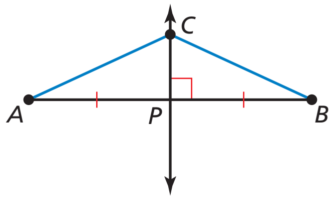
$\overleftrightarrow{CP}$ is a ⊥ bisector of $\overline{AB}$.

THEOREMS

6.1 Perpendicular Bisector Theorem

In a plane, if a point lies on the perpendicular bisector of a segment, then it is equidistant from the endpoints of the segment.

If $\overleftrightarrow{CP}$ is the ⊥ bisector of $\overline{AB}$, then $CA = CB$.

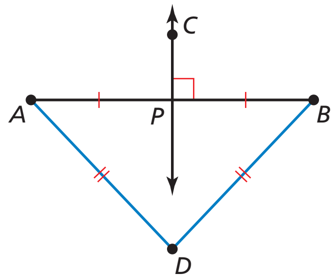


Prove this Theorem Exercise 1, page 293

6.2 Converse of the Perpendicular Bisector Theorem

In a plane, if a point is equidistant from the endpoints of a segment, then it lies on the perpendicular bisector of the segment.

If $DA = DB$, then point D lies on the ⊥ bisector of $\overline{AB}$.

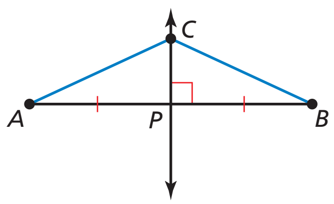


Prove this Theorem Exercise 30, page 297

PROOF Perpendicular Bisector Theorem

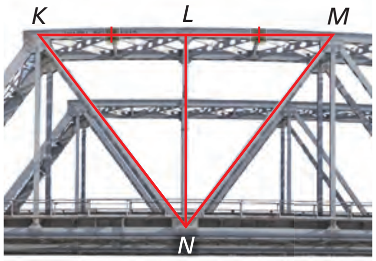
Given $\overleftrightarrow{CP}$ is the perpendicular bisector of $\overline{AB}$.

Prove $CA = CB$



Paragraph Proof Because $\overleftrightarrow{CP}$ is the perpendicular bisector of $\overline{AB}$, $\overleftrightarrow{CP}$ is perpendicular to $\overline{AB}$ and point P is the midpoint of $\overline{AB}$. By the definition of midpoint, $AP = BP$, and by the definition of perpendicular lines, $m\angle CPA = m\angle CPB = 90°$. Then by the definition of segment congruence, $\overline{AP} \cong \overline{BP}$, and by the definition of angle congruence, $\angle CPA \cong \angle CPB$. By the Reflexive Property of Segment Congruence, $\overline{CP} \cong \overline{CP}$. So, $\triangle CPA \cong \triangle CPB$ by the SAS Congruence Theorem, and $\overline{CA} \cong \overline{CB}$ because corresponding parts of congruent triangles are congruent. So, $CA = CB$ by the definition of segment congruence.

EXAMPLE 1 Using a Diagram



Is there enough information in the diagram to conclude that point N lies on the perpendicular bisector of $\overline{KM}$?

SOLUTION

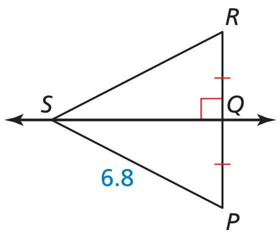
It is given that $\overline{KL} \cong \overline{ML}$. So, $\overline{LN}$ is a segment bisector of $\overline{KM}$. You do not know whether $\overline{LN}$ is perpendicular to $\overline{KM}$ because it is not indicated in the diagram.

▶ So, you cannot conclude that point N lies on the perpendicular bisector of $\overline{KM}$.

EXAMPLE 2 Using the Perpendicular Bisector Theorems

Find each measure.

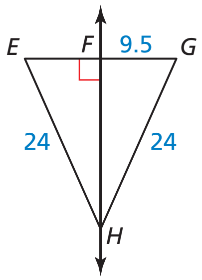
a. RS



From the figure, $\overleftrightarrow{SQ}$ is the perpendicular bisector of $\overline{PR}$. By the Perpendicular Bisector Theorem, $PS = RS$.

▶ So, $RS = PS = 6.8$.

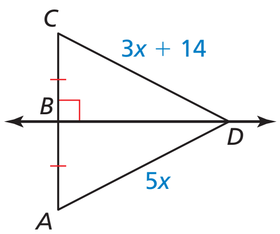
b. EG



Because $EH = GH$ and $\overleftrightarrow{HF} \perp \overline{EG}$, $\overleftrightarrow{HF}$ is the perpendicular bisector of $\overline{EG}$ by the Converse of the Perpendicular Bisector Theorem. So, F is the midpoint of $\overline{EG}$, and $EF = GF$.

▶ So, $EG = EF + GF = 9.5 + 9.5 = 19$.

c. AD



From the figure, $\overleftrightarrow{BD}$ is the perpendicular bisector of $\overline{AC}$.

$AD = CD$	Perpendicular Bisector Theorem
$5x = 3x + 14$	Substitute.
$x = 7$	Solve for x.

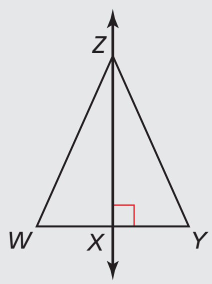
▶ So, $AD = 5x = 5(7) = 35$.

SELF-ASSESSMENT

1 I do not understand. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

1. Point M is the midpoint of $\overline{PQ}$, and $\overleftrightarrow{LM}$ is the perpendicular bisector of $\overline{PQ}$. Write a two-column proof to show that $LP = LQ$.

2. Is there enough information in the figure to conclude that point Z lies on the perpendicular bisector of $\overline{WY}$? Explain your reasoning.

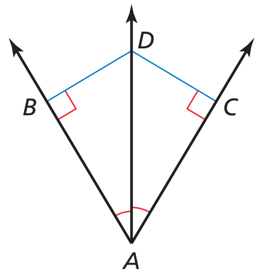


Use the figure and the given information to find the indicated measure.

3. $\overleftrightarrow{ZX}$ is the perpendicular bisector of $\overline{WY}$, and $YZ = 13.75$. Find WZ.

4. $\overleftrightarrow{ZX}$ is the perpendicular bisector of $\overline{WY}$, $WZ = 4n - 13$, and $YZ = n + 17$. Find YZ.

5. Find WX when $WZ = 20.5$, $WY = 14.8$, and $YZ = 20.5$.



Using Angle Bisectors

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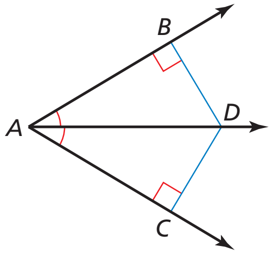
You know that an *angle bisector* is a ray that divides an angle into two congruent adjacent angles. You also know that the *distance from a point to a line* is the length of the perpendicular segment from the point to the line. So, in the figure, $\overrightarrow{AD}$ is the bisector of $\angle BAC$, and the distance from point D to $\overrightarrow{AB}$ is DB, where $\overline{DB} \perp \overrightarrow{AB}$.

THEOREMS

6.3 Angle Bisector Theorem

If a point lies on the bisector of an angle, then it is equidistant from the two sides of the angle.

If $\overrightarrow{AD}$ bisects $\angle BAC$ and $\overline{DB} \perp \overrightarrow{AB}$ and $\overline{DC} \perp \overrightarrow{AC}$, then $DB = DC$.

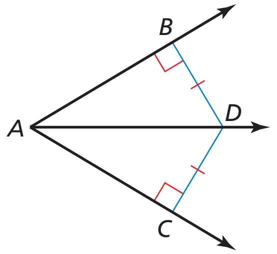


Prove this Theorem Exercise 31(a), page 297

6.4 Converse of the Angle Bisector Theorem

If a point is in the interior of an angle and is equidistant from the two sides of the angle, then it lies on the bisector of the angle.

If $\overline{DB} \perp \overrightarrow{AB}$ and $\overline{DC} \perp \overrightarrow{AC}$ and $DB = DC$, then $\overrightarrow{AD}$ bisects $\angle BAC$.

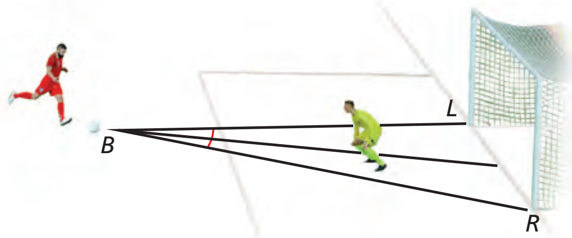


Prove this Theorem Exercise 31(b), page 297

EXAMPLE 3 Using a Diagram

WATCH INFO

A soccer goalie stands in the interior of $\angle LBR$, which is formed by $\overrightarrow{BL}$ and $\overrightarrow{BR}$, the paths from the ball to the goalposts. Why might the goalie want to stand on the bisector of $\angle LBR$?



SOLUTION

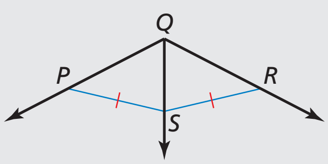
By the Angle Bisector Theorem, if the goalie stands on the bisector of $\angle LBR$, then the goalie is equidistant from $\overrightarrow{BL}$ and $\overrightarrow{BR}$.

▶ So, the goalie might want to stand on the bisector of $\angle LBR$ in order to move the same distance to block a shot toward either goalpost.

SELF-ASSESSMENT

1 I do not understand. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

6. Is there enough information in the figure to conclude that $\overrightarrow{QS}$ bisects $\angle PQR$? Explain.



EXAMPLE 4 Using the Angle Bisector Theorems

WATCH

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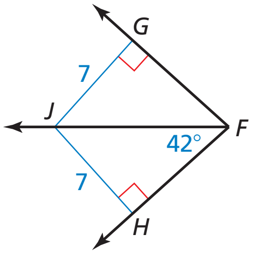
Find each measure.



a. $m\angle GFJ$

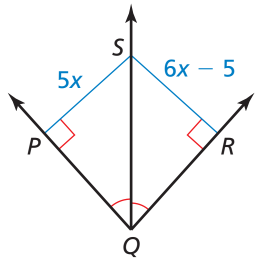
Because $\overline{JG} \perp \overrightarrow{FG}$ and $\overline{JH} \perp \overrightarrow{FH}$ and $JG = JH = 7$, $\overrightarrow{FJ}$ bisects $\angle GFH$ by the Converse of the Angle Bisector Theorem.

▶ So, $m\angle GFJ = m\angle HFJ = 42°$.

b. RS

From the figure, $\overrightarrow{QS}$ is the angle bisector of $\angle PQR$.

$PS = RS$	Angle Bisector Theorem
$5x = 6x - 5$	Substitute.
$5 = x$	Solve for x.



▶ So, $RS = 6x - 5 = 6(5) - 5 = 25$.

Writing Equations of Perpendicular Bisectors

EXAMPLE 5 Writing an Equation of a Bisector

WATCH

Write an equation of the perpendicular bisector of the segment with endpoints $P(-2, 3)$ and $Q(4, 1)$.

SOLUTION

Step 1 Graph $\overline{PQ}$. By definition, the perpendicular bisector of $\overline{PQ}$ is perpendicular to $\overline{PQ}$ at its midpoint.

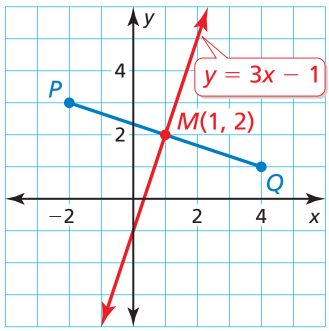
Step 2 Find the midpoint M of $\overline{PQ}$.

$$M\left(\frac{-2+4}{2}, \frac{3+1}{2}\right) = M\left(\frac{2}{2}, \frac{4}{2}\right) = M(1, 2)$$

Step 3 Find the slope of the perpendicular bisector.

$$\text{slope of } \overline{PQ} = \frac{1-3}{4-(-2)} = \frac{-2}{6} = -\frac{1}{3}$$

Because the slopes of perpendicular lines are negative reciprocals, the slope of the perpendicular bisector is 3.



Step 4 Write an equation. The perpendicular bisector of $\overline{PQ}$ has slope 3 and passes through (1, 2).

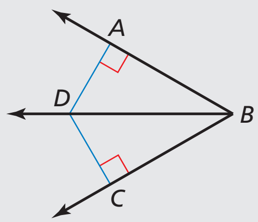
$y = mx + b$	Use slope-intercept form.
$2 = 3(1) + b$	Substitute for m, x, and y.
$-1 = b$	Solve for b.

▶ So, an equation of the perpendicular bisector of $\overline{PQ}$ is $y = 3x - 1$.

SELF-ASSESSMENT

1 I do not understand. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

Use the figure and the given information to find the indicated measure.



7. $\overrightarrow{BD}$ bisects $\angle ABC$, $AD = 3z + 7$, and $CD = 2z + 11$. Find CD.

8. Find $m\angle ABC$ when $AD = 3.2$, $CD = 3.2$, and $m\angle DBC = 39°$.

9. Write an equation of the perpendicular bisector of the segment with endpoints $(-1, -5)$ and $(3, -1)$.

6.1 Practice WITH CalcChat® AND CalcView®

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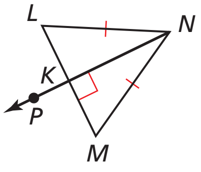
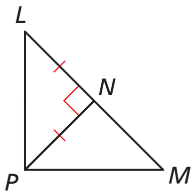
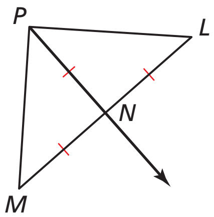
In Exercises 1–4, tell whether the information in the figure allows you to conclude that point P lies on the perpendicular bisector of $\overline{LM}$. Explain your reasoning. *Example 1*

1.


2.


3.


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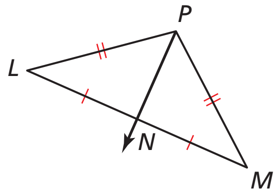
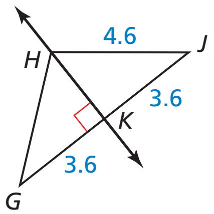
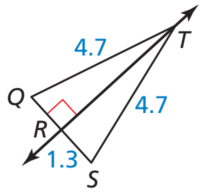
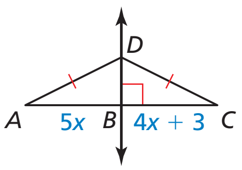
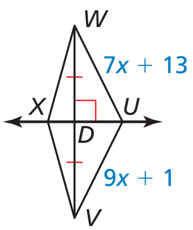


In Exercises 5–8, find the indicated measure. Explain your reasoning. *Example 2*

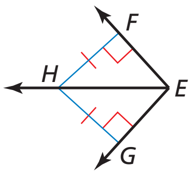
5. GH


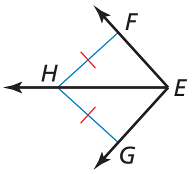
6. QR


7. AB


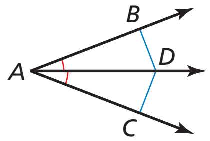
8. UW


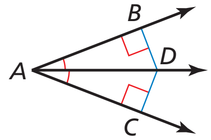
In Exercises 9 and 10, tell whether the information in the figure allows you to conclude that $\overrightarrow{EH}$ bisects $\angle FEG$. Explain your reasoning. *Example 3*

9.


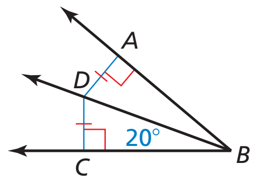
10.


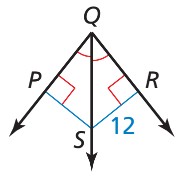
In Exercises 11 and 12, tell whether the information in the figure allows you to conclude that $DB = DC$. Explain your reasoning.

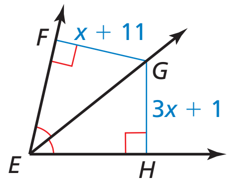
11.


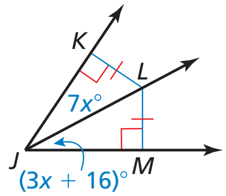
12.


In Exercises 13–16, find the indicated measure. Explain your reasoning. *Example 4*

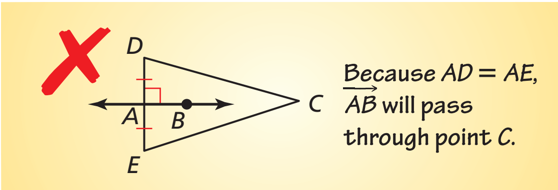
13. $m\angle ABD$


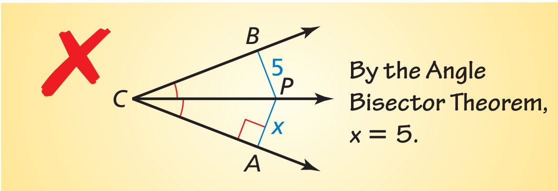
14. PS


15. FG


16. $m\angle KJL$


ERROR ANALYSIS In Exercises 17 and 18, describe and correct the error in the student's reasoning.

17.


18.


In Exercises 19–22, write an equation of the perpendicular bisector of the segment with the given endpoints. *Example 5*

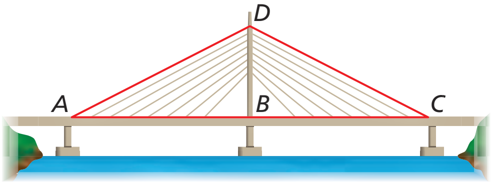
19. $M(1, 5), N(7, -1)$

20. $Q(-2, 0), R(6, 12)$

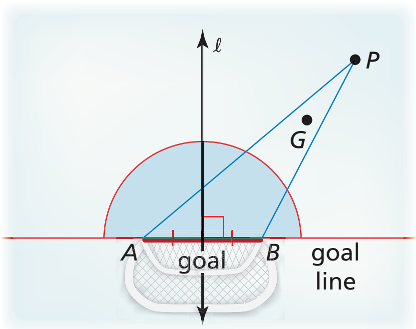
21. $U(-3, 4), V(9, 8)$

22. $Y(10, -7), Z(-4, 1)$

23. MODELING REAL LIFE In the diagram, the road is perpendicular to the support beam and $\overline{AB} \cong \overline{CB}$. Can you conclude that $\overline{AD} \cong \overline{CD}$? Explain.



24. MODELING REAL LIFE The diagram shows the position of the goalie and the puck during a hockey game. The goalie is at point G, and the puck is at point P. $\overrightarrow{PA}$ and $\overrightarrow{PB}$ represent the paths from the puck to the goalposts.



a. What should be the relationship between $\overrightarrow{PG}$ and $\angle APB$ so that the goalie is equidistant from $\overrightarrow{PA}$ and $\overrightarrow{PB}$?

b. How does $m\angle APB$ change as the puck gets closer to the goal? Does this change make it easier or more difficult for the goalie to defend the goal? Explain your reasoning.

25. CONSTRUCTION Use a compass and straightedge to construct a copy of $\overline{XY}$. Construct a perpendicular bisector and plot a point Z on the bisector so that the distance between point Z and $\overline{XY}$ is 3 centimeters. Measure $\overline{XZ}$ and $\overline{YZ}$. Which theorem does this construction demonstrate?

26. WRITING Explain how the Converse of the Perpendicular Bisector Theorem is related to the construction of a perpendicular bisector.

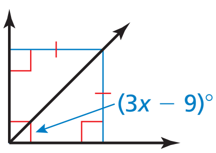
27. COLLEGE PREP What is the value of x in the figure?

Ⓐ 13

Ⓑ 18

Ⓒ 33

Ⓓ not enough information



28. COLLEGE PREP Which point lies on the perpendicular bisector of the segment with endpoints $M(7, 5)$ and $N(-1, 5)$?

Ⓐ (2, 0) Ⓑ (3, 9)

Ⓒ (4, 1) Ⓓ (1, 3)

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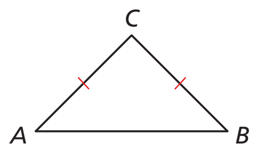
29. MAKING AN ARGUMENT Is it possible for an angle bisector of a triangle to be the same line as the perpendicular bisector of the opposite side? Explain.

30. PROVING A THEOREM Prove the Converse of the Perpendicular Bisector Theorem. (*Hint:* Use an auxiliary line.)



Given $CA = CB$

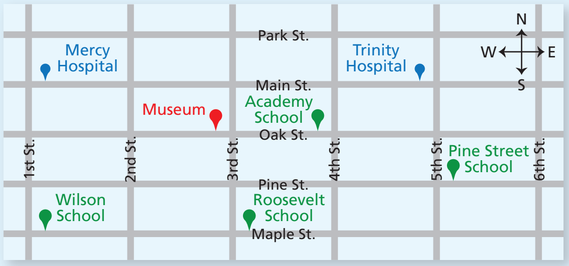
Prove Point C lies on the perpendicular bisector of $\overline{AB}$.

31. PROVING A THEOREM Use a congruence theorem to prove each theorem.

a. Angle Bisector Theorem

b. Converse of the Angle Bisector Theorem

32. HOW DO YOU SEE IT?
The map of a city is shown. The city is arranged so each block north to south is the same length and each block east to west is the same length. You are equidistant from the two hospitals. Describe your possible locations. Use a theorem to explain your reasoning.

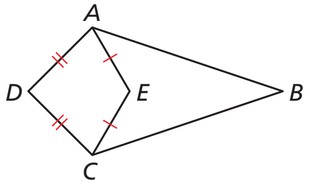


33. CONNECTING CONCEPTS Write an equation whose graph consists of all the points in the given quadrants that are equidistant from the x- and y-axes.

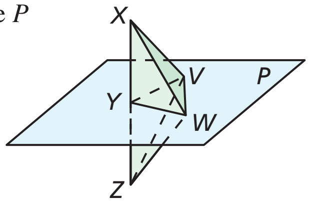
a. I and III **b.** II and IV

c. I and II **d.** III and IV

34. **PROOF** Use the information in the figure to prove that $\overline{AB} \cong \overline{CB}$ if and only if points D, E, and B are collinear.



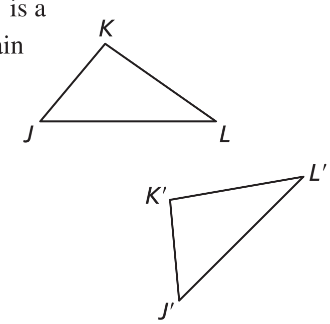
35. **PROOF** In the figure, plane P is a perpendicular bisector of $\overline{XZ}$ at point Y. Prove that $\angle VXW \cong \angle VZW$.



36. **THOUGHT PROVOKING** The postulates and theorems in this book represent Euclidean geometry. In spherical geometry, all points are on the surface of a sphere. A line is a circle on the sphere whose diameter is equal to the diameter of the sphere. In spherical geometry, is it possible for two lines to be perpendicular but not bisect each other? Explain your reasoning.

GO DIGITAL

37. **DIG DEEPER** $\triangle J'K'L'$ is a rotation of $\triangle JKL$. Explain how to find the center of rotation.

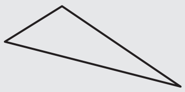


REVIEW & REFRESH

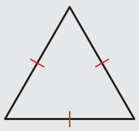
WATCH

In Exercises 38 and 39, classify the triangle by its sides.

38.

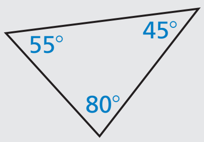


39.

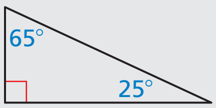


In Exercises 40 and 41, classify the triangle by its angles.

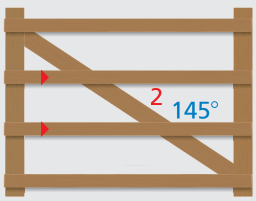
40.



41.



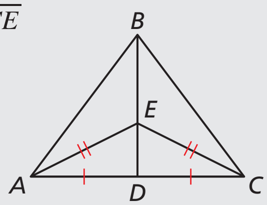
42. **MODELING REAL LIFE** A wooden gate is designed as shown. Find $m\angle 2$. Explain your reasoning.



43. Use the given information to write a plan for proof.

Given $\overline{AD} \cong \overline{CD}$, $\overline{AE} \cong \overline{CE}$

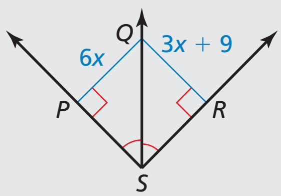
Prove $\triangle BDA \cong \triangle BDC$



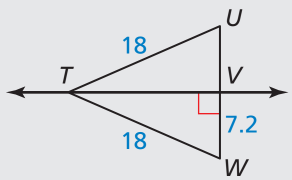
44. Find the product of $-2x^2$ and $3x^4 + 12x^3 - 14$.

In Exercises 45 and 46, find the indicated measure. Explain your reasoning.

45. QP



46. UW



47. In $\triangle RST$ and $\triangle XYZ$, $\overline{RS} \cong \overline{XY}$ and $\angle R \cong \angle X$. What is the third congruence statement that is needed to prove that $\triangle RST \cong \triangle XYZ$ using the ASA Congruence Theorem? the AAS Congruence Theorem?

48. Graph the square with the vertices $A(0, 0)$, $B(0, k)$, $C(k, k)$, and $D(k, 0)$. Then find the coordinates of the midpoint of each side.

49. Evaluate $4096^{1/6}$.