5.4 Equilateral and Isosceles Triangles

GO DIGITAL

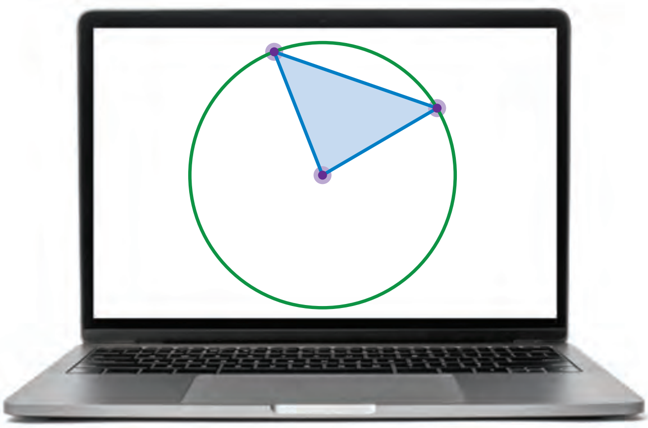
Learning Target Prove and use theorems about isosceles and equilateral triangles.

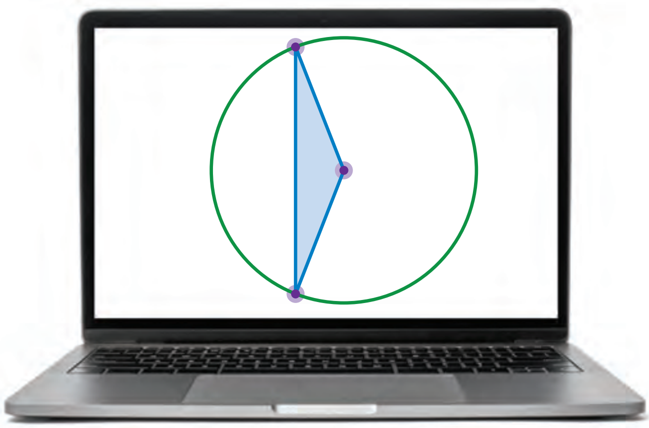
Success Criteria
- I can prove and use theorems about isosceles triangles.
- I can prove and use theorems about equilateral triangles.

EXPLORE IT! Reasoning about Isosceles Triangles

Work with a partner.

a. MP **CHOOSE TOOLS** Construct several circles. For each circle, draw a triangle with one vertex at the center of the circle and two vertices on the circle. Recall that a triangle is *isosceles* when it has at least two congruent sides. Explain why the triangles are isosceles. Then find the angle measures of each triangle. What do you notice? Make a conjecture about the angle measures of an isosceles triangle.

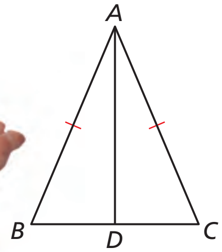




b. $\triangle ABC$ is an isosceles triangle. Given that $\overline{AB} \cong \overline{AC}$, show that $\angle B \cong \angle C$ when

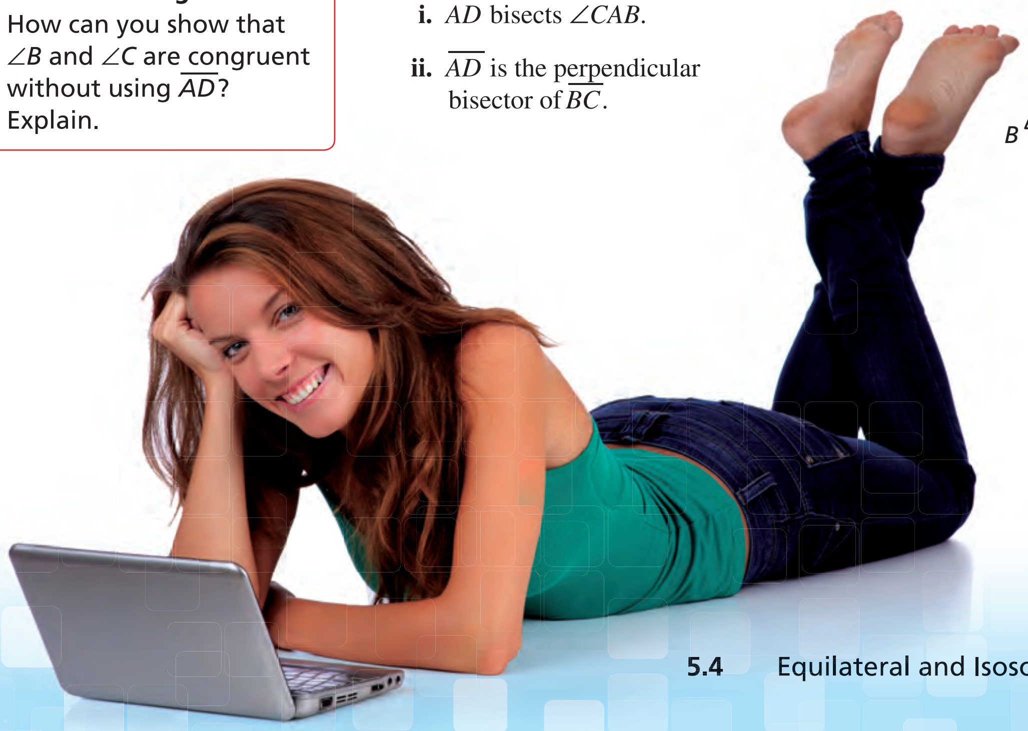
i. $\overline{AD}$ bisects $\angle CAB$.

ii. $\overline{AD}$ is the perpendicular bisector of $\overline{BC}$.



Math Practice

Construct Arguments
How can you show that $\angle B$ and $\angle C$ are congruent without using $\overline{AD}$? Explain.



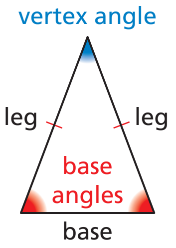
Vocabulary Az VOCAB

legs of an isosceles triangle, *p. 244*
vertex angle, *p. 244*
base, *p. 244*
base angles, *p. 244*

GO DIGITAL

Using the Base Angles Theorem

A triangle is isosceles when it has at least two congruent sides. When an isosceles triangle has exactly two congruent sides, these two sides are the **legs**. The angle formed by the legs is the **vertex angle**. The third side is the **base** of the isosceles triangle. The two angles adjacent to the base are called **base angles**.



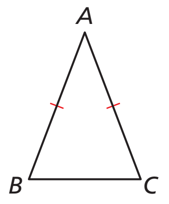
THEOREMS

5.6 Base Angles Theorem

If two sides of a triangle are congruent, then the angles opposite them are congruent.

If $\overline{AB} \cong \overline{AC}$, then $\angle B \cong \angle C$.

Prove this Theorem Explore It! part (b), page 243

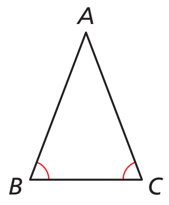


5.7 Converse of the Base Angles Theorem

If two angles of a triangle are congruent, then the sides opposite them are congruent.

If $\angle B \cong \angle C$, then $\overline{AB} \cong \overline{AC}$.

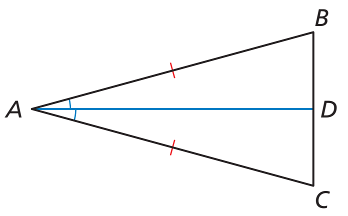
Prove this Theorem Exercise 20, page 265



PROOF Base Angles Theorem

Given $\overline{AB} \cong \overline{AC}$

Prove $\angle B \cong \angle C$



Plan for Proof

a. Draw $\overline{AD}$ so that it bisects $\angle CAB$.

b. Use the SAS Congruence Theorem to show that $\triangle ADB \cong \triangle ADC$.

c. Use properties of congruent triangles to show that $\angle B \cong \angle C$.

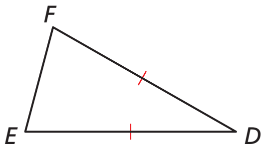
Plan in Action

	STATEMENTS	REASONS
a.	**1.** Draw $\overline{AD}$, the angle bisector of $\angle CAB$.	**1.** Construction of angle bisector
	2. $\angle CAD \cong \angle BAD$	**2.** Definition of angle bisector
	3. $\overline{AB} \cong \overline{AC}$	**3.** Given
	4. $\overline{DA} \cong \overline{DA}$	**4.** Reflexive Property of Segment Congruence
b.	**5.** $\triangle ADB \cong \triangle ADC$	**5.** SAS Congruence Theorem
c.	**6.** $\angle B \cong \angle C$	**6.** Corresponding parts of congruent triangles are congruent.

EXAMPLE 1 Using the Base Angles Theorem

WATCH

In $\triangle DEF$, $\overline{DE} \cong \overline{DF}$. Name two congruent angles.



SOLUTION

▶ $\overline{DE} \cong \overline{DF}$, so by the Base Angles Theorem, $\angle E \cong \angle F$.

COROLLARIES

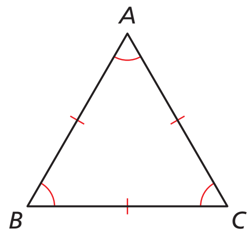
READING

The corollaries state that a triangle is *equilateral* if and only if it is *equiangular*.

5.2 Corollary to the Base Angles Theorem

If a triangle is equilateral, then it is equiangular.

Prove this Corollary Exercise 29, page 249; Exercise 10, page 340



5.3 Corollary to the Converse of the Base Angles Theorem

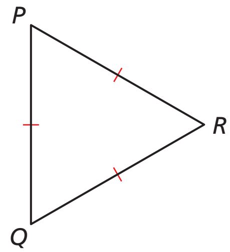
If a triangle is equiangular, then it is equilateral.

Prove this Corollary Exercise 31, page 250

EXAMPLE 2 Finding Measures in a Triangle

WATCH

Find the measures of $\angle P$, $\angle Q$, and $\angle R$.



SOLUTION

The diagram shows that $\triangle PQR$ is equilateral. So, by the Corollary to the Base Angles Theorem, $\triangle PQR$ is equiangular. So, $m\angle P = m\angle Q = m\angle R$.

$3(m\angle P) = 180^\circ$ Triangle Sum Theorem

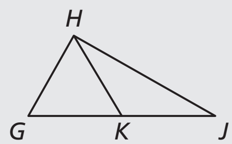
$m\angle P = 60^\circ$ Divide each side by 3.

▶ The measures of $\angle P$, $\angle Q$, and $\angle R$ are all 60°.

SELF-ASSESSMENT

1 I do not understand. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

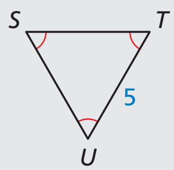
Complete the statement.



1. If $\overline{HG} \cong \overline{HK}$, then $\angle$_____ $\cong \angle$_____.

2. If $\angle KHJ \cong \angle KJH$, then _____ $\cong$ _____.

3. Find the length of $\overline{ST}$ for the triangle.



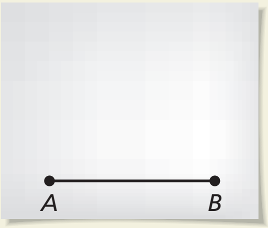
Using Isosceles and Equilateral Triangles

CONSTRUCTION Constructing an Equilateral Triangle

Construct an equilateral triangle that has side lengths congruent to $\overline{AB}$. Use a compass and straightedge.

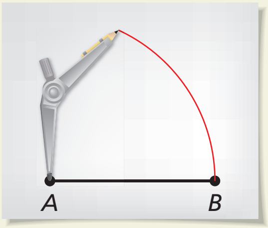
SOLUTION

Step 1



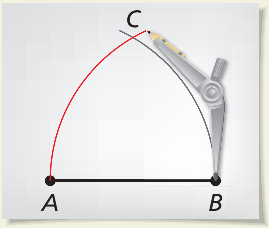
Copy a segment Copy $\overline{AB}$.

Step 2



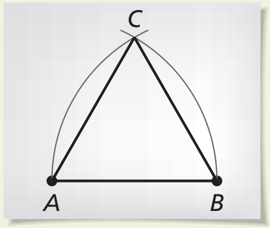
Draw an arc Draw an arc with center A and radius AB.

Step 3



Draw an arc Draw an arc with center B and radius AB. Label the intersection of the arcs from Steps 2 and 3 as C.

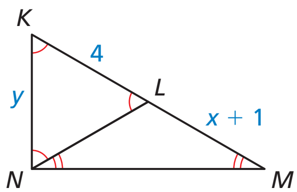
Step 4



Draw a triangle Draw $\triangle ABC$. Because $\overline{AB}$ and $\overline{AC}$ are radii of the same circle, $\overline{AB} \cong \overline{AC}$. Because $\overline{AB}$ and $\overline{BC}$ are radii of the same circle, $\overline{AB} \cong \overline{BC}$. By the Transitive Property of Segment Congruence, $\overline{AC} \cong \overline{BC}$. So, $\triangle ABC$ is equilateral.

EXAMPLE 3 Using Isosceles and Equilateral Triangles

Find the values of x and y in the diagram.



SOLUTION

Step 1 Find the value of y. Because $\triangle KLN$ is equiangular, it is also equilateral and $\overline{KN} \cong \overline{KL}$. So, $y = 4$.

Step 2 Find the value of x. Because $\angle LNM \cong \angle LMN$, $\overline{LN} \cong \overline{LM}$, and $\triangle LMN$ is isosceles. You also know that $LN = 4$ because $\triangle KLN$ is equilateral.

$LN = LM$	Definition of congruent segments
$4 = x + 1$	Substitute 4 for LN and $x + 1$ for LM.
$3 = x$	Subtract 1 from each side.

COMMON ERROR

You cannot use N to refer to $\angle LNM$ because three angles have N as their vertex.

EXAMPLE 4 Modeling Real Life

WATCH INFO

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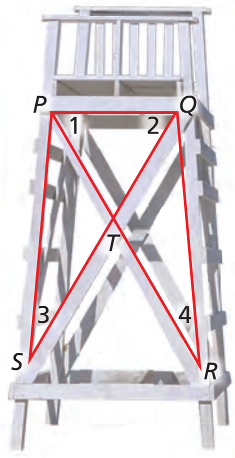
In the lifeguard tower, $\overline{PS} \cong \overline{QR}$ and $\angle QPS \cong \angle PQR$.

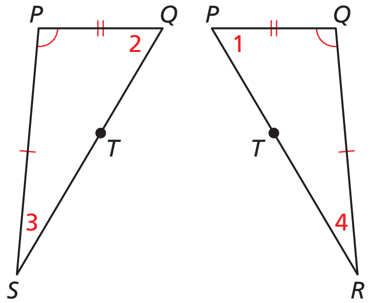


a. Explain how to prove that $\triangle QPS \cong \triangle PQR$.

b. Explain why $\triangle PQT$ is isosceles.

SOLUTION

a. Draw and label $\triangle QPS$ and $\triangle PQR$ so that they do not overlap. You can see that $\overline{PQ} \cong \overline{QP}$, $\overline{PS} \cong \overline{QR}$, and $\angle QPS \cong \angle PQR$. So, by the SAS Congruence Theorem, $\triangle QPS \cong \triangle PQR$.



COMMON ERROR

When you redraw the triangles so that they do not overlap, be careful to copy all given information and labels correctly.

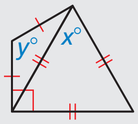
b. From part (a), you know that $\angle 1 \cong \angle 2$ because corresponding parts of congruent triangles are congruent. By the Converse of the Base Angles Theorem, $\overline{PT} \cong \overline{QT}$, and $\triangle PQT$ is isosceles.

SELF-ASSESSMENT

1 I do not understand. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

Find the values of x and y in the diagram.

4.



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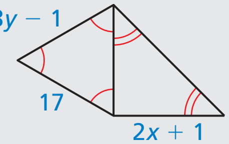

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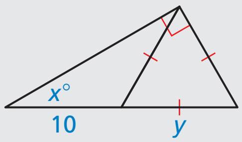

7. In Example 4, show that $\triangle PTS \cong \triangle QTR$.

5.4 Practice WITH CalcChat® AND CalcView®

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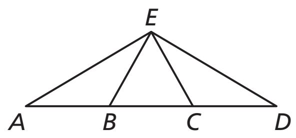
In Exercises 1–4, complete the statement. State which theorem you used. *Example 1*



1. If $\overline{AE} \cong \overline{DE}$, then $\angle$___ $\cong \angle$___.

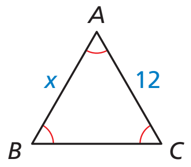
2. If $\overline{AB} \cong \overline{EB}$, then $\angle$___ $\cong \angle$___.

3. If $\angle D \cong \angle CED$, then ___ $\cong$ ___.

4. If $\angle EBC \cong \angle ECB$, then ___ $\cong$ ___.

In Exercises 5–8, find the value of *x*. *Example 2*

5.



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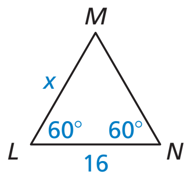

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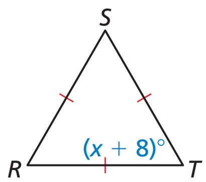

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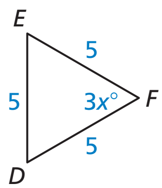


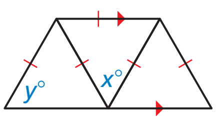
CONSTRUCTION **In Exercises 9 and 10, construct an equilateral triangle whose sides are the given length.**

9. 3 inches

10. 1.25 inches

In Exercises 11–14, find the values of *x* and *y*. *Example 3*

11.



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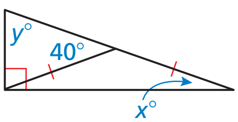

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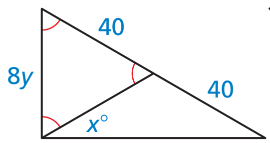
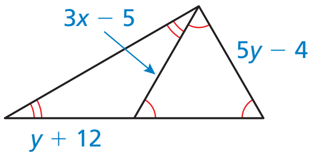
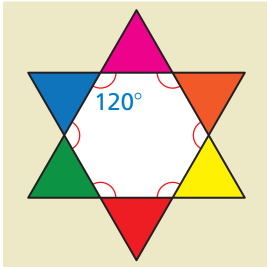


14.

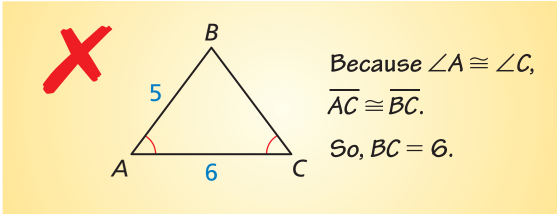


15. **MODELING REAL LIFE** Find the values of *x* and *y* in the diagram of the sports pennant.

16. **MODELING REAL LIFE** The image printed on a Chinese checkers board is shown. Without measuring, classify each of the six triangles surrounding the hexagon by sides and angle measures. Explain your reasoning.



17. **ERROR ANALYSIS** Describe and correct the error in finding the length of $\overline{BC}$.



18. **COLLEGE PREP** The base of isosceles $\triangle XYZ$ is $\overline{YZ}$. What can you prove? Select all that apply.

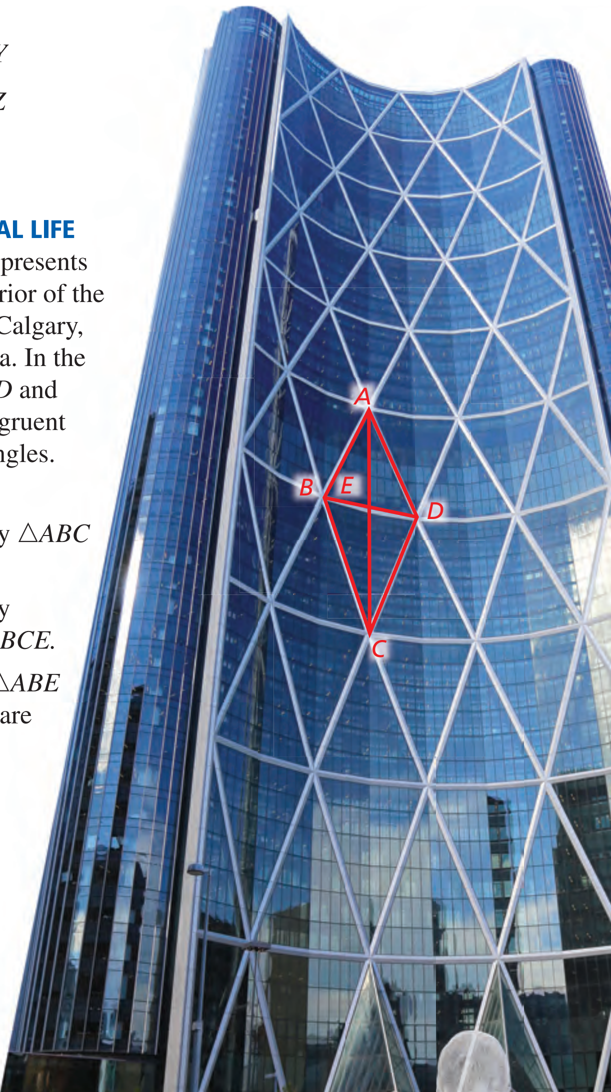
Ⓐ $\overline{XY} \cong \overline{XZ}$

Ⓑ $\angle X \cong \angle Y$

Ⓒ $\angle Y \cong \angle Z$

Ⓓ $\overline{YZ} \cong \overline{ZX}$

19. **MODELING REAL LIFE** The diagram represents part of the exterior of the Bow Tower in Calgary, Alberta, Canada. In the diagram, $\triangle ABD$ and $\triangle CBD$ are congruent equilateral triangles. *Example 4*



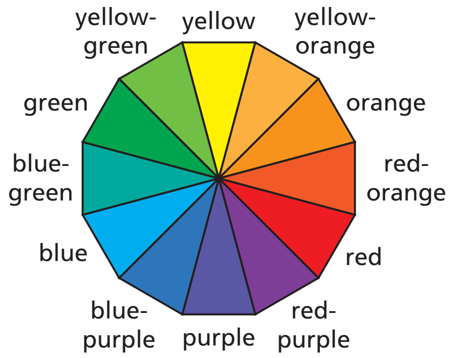
a. Explain why $\triangle ABC$ is isosceles.

b. Explain why $\angle BAE \cong \angle BCE$.

c. Show that $\triangle ABE$ and $\triangle CBE$ are congruent.

d. Find the measure of $\angle BAE$.

20. **MODELING REAL LIFE** The diagram is based on a color wheel. The 12 triangles in the diagram are isosceles triangles with congruent vertex angles.



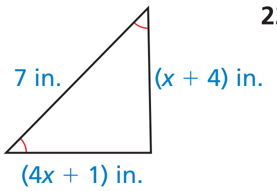
a. Complementary colors lie directly opposite each other on the color wheel. Explain how you know that the yellow triangle is congruent to the purple triangle.

b. The measure of the vertex angle of the yellow triangle is 30°. Find the measures of the base angles.

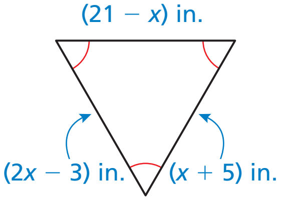
c. Trace the color wheel. Then form a triangle whose vertices are the midpoints of the bases of the red, yellow, and blue triangles. (These colors are the *primary colors*.) What type of triangle is this?

In Exercises 21 and 22, find the perimeter of the triangle.

21.



22.



23. **CONNECTING CONCEPTS** The lengths of the sides of a triangle are $3t$, $5t - 12$, and $t + 20$. Find the values of t that make the triangle isosceles. Explain your reasoning.

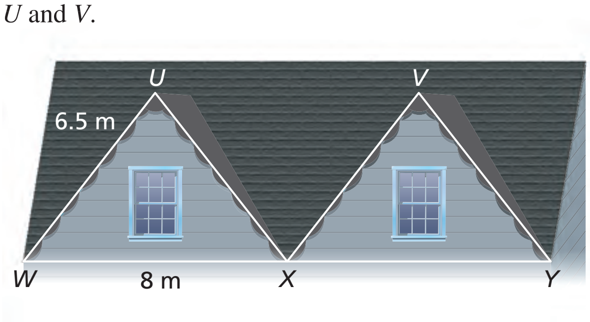
24. **CONNECTING CONCEPTS** The measure of an exterior angle of an isosceles triangle is $x°$. Write expressions representing the possible angle measures of the triangle in terms of x.

25. **WRITING** Explain why the measure of the vertex angle of an isosceles triangle must be an even number of degrees when the measures of all the angles of the triangle are whole numbers.

26. **OPEN-ENDED** Find and draw an object (or part of an object) that can be modeled by an isosceles or equilateral triangle. Describe the relationship between the interior angles and sides of the triangle in terms of the object.

GO DIGITAL

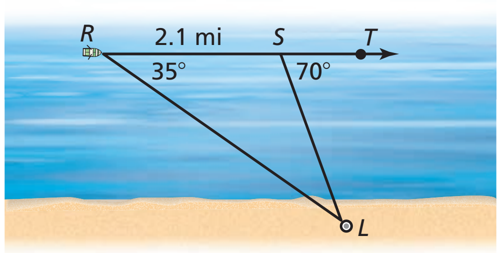
27. **MP PROBLEM SOLVING** The triangular faces of the peaks on a roof are congruent isosceles triangles with vertex angles U and V.



a. Name two angles congruent to $\angle WUX$. Explain your reasoning.

b. Find the distance between points U and V.

28. **MP REPEATED REASONING** A boat is traveling parallel to the shore along $\overrightarrow{RT}$. When the boat is at point R, the captain measures the angle to the lighthouse L as 35°. After the boat has traveled 2.1 miles, the captain measures the angle to the lighthouse to be 70°.



a. Find SL. Explain your reasoning.

b. Explain how the captain can use a similar process to find the distance between the boat and the shoreline.

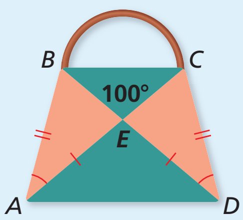
29. **PROVING A COROLLARY** Prove that the Corollary to the Base Angles Theorem follows from the Base Angles Theorem.

30. **HOW DO YOU SEE IT?**
Use the image of the purse shown.

a. Explain why $\triangle ABE \cong \triangle DCE$.

b. Name the isosceles triangles in the purse.

c. Name three angles that are congruent to $\angle EAD$.



31. **PROVING A COROLLARY** Prove that the Corollary to the Converse of the Base Angles Theorem follows from the Converse of the Base Angles Theorem.

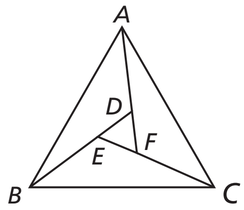
32. **THOUGHT PROVOKING**
The postulates and theorems in this book represent Euclidean geometry. In spherical geometry, all points are points on the surface of a sphere. A line is a circle on the sphere whose diameter is equal to the diameter of the sphere. In spherical geometry, do all equiangular triangles have the same angle measures? Justify your answer.

33. **MAKING AN ARGUMENT** The coordinates of two points are $T(0, 6)$ and $U(6, 0)$. Will the points T, U, and V always be the vertices of an isosceles triangle when V is any point on the line $y = x$? Explain.

34. **CONNECTING CONCEPTS** Use the construction of an equilateral triangle and rotations to draw a hexagon whose sides are 2 inches long. Explain your process.

GO DIGITAL

35. **PROOF** Use the diagram to prove that $\triangle DEF$ is equilateral.



Given $\triangle ABC$ is equilateral.
$\angle CAD \cong \angle ABE \cong \angle BCF$

Prove $\triangle DEF$ is equilateral.

REVIEW & REFRESH

WATCH

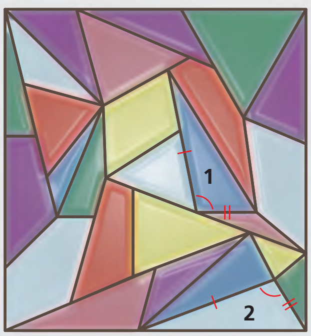
In Exercises 36–38, use the given property to complete the statement.

36. Reflexive Property of Segment Congruence:
____ $\cong \overline{SE}$

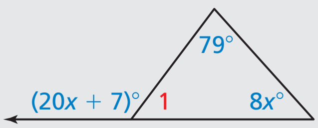
37. Symmetric Property of Segment Congruence:
If ____ $\cong$ ____, then $\overline{RS} \cong \overline{JK}$.

38. Transitive Property of Segment Congruence:
If $\overline{EF} \cong \overline{PQ}$, and $\overline{PQ} \cong \overline{UV}$, then ____ $\cong$ ____.

39. **MODELING REAL LIFE** The figure shows a stained glass window. Is there enough information given to prove that $\triangle 1 \cong \triangle 2$? Explain.



40. Find $m\angle 1$.

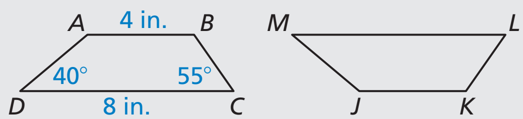


In Exercises 41 and 42, graph $\overline{YZ}$ with endpoints $Y(-4, 5)$ and $Z(-2, 1)$ and its image after the composition.

41. **Rotation:** 270° about the origin
Translation: $(x, y) \rightarrow (x - 3, y - 2)$

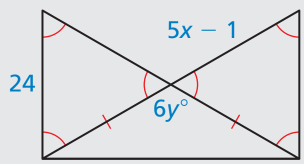
42. **Reflection:** in the line $y = -x$
Rotation: 180° about the origin

43. In the diagram, $ABCD \cong JKLM$. Find $m\angle L$ and JK.



44. Find the distance from the point $(-4, -7)$ to the line $y = -\frac{1}{2}x - 4$.

45. Find the values of x and y.



46. Find the mean, median, mode, range, and standard deviation of the data set.

13, 18, 17, 13, 15, 14