8.1 Sample Spaces and Probability

GO DIGITAL

Learning Target Find sample spaces and probabilities of events.

Success Criteria
- I can list the possible outcomes in a sample space.
- I can find theoretical probabilities.
- I can find experimental probabilities.

EXPLORE IT! Finding Sample Spaces and Describing Events

Work with a partner.

a. Describe the set of all possible outcomes for each experiment.

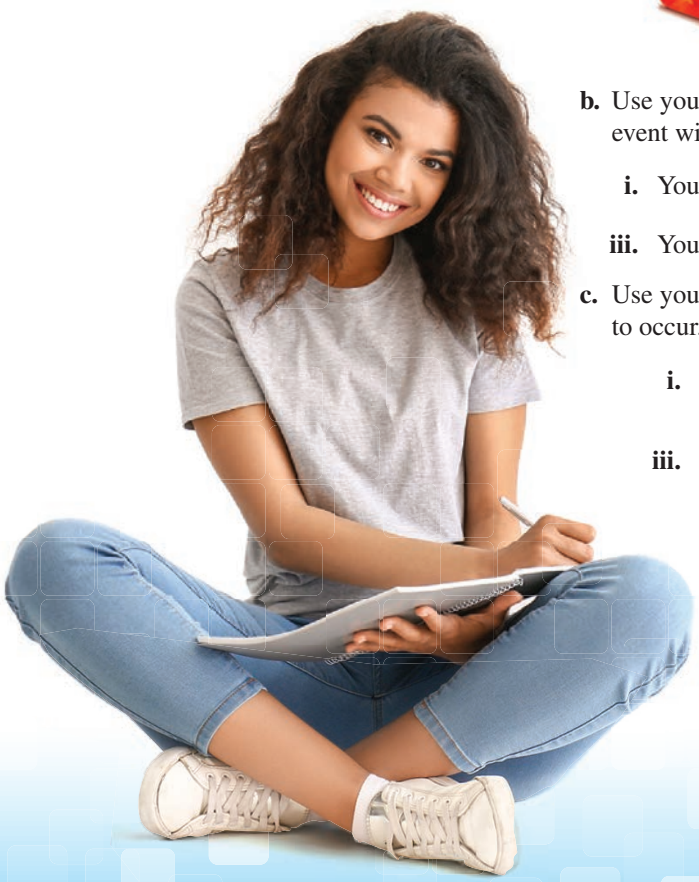
i. Three coins are flipped.

ii. One six-sided die is rolled.

iii. Two six-sided dice are rolled.

Math Practice

Understand Mathematical Terms
Can the likelihood of an event be impossible? Can the likelihood of an event be certain?



b. Use your results in part (a) to describe the likelihood that the given event will occur. Explain your reasoning.

i. You flip three tails.

ii. You roll an odd number.

iii. You roll a sum greater than 3.

c. Use your results in part (a) to determine which event is more likely to occur. Explain your reasoning.

i. **Event *A*:** flip exactly two heads
Event *B*: flip three heads

ii. **Event *A*:** roll an even number
Event *B*: roll a number less than 3

iii. **Event *A*:** roll "doubles"
Event *B*: roll a sum less than 6

d. Describe a real-life situation where it is important to know the likelihood of an event.

Vocabulary

probability experiment, *p. 404*
outcome, *p. 404*
event, *p. 404*
sample space, *p. 404*
probability of an event, *p. 404*
theoretical probability, *p. 405*
geometric probability, *p. 406*
experimental probability, *p. 407*

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Sample Spaces

A **probability experiment** is an action, or trial, that has varying results. The possible results of a probability experiment are **outcomes**. A collection of one or more outcomes is an **event**. The set of all possible outcomes is called a **sample space**. Here are some examples.

Probability experiment: rolling a six-sided die
Sample space: 1, 2, 3, 4, 5, 6
Event: rolling an even number
Outcome: rolling a 4

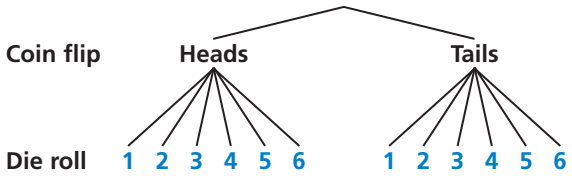
EXAMPLE 1 Finding a Sample Space

WATCH

You flip a coin and roll a six-sided die. How many possible outcomes are in the sample space? List the possible outcomes.

SOLUTION

Use a tree diagram to find the outcomes in the sample space.



The sample space has 12 possible outcomes. They are listed below.

Heads, 1 Heads, 2 Heads, 3 Heads, 4 Heads, 5 Heads, 6
Tails, 1 Tails, 2 Tails, 3 Tails, 4 Tails, 5 Tails, 6

Math Practice

State the Meaning of Symbols

Using H for "heads" and T for "tails," you can list the outcomes as shown below.

H1 H2 H3 H4 H5 H6
T1 T2 T3 T4 T5 T6

SELF-ASSESSMENT

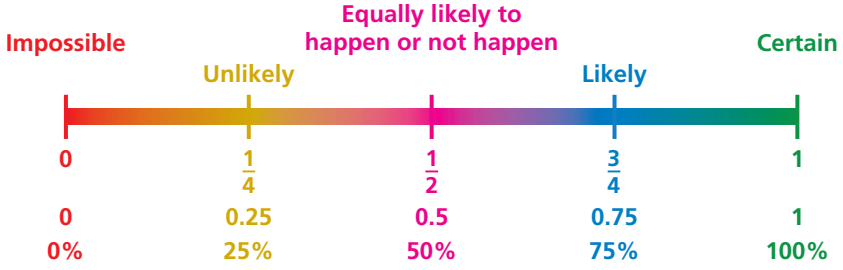
1 I do not understand. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

Find the number of possible outcomes in the sample space. Then list the possible outcomes.

1. You flip two coins.
2. You flip two coins and roll a six-sided die.
3. **WRITING** Explain the difference between an outcome and an event.

Theoretical Probabilities

The **probability of an event** is a measure of the likelihood, or chance, that the event will occur. Probability is a number from 0 to 1, including 0 and 1, and can be expressed as a decimal, fraction, or percent.



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The outcomes for a specified event are called *favorable outcomes*. When all outcomes are equally likely, the **theoretical probability** of the event can be found using the following.

$$\text{Theoretical probability} = \frac{\text{Number of favorable outcomes}}{\text{Total number of outcomes}}$$

The probability of event A is written as $P(A)$.

EXAMPLE 2 Finding a Theoretical Probability

WATCH

A student taking a quiz randomly guesses the answers to four true-false questions. What is the probability of the student guessing exactly two correct answers?

Math Practice

Communicate Precisely

Why is it more precise to use the phrase "exactly two answers" than the phrase "two answers?"

SOLUTION

Step 1 Identify the sample space. Let C represent a correct answer and I represent an incorrect answer. The possible outcomes are shown in the table.

Number correct	Outcome
0	IIII
1	CIII ICII IICI IIIC
2	IICC ICIC ICCI CIIC CICI CCII
3	ICCC CICC CCIC CCCI
4	CCCC

exactly two correct

Step 2 Identify the number of favorable outcomes and the total number of outcomes. There are 6 favorable outcomes with exactly two correct answers and the total number of outcomes is 16.

Step 3 Find the probability of the student guessing exactly two correct answers. Because the student is randomly guessing, the outcomes should be equally likely. So, use the theoretical probability formula.

$$P(\text{exactly two correct answers}) = \frac{\text{Number of favorable outcomes}}{\text{Total number of outcomes}}$$
$$= \frac{6}{16}$$
$$= \frac{3}{8}$$

▶ The probability of the student guessing exactly two correct answers is $\frac{3}{8}$, or 37.5%.

The sum of the probabilities of all outcomes in a sample space is 1. So, when you know the probability of event A, you can find the probability of the *complement* of event A. The *complement* of event A consists of all outcomes that are not in A and is denoted by $\overline{A}$. The notation $\overline{A}$ is read as "A bar." You can use the following formula to find $P(\overline{A})$.

WORDS AND MATH

Complements are parts that combine to create a whole. The probabilities of an event and its complement sum to 1.

KEY IDEA

Probability of the Complement of an Event

The probability of the complement of event A is

$$P(\overline{A}) = 1 - P(A).$$

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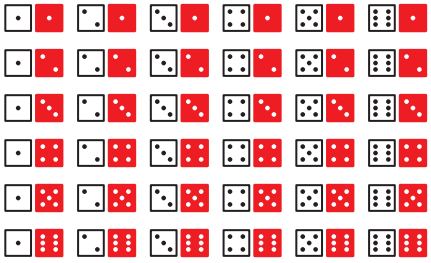
EXAMPLE 3 Finding Probabilities of Complements

When two six-sided dice are rolled, there are 36 possible outcomes, as shown. Find the probability of each event.



a. The sum is *not* 6.

b. The sum is less than or equal to 9.

SOLUTION

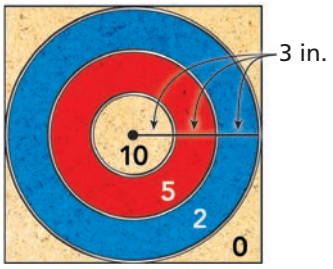
a. $P(\text{sum is not } 6) = 1 - P(\text{sum is } 6) = 1 - \frac{5}{36} = \frac{31}{36} \approx 0.861$

b. $P(\text{sum} \leq 9) = 1 - P(\text{sum} > 9) = 1 - \frac{6}{36} = \frac{30}{36} = \frac{5}{6} \approx 0.833$

Some probabilities are found by calculating a ratio of two lengths, areas, or volumes. Such probabilities are called **geometric probabilities**.

EXAMPLE 4 Using Area to Find Probability

You throw a dart at the board shown. Your dart is equally likely to hit any point inside the square board. Are you more likely to get 10 points or 0 points?



SOLUTION

The radius of the largest circle is 9 inches, so the side length of the board is 18 inches.

The probability of getting 10 points is

$$P(\text{10 points}) = \frac{\text{Area of smallest circle}}{\text{Area of entire board}} = \frac{\pi \cdot 3^2}{18^2} = \frac{9\pi}{324} \approx 0.087.$$

REMEMBER
The area A of a circle with radius r is $A = \pi r^2$.

The probability of getting 0 points is

$$P(\text{0 points}) = \frac{\text{Area outside largest circle}}{\text{Area of entire board}}$$

$$= \frac{18^2 - (\pi \cdot 9^2)}{18^2}$$

$$= \frac{324 - 81\pi}{324}$$

$$\approx 0.215.$$

▶ Because $0.215 > 0.087$, you are more likely to get 0 points.

SELF-ASSESSMENT

1 I do not understand. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

4. You flip a coin and roll a six-sided die. What is the probability that the coin shows tails and the die shows 4?

Use the information in Example 3 to find the probability of each event.

5. The sum is *not* 11.

6. The sum is greater than 3.

7. In Example 4, are you more likely to get 5 points or 0 points?

8. In Example 4, are you more likely to score more than 2 points or exactly 2 points?

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Experimental Probabilities

An **experimental probability** is based on repeated *trials* of a probability experiment. The number of trials is the number of times the probability experiment is performed. Each trial in which a favorable outcome occurs is called a *success*. The experimental probability can be found using the following.

$$\text{Experimental probability} = \frac{\text{Number of successes}}{\text{Number of trials}}$$

EXAMPLE 5 Finding an Experimental Probability

WATCH

Spinner Results			
red	**green**	**blue**	**yellow**
5	9	3	3

Each section of the spinner shown has the same area. The spinner is spun 20 times. The table shows the results. For which color is the experimental probability of stopping on the color the same as the theoretical probability?

SOLUTION

The theoretical probability of stopping on each of the four colors is $\frac{1}{4}$. Use the outcomes in the table to find the experimental probabilities.

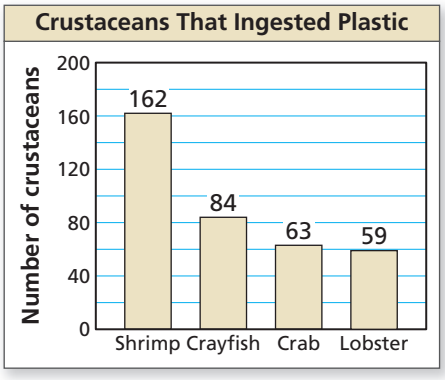
$$P(\text{red}) = \frac{5}{20} = \frac{1}{4} \qquad P(\text{green}) = \frac{9}{20} \qquad P(\text{blue}) = \frac{3}{20} \qquad P(\text{yellow}) = \frac{3}{20}$$

▶ The experimental probability of stopping on red is the same as the theoretical probability.

EXAMPLE 6 Modeling Real Life

WATCH INFO

A research team finds that 368 out of 490 crustaceans have ingested plastic. The types of crustaceans that ingested plastic are shown. The team randomly selects a crustacean that ingested plastic to demonstrate their findings. What is the probability that they choose a crayfish?



SOLUTION

The number of trials is the number of crustaceans that ingested plastic, 368. A success is a crustacean that ingested plastic is a crayfish. From the figure, there are 84 crayfish.

$$P(\text{crustacean that ingested plastic is a crayfish}) = \frac{84}{368} = \frac{21}{92} \approx 0.228$$

▶ The probability that they choose a crayfish is about 23%.

SELF-ASSESSMENT

1 I do not understand. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

9. In Example 5, for which color is the experimental probability of stopping on the color greater than the theoretical probability?

10. WHAT IF? In Example 6, what is the probability that they randomly select a lobster?

8.1 Practice WITH CalcChat® AND CalcView®

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In Exercises 1–4, find the number of possible outcomes in the sample space. Then list the possible outcomes.
Example 1

1. You flip a coin and draw a marble at random from a bag containing two purple marbles and one white marble.

2. You flip four coins.

3. You randomly choose a letter from A to F and a whole number from 1 to 3.

4. You draw two marbles without replacement from a bag containing three green marbles and three black marbles.

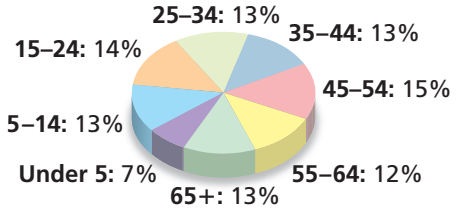
5. FINDING A THEORETICAL PROBABILITY A game show airs five days per week. Each day, a prize is randomly placed behind one of two doors. The contestant wins the prize by selecting the correct door. What is the probability that exactly two of the five contestants win a prize during a week?
Example 2

6. FINDING A THEORETICAL PROBABILITY Your friend has two standard decks of 52 playing cards and asks you to randomly draw one card from each deck. What is the probability that you will draw two spades?

7. FINDING PROBABILITIES OF COMPLEMENTS When two six-sided dice are rolled, there are 36 possible outcomes. Find the probability that (a) the sum is *not* 4 and (b) the sum is greater than 5. *Example 3*

8. FINDING PROBABILITIES OF COMPLEMENTS The age distribution of guests at a cultural festival is shown. Find the probability that (a) a person chosen at random is at least 15 years old and (b) a person chosen at random is *not* 25 to 44 years old.

Age Distribution



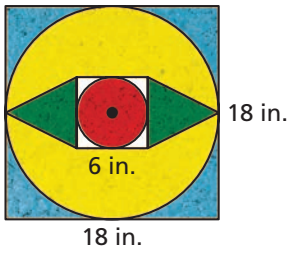
9. ERROR ANALYSIS A student randomly guesses the answers to two true-false questions. Describe and correct the error in finding the probability of the student guessing both answers correctly.

✗ The student can either guess two incorrect answers, two correct answers, or one of each. So the probability of guessing both answers correctly is $\frac{1}{3}$.

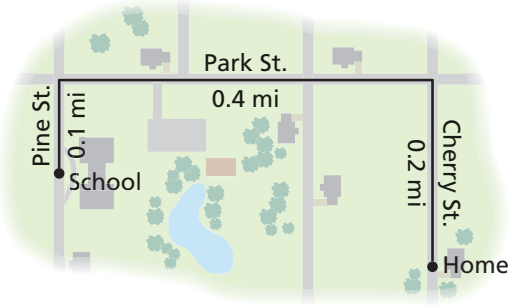
10. ERROR ANALYSIS A student randomly draws a whole number between 1 and 30. Describe and correct the error in finding the probability that the number drawn is greater than 4.

✗ $P(\text{number} > 4) = 1 - P(\text{number} < 4)$
$= 1 - \frac{3}{30}$
$= \frac{27}{30}$
$= \frac{9}{10}$

11. FINDING A GEOMETRIC PROBABILITY You throw a dart at the board shown. Your dart is equally likely to hit any point inside the square board. What is the probability your dart lands in the yellow region?
Example 4



12. FINDING A GEOMETRIC PROBABILITY A student loses his earbuds while walking home from school. The earbuds are equally likely to be at any point along the path shown. What is the probability that the earbuds are on Cherry Street?



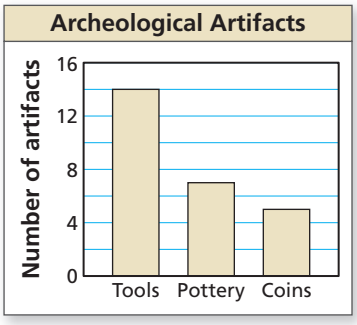
13. **DRAWING CONCLUSIONS** You roll a six-sided die 60 times. The table shows the results. For which number is the experimental probability of rolling the number the same as the theoretical probability? *Example 5*

Six-sided Die Results					
⚀	⚁	⚂	⚃	⚄	⚅
11	14	7	10	6	12

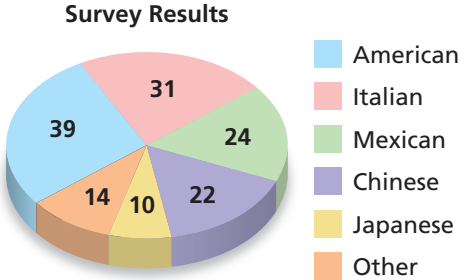
14. **DRAWING CONCLUSIONS** A bag contains 5 marbles that are each a different color. A marble is drawn, its color is recorded, and then the marble is placed back in the bag. The table shows the results after 30 draws. For which marble(s) is the experimental probability of drawing the marble greater than the theoretical probability?

Drawing Results				
white	**black**	**red**	**green**	**blue**
5	6	8	2	9

15. **MODELING REAL LIFE** An archaeologist uncovers 26 artifacts from a site. The types of artifacts are shown. An artifact is randomly selected for display. What is the probability that a piece of pottery is selected? *Example 6*



16. **MODELING REAL LIFE** A survey of 140 teenagers asked what type of food they like best. The results are shown. What is the probability that a randomly selected teenager from the survey likes Mexican food best?



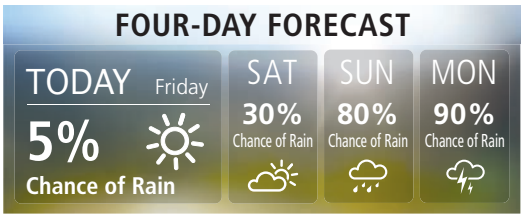
17. **MAKING AN ARGUMENT** You flip a coin three times. It lands on heads twice and on tails once. Your friend concludes that the theoretical probability of the coin landing heads up is $\frac{2}{3}$. Is your friend correct? Explain your reasoning.

18. **OPEN-ENDED** Describe a real-life event that has a probability of 0. Then describe a real-life event that has a probability of 1.

19. **ANALYZING RELATIONSHIPS** Refer to the board in Exercise 11. Order the likelihoods that the dart lands in the given region from least likely to most likely.

A. green
B. *not* blue
C. red
D. *not* yellow

20. **ANALYZING RELATIONSHIPS** Refer to the chart below. Order the following events from least likely to most likely.



A. It rains on Sunday.
B. It does *not* rain on Saturday.
C. It rains on Monday.
D. It does *not* rain on Friday.

21. **MP** **USING TOOLS** Use the figure in Example 3.

a. List the possible sums that result from rolling two six-sided dice.
b. Find the theoretical probability of rolling each sum.
c. The table shows a simulation of rolling two six-sided dice three times. Use a random number generator to simulate rolling two six-sided dice 50 times. Compare the experimental probabilities of rolling each possible sum with the theoretical probabilities.

	A	B	C
1	**First Die**	**Second Die**	**Sum**
2	4	6	10
3	3	5	8
4	1	6	7
5			

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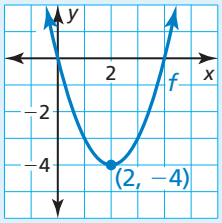
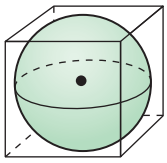
22. HOW DO YOU SEE IT?
Consider the graph of f shown. What is the probability that the graph of $y = f(x) + c$ intersects the x-axis when c is a randomly chosen integer from 1 to 6? Explain.



23. CONNECTING CONCEPTS A sphere fits inside a cube so that it touches each side, as shown. What is the probability a point chosen at random inside the cube is also inside the sphere?



24. THOUGHT PROVOKING
Describe a probability experiment that involves more than one action and has 48 possible outcomes in the sample space.

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25. DRAWING CONCLUSIONS
A manufacturer tests 1200 computers and finds that 9 of them have defects. Predict the number of computers with defects in a shipment of 15,000 computers. Explain your reasoning.

26. DIG DEEPER A test contains n true-false questions. A student randomly guesses the answer to each question. Write an expression that gives the probability of correctly answering all n questions.

27. PERFORMANCE TASK You are in charge of designing a game of chance for a fundraising event. You will charge a fee to play, and each winner will receive a cash prize. You expect about 200 people to play. Write a proposal in which you describe your game. Be sure to include how much you will charge to play, how much each winner will receive, the theoretical probability of winning, and how much you expect to raise (after prizes are deducted).

REVIEW & REFRESH

WATCH

In Exercises 28 and 29, simplify the expression.

28. $e^{\ln 9}$

29. $\log_3 81^{-2x}$

In Exercises 30 and 31, describe the transformation of f represented by g. Then graph each function.

30. $f(x) = x^3$, $g(x) = -0.5x^3$

31. $f(x) = x^4$, $g(x) = (x + 1)^4 - 2$

32. The spinner is divided into sections with the same area. You spin the spinner 25 times. It stops on a multiple of 3 twenty times. Compare the experimental probability of spinning a multiple of 3 with the theoretical probability.

33. Solve $\frac{x}{x-3} - \frac{4}{x} = \frac{12}{x}$.

34. MP REASONING A polynomial function f has rational coefficients and zeros of $2i$, $5 - i$, and -3. What is the least possible degree of f?

In Exercises 35–40, perform the indicated operation.

35. $\frac{4x^9y}{3x^3} \cdot \frac{2xy}{8y^2}$

36. $\frac{2y}{5x} \div \frac{y}{6x}$

37. $\frac{7}{x-2} + \frac{3x}{x+1}$

38. $\frac{4}{3x^2} - \frac{1}{x^2+5x}$

39. $\frac{x+8}{x^2-x-30} \cdot (x^2 - 7x + 6)$

40. $\frac{x^2+9x+14}{x^2-9} \div \frac{x+7}{5x+15}$

41. MODELING REAL LIFE
You want to order a pizza from one of two pizza shops. The table shows the total costs of a pizza with different numbers of toppings at Shop A. The total cost y (in dollars) of a pizza with x toppings at Shop B is represented by the equation $y = 1.75x + 11$. Which pizza shop charges less per topping? How many toppings must you order for the total costs to be the same?

Shop A	
Number of toppings, x	Total cost, y
1	13.50
2	15.00
3	16.50
4	18.00
5	19.50

42. Evaluate $625^{3/4}$.