7.5 Properties of Trapezoids and Kites

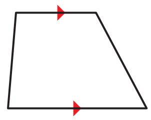
Learning Target Use properties of trapezoids and kites to find measures.

Success Criteria

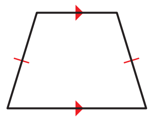
- I can identify trapezoids and kites.
- I can use properties of trapezoids and kites to solve problems.
- I can find the length of the midsegment of a trapezoid.
- I can explain the hierarchy of quadrilaterals.

EXPLORE IT! Discovering Properties of Trapezoids and Kites

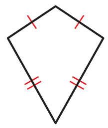
Work with a partner. Recall the types of quadrilaterals shown.



Trapezoid



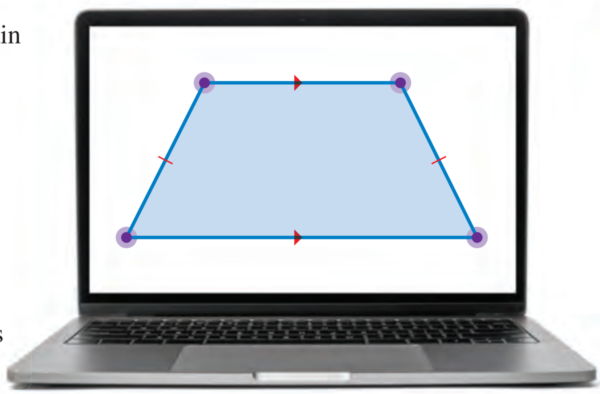
Isosceles Trapezoid



Kite

a. Use the diagrams to define each type of quadrilateral.

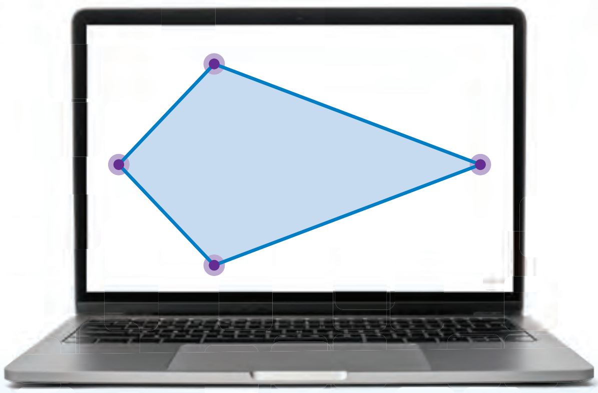
b. Use technology to construct an isosceles trapezoid. Explain your method.



c. Find the angle measures and diagonal lengths of the trapezoid. What do you observe?

d. Repeat parts (b) and (c) for several other isosceles trapezoids. Make conjectures based on your results.

e. Use technology to construct a kite. Explain your method. Make a conjecture about interior angle measures and a conjecture about the diagonals of the kite. Provide examples to support your reasoning.



Math Practice

Build Arguments
Can you construct a kite using its diagonals? Explain your reasoning.

Using Properties of Trapezoids

GO DIGITAL

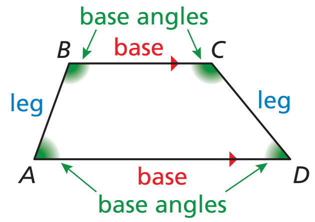
Vocabulary

AZ VOCAB

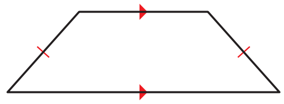
trapezoid, *p. 384*
bases, *p. 384*
base angles, *p. 384*
legs, *p. 384*
isosceles trapezoid, *p. 384*
midsegment of a trapezoid, *p. 386*
kite, *p. 387*

A **trapezoid** is a quadrilateral with exactly one pair of parallel sides. The parallel sides are the **bases**.

Base angles of a trapezoid are two consecutive angles whose common side is a base. A trapezoid has two pairs of base angles. For example, in trapezoid *ABCD*, $\angle A$ and $\angle D$ are one pair of base angles, and $\angle B$ and $\angle C$ are the second pair. The nonparallel sides are the **legs** of the trapezoid.



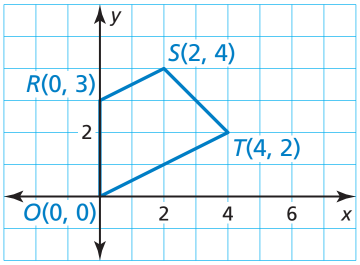
If the legs of a trapezoid are congruent, then the trapezoid is an **isosceles trapezoid**.



Isosceles trapezoid

EXAMPLE 1 Identifying a Trapezoid in the Coordinate Plane

WATCH



Show that *ORST* is a trapezoid. Then decide whether it is isosceles.

SOLUTION

Step 1 Compare the slopes of opposite sides.

$$\text{slope of } \overline{RS} = \frac{4-3}{2-0} = \frac{1}{2} \qquad \text{slope of } \overline{OT} = \frac{2-0}{4-0} = \frac{2}{4} = \frac{1}{2}$$

The slopes of $\overline{RS}$ and $\overline{OT}$ are the same, so $\overline{RS} \parallel \overline{OT}$.

$$\text{slope of } \overline{ST} = \frac{2-4}{4-2} = \frac{-2}{2} = -1 \qquad \text{slope of } \overline{RO} = \frac{3-0}{0-0} = \frac{3}{0} \quad \text{Undefined}$$

The slopes of $\overline{ST}$ and $\overline{RO}$ are not the same, so $\overline{ST}$ is not parallel to $\overline{RO}$.

▶ Because *ORST* has exactly one pair of parallel sides, it is a trapezoid.

Step 2 Compare the lengths of legs $\overline{RO}$ and $\overline{ST}$.

$$RO = |3-0| = 3 \qquad ST = \sqrt{(2-4)^2 + (4-2)^2} = \sqrt{8} \approx 2.83$$

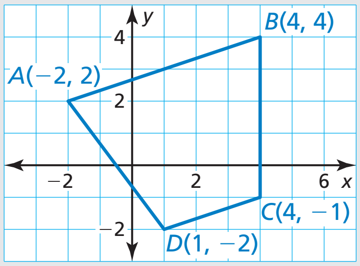
Because $RO \neq ST$, legs $\overline{RO}$ and $\overline{ST}$ are *not* congruent.

▶ So, *ORST* is not an isosceles trapezoid.

SELF-ASSESSMENT

1 I do not understand. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

In Exercises 1 and 2, use trapezoid *ABCD*.



1. Show that *ABCD* is a trapezoid. Then decide whether it is isosceles.

2. MP **REASONING** Vertex *B* moves to *B*(1, 3). Is *ABCD* a trapezoid? Explain your reasoning.

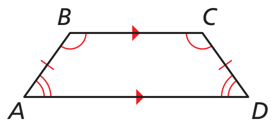
GO DIGITAL

THEOREMS

7.14 Isosceles Trapezoid Base Angles Theorem

If a trapezoid is isosceles, then each pair of base angles is congruent.

If trapezoid *ABCD* is isosceles, then $\angle A \cong \angle D$ and $\angle B \cong \angle C$.

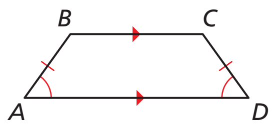


Prove this Theorem Exercise 41, page 391

7.15 Isosceles Trapezoid Base Angles Converse

If a trapezoid has a pair of congruent base angles, then it is an isosceles trapezoid.

If $\angle A \cong \angle D$ (or if $\angle B \cong \angle C$), then trapezoid *ABCD* is isosceles.

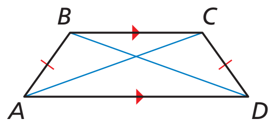


Prove this Theorem Exercise 42, page 391

7.16 Isosceles Trapezoid Diagonals Theorem

A trapezoid is isosceles if and only if its diagonals are congruent.

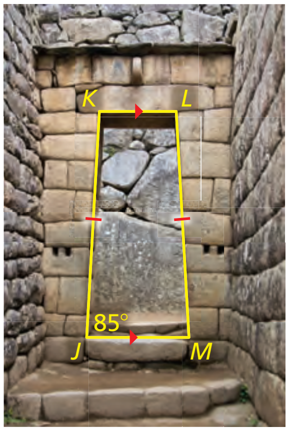
Trapezoid *ABCD* is isosceles if and only if $\overline{AC} \cong \overline{BD}$.



Prove this Theorem Exercise 51, page 392

EXAMPLE 2 Using Properties of Isosceles Trapezoids

WATCH INFO



Incan architecture often features trapezoidal doorways and windows. Find $m\angle M$, $m\angle K$, and $m\angle L$ in the doorway.

SOLUTION

JKLM is an isosceles trapezoid because there is exactly one pair of parallel sides and the legs are congruent.

Step 1 Find $m\angle M$. Because $\angle J$ and $\angle M$ are a pair of base angles, they are congruent. So, $m\angle M = m\angle J = 85°$.

Step 2 Find $m\angle K$. Because $\angle J$ and $\angle K$ are consecutive interior angles formed by $\overleftrightarrow{JK}$ intersecting two parallel lines, they are supplementary. So, $m\angle K = 180° - 85° = 95°$.

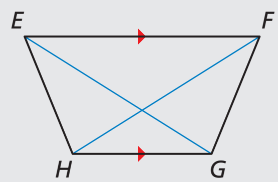
Step 3 Find $m\angle L$. Because $\angle K$ and $\angle L$ are a pair of base angles, they are congruent. So, $m\angle L = m\angle K = 95°$.

▶ So, $m\angle M = 85°$, $m\angle K = 95°$, and $m\angle L = 95°$.

SELF-ASSESSMENT

1 I do not understand. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

In Exercises 3 and 4, use trapezoid *EFGH*.



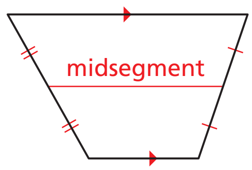
3. If $EG = FH$, is trapezoid *EFGH* isosceles? Explain.

4. If $m\angle HEF = 70°$ and $m\angle FGH = 110°$, is trapezoid *EFGH* isosceles? Explain.

Using the Trapezoid Midsegment Theorem

GO DIGITAL

Recall that a midsegment of a triangle is a segment that connects the midpoints of two sides of the triangle. The **midsegment of a trapezoid** is the segment that connects the midpoints of its legs. The theorem below is similar to the Triangle Midsegment Theorem.



READING

The midsegment of a trapezoid is sometimes called the *median* of the trapezoid.

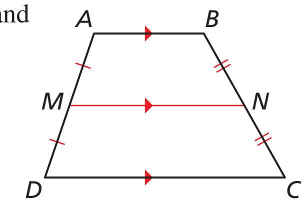
THEOREM

7.17 Trapezoid Midsegment Theorem

The midsegment of a trapezoid is parallel to each base, and its length is one-half the sum of the lengths of the bases.

If $\overline{MN}$ is the midsegment of trapezoid $ABCD$, then $\overline{MN} \parallel \overline{AB}$, $\overline{MN} \parallel \overline{DC}$, and $MN = \frac{1}{2}(AB + CD)$.

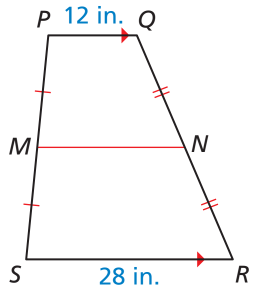
Prove this Theorem Exercise 50, page 391



EXAMPLE 3 Using the Midsegment of a Trapezoid

WATCH

In the diagram, $\overline{MN}$ is the midsegment of trapezoid $PQRS$. Find MN.



SOLUTION

$MN = \frac{1}{2}(PQ + SR)$ — Trapezoid Midsegment Theorem

$= \frac{1}{2}(12 + 28)$ — Substitute 12 for *PQ* and 28 for *SR*.

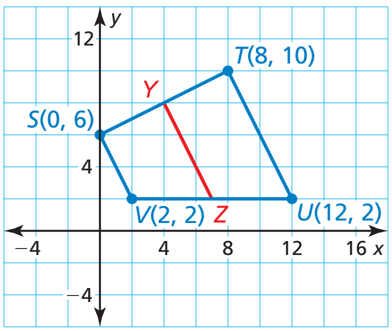
$= 20$ — Simplify.

▶ So, MN is 20 inches.

EXAMPLE 4 Using a Midsegment in the Coordinate Plane

WATCH

Find the length of midsegment $\overline{YZ}$ in trapezoid $STUV$.



SOLUTION

Step 1 Find the lengths of $\overline{SV}$ and $\overline{TU}$.

$$SV = \sqrt{(0 - 2)^2 + (6 - 2)^2} = \sqrt{20} = 2\sqrt{5}$$

$$TU = \sqrt{(8 - 12)^2 + (10 - 2)^2} = \sqrt{80} = 4\sqrt{5}$$

Step 2 Use the Trapezoid Midsegment Theorem.

$$YZ = \frac{1}{2}(SV + TU) = \frac{1}{2}\left(2\sqrt{5} + 4\sqrt{5}\right) = \frac{1}{2}\left(6\sqrt{5}\right) = 3\sqrt{5}$$

▶ So, the length of $\overline{YZ}$ is $3\sqrt{5}$ units.

SELF-ASSESSMENT

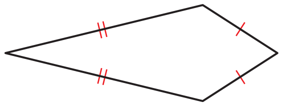
1 I do not understand. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

5. In trapezoid $JKLM$, $\angle J$ and $\angle M$ are right angles, and $JK = 9$ centimeters. The length of midsegment $\overline{NP}$ of trapezoid $JKLM$ is 12 centimeters. Sketch trapezoid $JKLM$ and its midsegment. Find ML. Explain your reasoning.

6. Use a different method to find the length of $\overline{YZ}$ in Example 4.

Using Properties of Kites

A **kite** is a quadrilateral that has two pairs of consecutive congruent sides, but opposite sides are not congruent.



WORDS AND MATH

The most common type of toy kite is shaped like the quadrilateral of the same name. The quadrilateral was named after the toy.

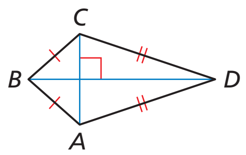
GO DIGITAL

THEOREMS

7.18 Kite Diagonals Theorem

If a quadrilateral is a kite, then its diagonals are perpendicular.

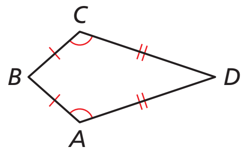
If quadrilateral $ABCD$ is a kite, then $\overline{AC} \perp \overline{BD}$.



7.19 Kite Opposite Angles Theorem

If a quadrilateral is a kite, then exactly one pair of opposite angles are congruent.

If quadrilateral $ABCD$ is a kite and $\overline{BC} \cong \overline{BA}$, then $\angle A \cong \angle C$ and $\angle B \ncong \angle D$.



Prove this Theorem Exercise 49, page 391

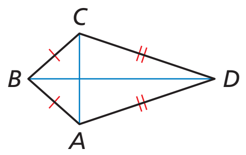
STUDY TIP

The congruent angles of a kite are formed by the noncongruent adjacent sides.

PROOF Kite Diagonals Theorem

Given $ABCD$ is a kite, $\overline{BC} \cong \overline{BA}$, and $\overline{DC} \cong \overline{DA}$.

Prove $\overline{AC} \perp \overline{BD}$

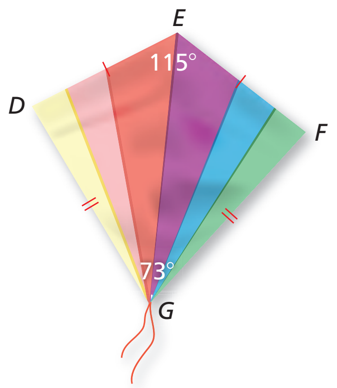


STATEMENTS	REASONS
1. $ABCD$ is a kite with $\overline{BC} \cong \overline{BA}$ and $\overline{DC} \cong \overline{DA}$.	**1.** Given
2. B and D lie on the $\perp$ bisector of $\overline{AC}$.	**2.** Converse of the $\perp$ Bisector Theorem
3. $\overline{BD}$ is the $\perp$ bisector of $\overline{AC}$.	**3.** Through any two points, there exists exactly one line.
4. $\overline{AC} \perp \overline{BD}$	**4.** Definition of $\perp$ bisector

EXAMPLE 5 Finding Angle Measures in a Kite

WATCH

Find $m\angle D$ in the kite shown.



SOLUTION

By the Kite Opposite Angles Theorem, $DEFG$ has exactly one pair of congruent opposite angles. Because $\angle E \ncong \angle G$, $\angle D$ and $\angle F$ must be congruent. So, $m\angle D = m\angle F$. Write and solve an equation to find $m\angle D$.

$m\angle D + m\angle F + 115° + 73° = 360°$ — Corollary to the Polygon Interior Angles Theorem

$m\angle D + m\angle D + 115° + 73° = 360°$ — Substitute $m\angle D$ for $m\angle F$.

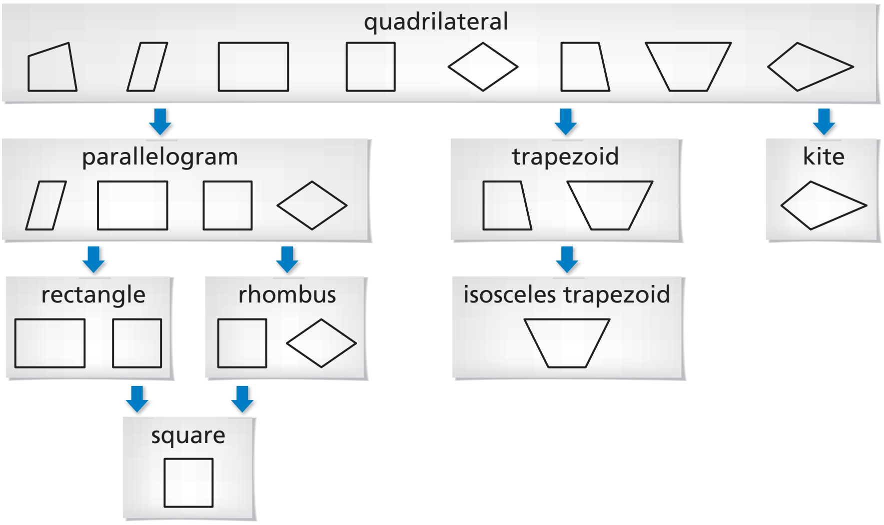
$2(m\angle D) + 188° = 360°$ — Combine like terms.

$m\angle D = 86°$ — Solve for $m\angle D$.

Identifying Special Quadrilaterals

GO DIGITAL

The diagram shows relationships among the special quadrilaterals you have studied in this chapter. Each shape in the diagram has the properties of the shapes linked above it. For example, a rhombus has the properties of a parallelogram and a quadrilateral.

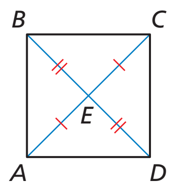


EXAMPLE 6 **Identifying a Quadrilateral**

WATCH

> **REMEMBER**
> In Example 6, *ABCD* looks like a square. But you must rely only on marked information when you interpret a diagram.

What is the most specific name for quadrilateral *ABCD*?



SOLUTION

The diagram shows $\overline{AE} \cong \overline{CE}$ and $\overline{BE} \cong \overline{DE}$. So, the diagonals bisect each other. By the Parallelogram Diagonals Converse, *ABCD* is a parallelogram.

Rectangles, rhombuses, and squares are also parallelograms. However, there is no information given about the side lengths or angle measures of *ABCD*. So, you cannot determine whether it is a rectangle, a rhombus, or a square.

▶ So, the most specific name for *ABCD* is a parallelogram.

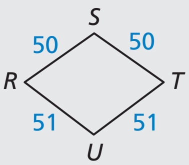
SELF-ASSESSMENT 1 I do not understand. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

7. In a kite, the measures of a pair of opposite angles are 50° and 108°. Find the measure of one of the other angles in the kite.

8. Quadrilateral *DEFG* has at least one pair of opposite sides congruent. What types of quadrilaterals meet this condition?

Give the most specific name for the quadrilateral. Explain your reasoning.

9.



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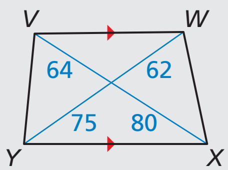

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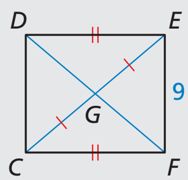


7.5 Practice WITH CalcChat® AND CalcView®

GO DIGITAL

In Exercises 1–4, show that the quadrilateral with the given vertices is a trapezoid. Then decide whether it is isosceles. *Example 1*

1. $W(1, 4)$, $X(1, 8)$, $Y(-3, 9)$, $Z(-3, 3)$

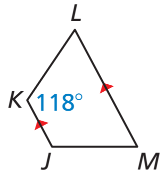
2. $D(-3, 3)$, $E(-1, 1)$, $F(1, -4)$, $G(-3, 0)$

3. $M(-2, 0)$, $N(0, 4)$, $P(5, 4)$, $Q(8, 0)$

4. $H(1, 9)$, $J(4, 2)$, $K(5, 2)$, $L(8, 9)$

In Exercises 5 and 6, find the measure of each angle in the isosceles trapezoid. *Example 2*

5.



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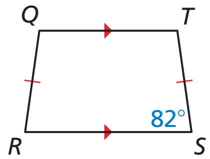
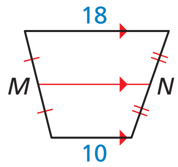


In Exercises 7 and 8, find the length of the midsegment of the trapezoid. *Example 3*

7.



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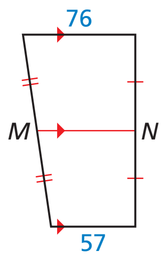
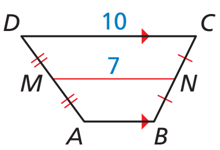


In Exercises 9 and 10, find AB.

9.



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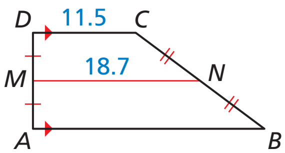


In Exercises 11–14, find the length of the midsegment of the trapezoid with the given vertices. *Example 4*

11. $S(0, 0)$, $T(2, 7)$, $U(6, 10)$, $V(8, 6)$

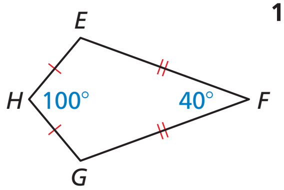
12. $A(0, 3)$, $B(2, 5)$, $C(6, 4)$, $D(2, 0)$

13. $A(2, 0)$, $B(8, -4)$, $C(12, 2)$, $D(0, 10)$

14. $S(-2, 4)$, $T(-2, -4)$, $U(3, -2)$, $V(13, 10)$

In Exercises 15–18, find $m\angle G$. *Example 5*

15.



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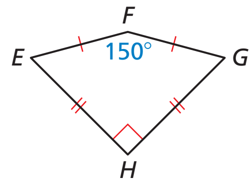

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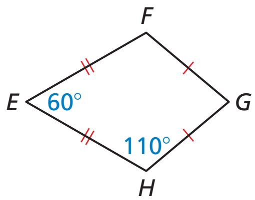

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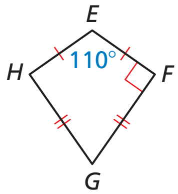
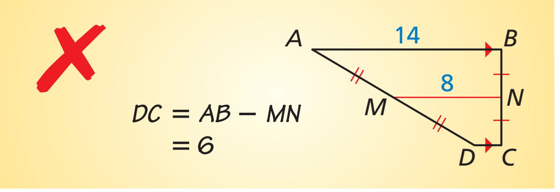
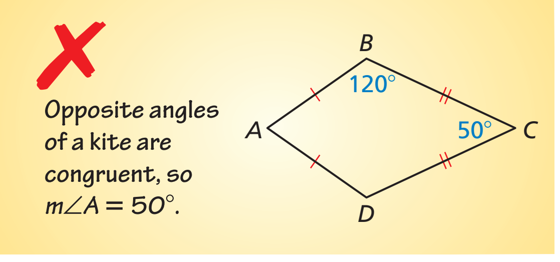


19. ERROR ANALYSIS Describe and correct the error in finding DC.



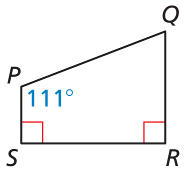
20. ERROR ANALYSIS Describe and correct the error in finding $m\angle A$.



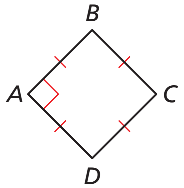
In Exercises 21–24, give the most specific name for the quadrilateral. Explain your reasoning. *Example 6*

21.

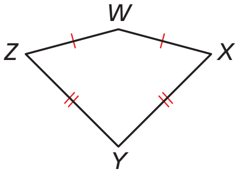
22.



23.

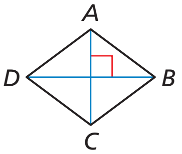


24.

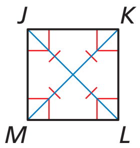


MP REASONING In Exercises 25 and 26, tell whether enough information is given in the diagram to classify the quadrilateral by the indicated name. Explain.

25. rhombus

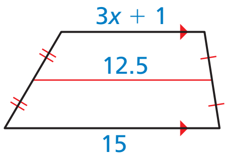


26. square

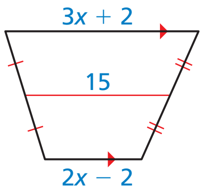


MP STRUCTURE In Exercises 27 and 28, find the value of *x*.

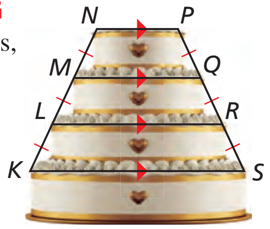
27.



28.



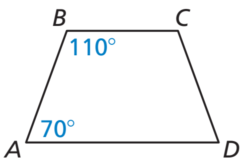
29. **MP REPEATED REASONING** In the diagram, $NP = 8$ inches, and $LR = 20$ inches. What is the diameter of the bottom layer of the cake?



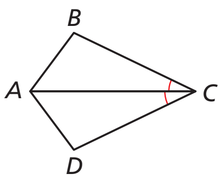
30. **CONNECTING CONCEPTS** You use 94 inches of plastic to frame the perimeter of a kite. One side of the kite has a length of 18 inches. Find the length of each of the three remaining sides.

MP REASONING In Exercises 31–34, determine which pairs of segments or angles must be congruent so that you can prove that *ABCD* is the indicated quadrilateral. Explain your reasoning. (There may be more than one correct answer.)

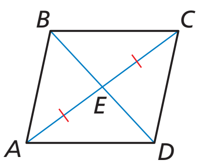
31. isosceles trapezoid



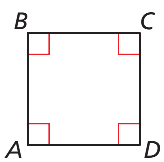
32. kite



33. parallelogram



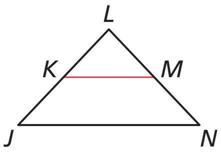
34. square



35. **PROOF** Write a proof.

Given $\overline{JL} \cong \overline{LN}$, $\overline{KM}$ is a midsegment of $\triangle JLN$.

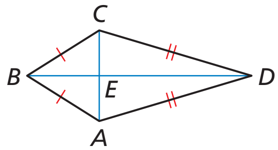
Prove Quadrilateral *JKMN* is an isosceles trapezoid.



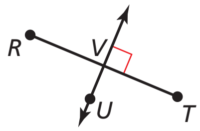
36. **PROOF** Write a proof.

Given *ABCD* is a kite.
$\overline{AB} \cong \overline{CB}$, $\overline{AD} \cong \overline{CD}$

Prove $\overline{CE} \cong \overline{AE}$

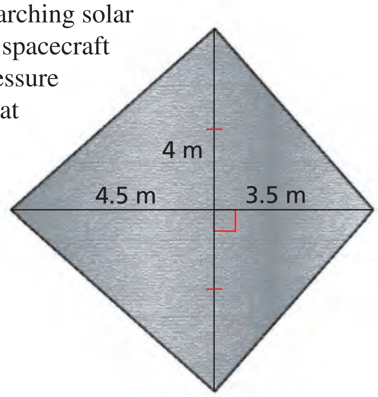


37. **ABSTRACT REASONING** Point *U* lies on the perpendicular bisector of $\overline{RT}$. Describe the set of points *S* for which *RSTU* is a kite.



38. **MP REASONING** Determine whether the points $A(4, 5)$, $B(-3, 3)$, $C(-6, -13)$, and $D(6, -2)$ are the vertices of a kite. Explain your reasoning.

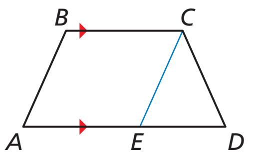
39. **MODELING REAL LIFE** Scientists are researching solar sails, which move spacecraft using radiation pressure from sunlight. What is the area of the solar sail shown? Explain your reasoning.



40. **PERFORMANCE TASK** You want to frame a 4-inch by 6-inch photo. Construct a rectangular frame using four pieces of wood, each in the shape of a trapezoid. Make a diagram showing how to arrange the trapezoids. Include side lengths and angle measures in your diagram.

GO DIGITAL

PROVING A THEOREM **In Exercises 41 and 42, use the diagram to prove the given theorem. In the diagram, $\overline{EC}$ is drawn parallel to $\overline{AB}$.**



41. Isosceles Trapezoid Base Angles Theorem

Given *ABCD* is an isosceles trapezoid.
$\overline{BC} \parallel \overline{AD}$

Prove $\angle A \cong \angle D$, $\angle B \cong \angle BCD$

42. Isosceles Trapezoid Base Angles Converse

Given *ABCD* is a trapezoid.
$\angle A \cong \angle D$, $\overline{BC} \parallel \overline{AD}$

Prove *ABCD* is an isosceles trapezoid.

43. **CONNECTING CONCEPTS** The bases of a trapezoid lie on the lines $y = 2x + 7$ and $y = 2x - 5$. Write the equation of the line that contains the midsegment of the trapezoid.

44. **MP PRECISION** In trapezoid *PQRS*, $\overline{PQ} \parallel \overline{RS}$ and $\overline{MN}$ is the midsegment of *PQRS*. If $RS = 5 \cdot PQ$, what is the ratio of *MN* to *RS*?

Ⓐ 3 : 5 Ⓑ 5 : 3

Ⓒ 1 : 2 Ⓓ 3 : 1

45. **CONSTRUCTION** $\overline{AC}$ and $\overline{BD}$ bisect each other.

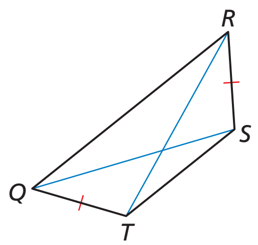
a. Construct quadrilateral *ABCD* so that $\overline{AC}$ and $\overline{BD}$ are congruent, but not perpendicular. Classify the quadrilateral. Justify your answer.

b. Construct quadrilateral *ABCD* so that $\overline{AC}$ and $\overline{BD}$ are perpendicular, but not congruent. Classify the quadrilateral. Justify your answer.

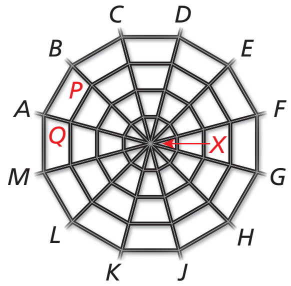
46. **PROOF** Write a proof.

Given *QRST* is an isosceles trapezoid.

Prove $\angle TQS \cong \angle SRT$



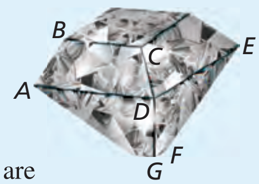
47. **MODELING REAL LIFE** A plastic web is made in the shape of a regular dodecagon (12-sided polygon). $\overline{AB} \parallel \overline{PQ}$, and *X* is equidistant from the vertices of the dodecagon.



a. Are you given enough information to prove that *ABPQ* is an isosceles trapezoid?

b. What is the measure of each interior angle of *ABPQ*?

48. **HOW DO YOU SEE IT?** One of the earliest shapes used for cut diamonds is called the *table cut*, as shown in the figure. Each face of a cut gem is called a *facet*.



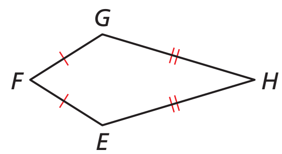
a. $\overline{BC} \parallel \overline{AD}$, and $\overline{AB}$ and $\overline{DC}$ are not parallel. What shape is the facet labeled *ABCD*?

b. $\overline{DE} \parallel \overline{GF}$, and $\overline{DG}$ and $\overline{EF}$ are congruent but not parallel. What shape is the facet labeled *DEFG*?

49. **PROVING A THEOREM** Use the plan for proof below to write a paragraph proof of the Kite Opposite Angles Theorem.

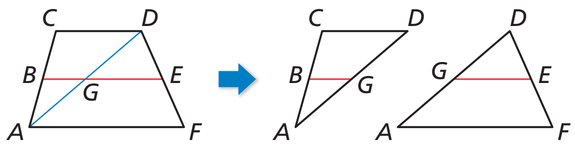
Given *EFGH* is a kite.
$\overline{EF} \cong \overline{FG}$, $\overline{EH} \cong \overline{GH}$

Prove $\angle E \cong \angle G$, $\angle F \ncong \angle H$



Plan for Proof First show that $\angle E \cong \angle G$. Then use an indirect argument to show that $\angle F \ncong \angle H$.

50. **PROVING A THEOREM** In the diagram below, $\overline{BG}$ is the midsegment of $\triangle ACD$, and $\overline{GE}$ is the midsegment of $\triangle ADF$. Use the diagram to prove the Trapezoid Midsegment Theorem.

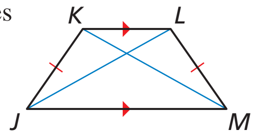


51. PROVING A THEOREM To prove the biconditional statement in the Isosceles Trapezoid Diagonals Theorem, you must prove both parts separately.

a. Prove part of the Isosceles Trapezoid Diagonals Theorem.

Given *JKLM* is an isosceles trapezoid, $\overline{KL} \parallel \overline{JM}$, and $\overline{JK} \cong \overline{LM}$.

Prove $\overline{JL} \cong \overline{KM}$



b. Write the other part of the Isosceles Trapezoid Diagonals Theorem as a conditional. Then prove the statement is true.

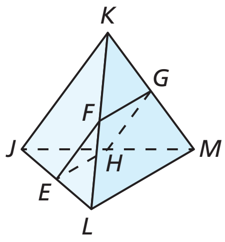
52. THOUGHT PROVOKING Is SSSSA a valid congruence theorem for kites? Justify your answer.

GO DIGITAL

53. PROOF What special type of quadrilateral is *EFGH*? Write a proof to show that your answer is correct.

Given In the three-dimensional figure, $\overline{JK} \cong \overline{LM}$. *E*, *F*, *G*, and *H* are the midpoints of $\overline{JL}$, $\overline{KL}$, $\overline{KM}$, and $\overline{JM}$, respectively.

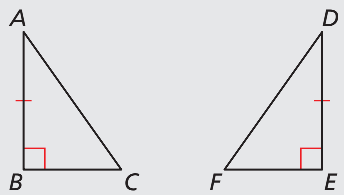
Prove *EFGH* is a ________.



REVIEW & REFRESH

WATCH

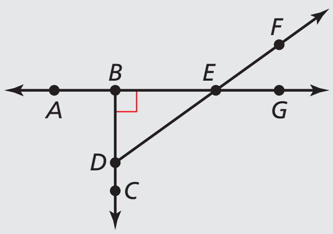
54. Decide whether enough information is given to prove that the triangles are congruent using the HL Congruence Theorem.



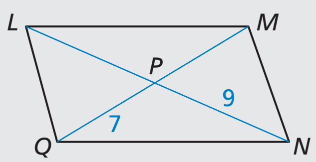
55. Find the distance from $(-4, 7)$ to the line $y = 2x$.

56. Classify the quadrilateral.

57. Identify the pair(s) of congruent angles in the diagram. Explain how you know they are congruent.



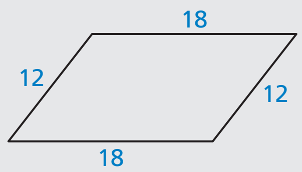
58. Find the measure of *LP* in ▱*LMNQ*. Explain your reasoning.



59. Write the conditional statement in if-then form.

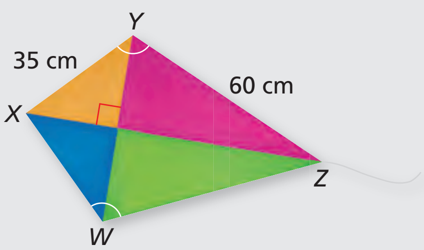
The polygon is a triangle because the sum of its interior angle measures is 180°.

60. State which theorem you can use to show that the quadrilateral is a parallelogram.



61. Graph $\overline{MN}$ with endpoints $M(1, 3)$ and $N(3, 5)$ and its image after a translation two units right, followed by a rotation 90° about the origin.

62. MODELING REAL LIFE Find the perimeter of the kite.



63. Sketch a diagram showing $\overline{AB}$ in plane *P* and $\overleftrightarrow{CD}$ not in plane *P*, such that $\overleftrightarrow{CD}$ bisects $\overline{AB}$.

64. Identify all pairs of alternate exterior angles.