3.3 Proofs with Parallel Lines

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Learning Target Prove and use theorems about identifying parallel lines.

Success Criteria
- I can use theorems to identify parallel lines.
- I can construct parallel lines.
- I can prove theorems about identifying parallel lines.

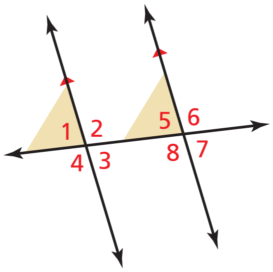
EXPLORE IT! Determining Whether Converses Are True

Math Practice

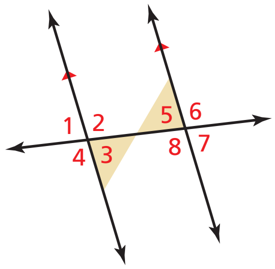
Construct Arguments
When the converse of one of the statements is true, what can you conclude about the inverse?

Work with a partner. Write the converse of each conditional statement. Determine whether the converse is true. Justify your conclusion.

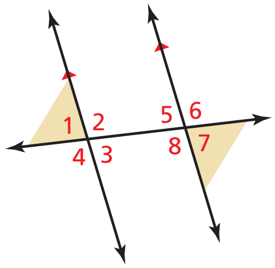
a. Corresponding Angles Theorem
If two parallel lines are cut by a transversal, then the pairs of corresponding angles are congruent.



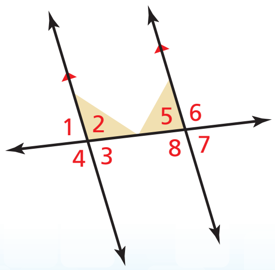
b. Alternate Interior Angles Theorem
If two parallel lines are cut by a transversal, then the pairs of alternate interior angles are congruent.



c. Alternate Exterior Angles Theorem
If two parallel lines are cut by a transversal, then the pairs of alternate exterior angles are congruent.



d. Consecutive Interior Angles Theorem
If two parallel lines are cut by a transversal, then the pairs of consecutive interior angles are supplementary.



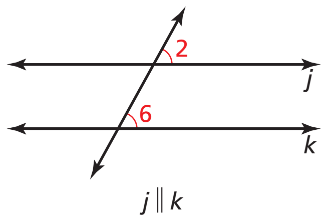
Using the Corresponding Angles Converse

Theorem 3.5 below is the converse of the Corresponding Angles Theorem. Similarly, the other theorems about angles formed when parallel lines are cut by a transversal have true converses. Remember that the converse of a true conditional statement is not necessarily true, so you must prove each converse of a theorem.

THEOREM

3.5 Corresponding Angles Converse

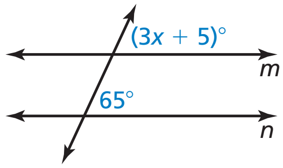
If two lines are cut by a transversal so the corresponding angles are congruent, then the lines are parallel.



Prove this Theorem Exercise 35, page 174

EXAMPLE 1 Using the Corresponding Angles Converse

Find the value of x that makes $m \parallel n$.



SOLUTION

Lines m and n are parallel when the marked corresponding angles are congruent.

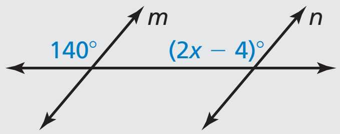
$(3x + 5)° = 65°$	Use the Corresponding Angles Converse to write an equation.
$3x = 60$	Subtract 5 from each side.
$x = 20$	Divide each side by 3.

▶ So, lines m and n are parallel when $x = 20$.

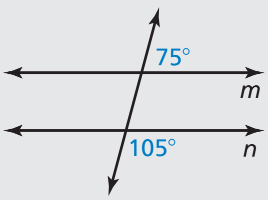
SELF-ASSESSMENT

1 I do not understand. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

1. Find the value of x that makes $m \parallel n$.



2. **MP REASONING** Is there enough information in the diagram to conclude that $m \parallel n$? Explain.



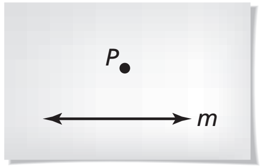
3. **MP PRECISION** Explain why the Corresponding Angles Converse is the converse of the Corresponding Angles Theorem.

Constructing Parallel Lines

The Corresponding Angles Converse justifies the construction of parallel lines, as shown below.

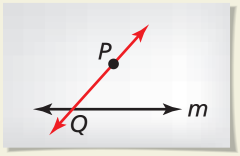
CONSTRUCTION **Constructing Parallel Lines** WATCH

Use a compass and straightedge to construct a line through point P that is parallel to line m.



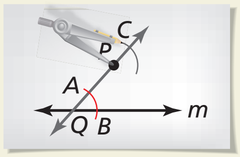
SOLUTION

Step 1



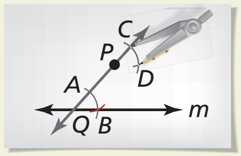
Draw a point and line Start by drawing point P and line m. Choose a point Q anywhere on line m and draw $\overleftrightarrow{QP}$.

Step 2



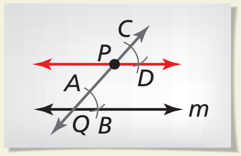
Draw arcs Draw an arc with center Q that crosses $\overleftrightarrow{QP}$ and line m. Label points A and B. Using the same compass setting, draw an arc with center P. Label point C.

Step 3



Copy angle Draw an arc with radius AB and center A. Using the same compass setting, draw an arc with center C. Label the intersection D.

Step 4



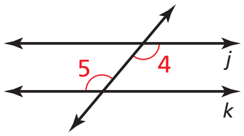
Draw parallel lines Draw $\overleftrightarrow{PD}$. This line is parallel to line m.

THEOREMS

3.6 Alternate Interior Angles Converse

If two lines are cut by a transversal so the alternate interior angles are congruent, then the lines are parallel.

Prove this Theorem Example 2, page 136

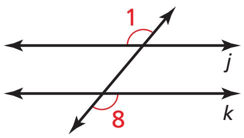


$j \parallel k$

3.7 Alternate Exterior Angles Converse

If two lines are cut by a transversal so the alternate exterior angles are congruent, then the lines are parallel.

Prove this Theorem Exercise 9, page 138

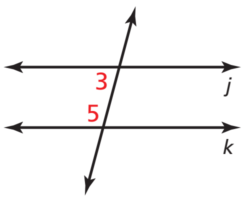


$j \parallel k$

3.8 Consecutive Interior Angles Converse

If two lines are cut by a transversal so the consecutive interior angles are supplementary, then the lines are parallel.

Prove this Theorem Exercise 10, page 138



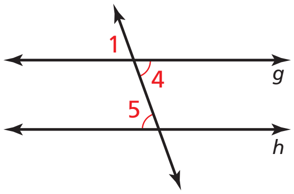
If $\angle 3$ and $\angle 5$ are supplementary, then $j \parallel k$.

Proving Theorems about Parallel Lines

EXAMPLE 2 Proving the Alternate Interior Angles Converse

Prove that if two lines are cut by a transversal so the alternate interior angles are congruent, then the lines are parallel.

SOLUTION

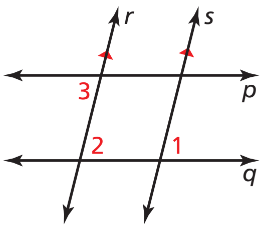


Given $\angle 4 \cong \angle 5$

Prove $g \parallel h$

STATEMENTS	REASONS
1. $\angle 4 \cong \angle 5$	**1.** Given
2. $\angle 1 \cong \angle 4$	**2.** Vertical Angles Congruence Theorem
3. $\angle 1 \cong \angle 5$	**3.** Transitive Property of Angle Congruence
4. $g \parallel h$	**4.** Corresponding Angles Converse

EXAMPLE 3 Determining Whether Lines Are Parallel



In the diagram, $r \parallel s$ and $\angle 1$ is congruent to $\angle 3$. Prove $p \parallel q$.

SOLUTION

Look at the diagram to make a plan. The diagram suggests that you look at angles 1, 2, and 3. Also, you may find it helpful to focus on one pair of lines and one transversal at a time.

Plan for Proof **a.** Look at $\angle 1$ and $\angle 2$. $\angle 1 \cong \angle 2$ because $r \parallel s$.

b. Look at $\angle 2$ and $\angle 3$. If $\angle 2 \cong \angle 3$, then $p \parallel q$.

Plan in Action **a.** It is given that $r \parallel s$, so by the Corresponding Angles Theorem, $\angle 1 \cong \angle 2$.

b. It is also given that $\angle 1 \cong \angle 3$. Then $\angle 2 \cong \angle 3$ by the Transitive Property of Angle Congruence.

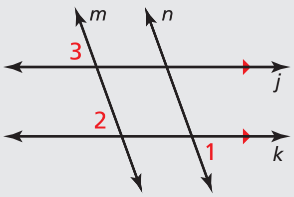
▶ So, by the Alternate Interior Angles Converse, $p \parallel q$.

SELF-ASSESSMENT 1 I do not understand. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

4. Complete the following paragraph proof of the Alternate Interior Angles Converse using the diagram in Example 2.

It is given that $\angle 4 \cong \angle 5$. By the ______, $\angle 1 \cong \angle 4$. Then by the Transitive Property of Angle Congruence, ______. So, by the ______, $g \parallel h$.

5. In the diagram, $j \parallel k$ and $\angle 1$ is congruent to $\angle 3$. Prove $m \parallel n$.



Using the Transitive Property of Parallel Lines

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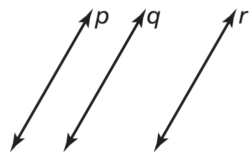
THEOREM

3.9 Transitive Property of Parallel Lines

If two lines are parallel to the same line, then they are parallel to each other.

Prove this Theorem Exercise 38, page 140;
Exercise 48, page 156

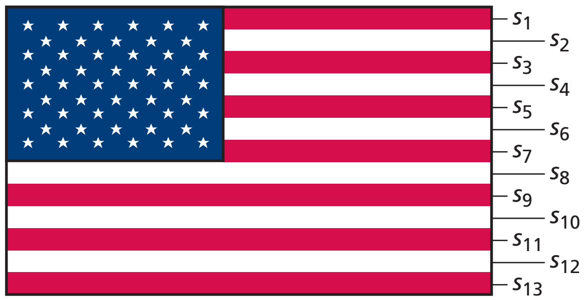


If $p \parallel q$ and $q \parallel r$, then $p \parallel r$.

EXAMPLE 4 Using the Transitive Property of Parallel Lines

WATCH INFO

The flag of the United States has 13 alternating red and white stripes. Each stripe is parallel to the stripe immediately below it. Explain why the top stripe is parallel to the bottom stripe.



SOLUTION

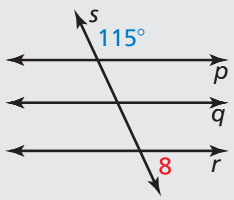
You can name the stripes from top to bottom as $s_1, s_2, s_3, \ldots, s_{13}$. Each stripe is parallel to the one immediately below it, so $s_1 \parallel s_2$, $s_2 \parallel s_3$, and so on. Then $s_1 \parallel s_3$ by the Transitive Property of Parallel Lines. Similarly, because $s_3 \parallel s_4$, it follows that $s_1 \parallel s_4$. By continuing this reasoning, $s_1 \parallel s_{13}$.

▶ So, the top stripe is parallel to the bottom stripe by the Transitive Property of Parallel Lines.

SELF-ASSESSMENT 1 I do not understand. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

6. Each step is parallel to the step immediately above it. The bottom step is parallel to the ground. Explain why the top step is parallel to the ground.

7. In the diagram below, $p \parallel q$ and $q \parallel r$. Find $m\angle 8$. Explain your reasoning.



3.3 Practice WITH CalcChat® AND CalcView®

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In Exercises 1–6, find the value of x that makes $m \parallel n$. Explain your reasoning. ▶ *Example 1*

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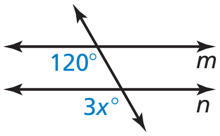

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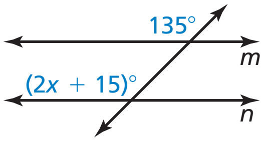

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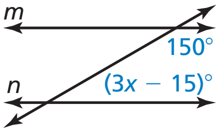

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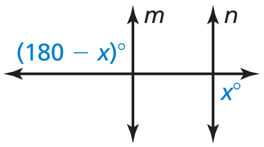

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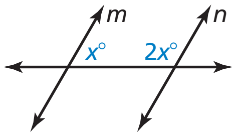

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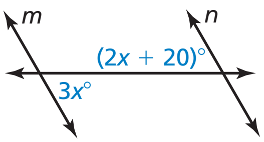
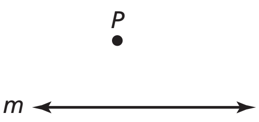


CONSTRUCTION **In Exercises 7 and 8, trace line m and point P. Then use a compass and straightedge to construct a line through point P that is parallel to line m.**

7.


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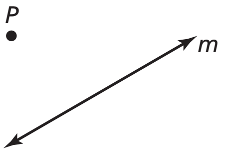
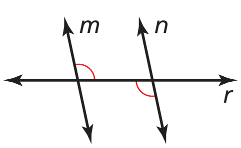


PROVING A THEOREM **In Exercises 9 and 10, prove the theorem.** ▶ *Example 2*

9. Alternate Exterior Angles Converse

10. Consecutive Interior Angles Converse

In Exercises 11–16, decide whether there is enough information to prove that $m \parallel n$. If so, state the theorem you can use. ▶ *Example 3*

11.


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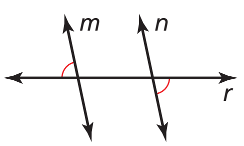

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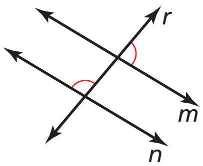

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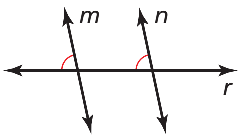

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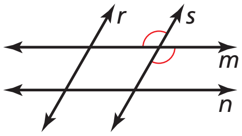

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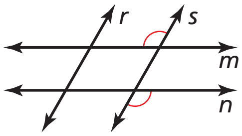

ERROR ANALYSIS **In Exercises 17 and 18, describe and correct the error in the reasoning.**

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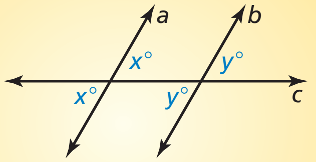
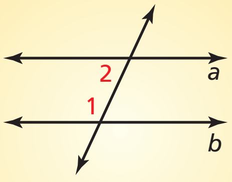
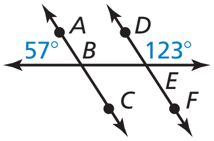


$a \parallel b$ by the Vertical Angles Congruence Theorem.

18.


$a \parallel b$ by the Consecutive Interior Angles Converse.

In Exercises 19–22, are $\overleftrightarrow{AC}$ and $\overleftrightarrow{DF}$ parallel? Explain your reasoning.

19.


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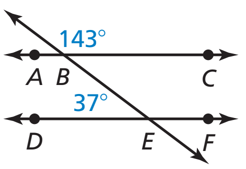

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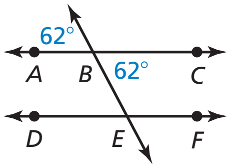

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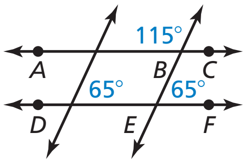

23. **MP REPEATED REASONING** Each rung of the ladder is parallel to the rung directly above it. Explain why the top rung is parallel to the bottom rung. ▶ *Example 4*

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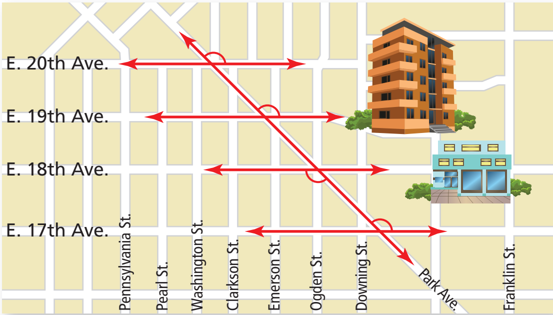
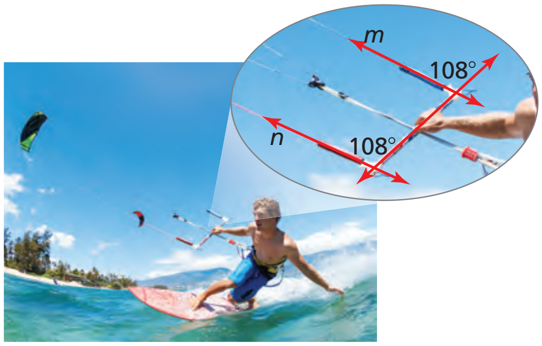
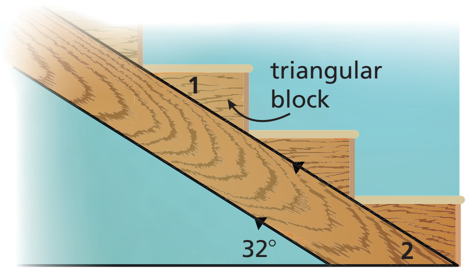
24. MP **REPEATED REASONING** The map shows part of Denver, Colorado. Use the markings on the map. Are the numbered streets parallel to one another? Explain.



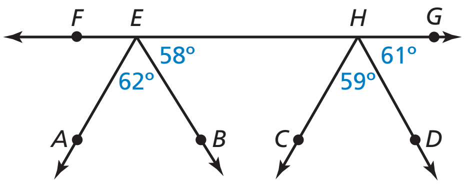
25. **MODELING REAL LIFE** The diagram of the control bar of the kite shows the angles formed between the control bar and the kite lines. How do you know that *n* is parallel to *m*?



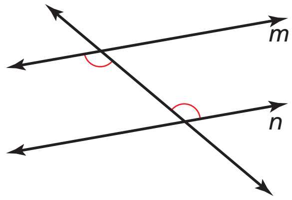
26. **MODELING REAL LIFE** One way to build stairs is to attach triangular blocks to an angled support, as shown. The sides of the angled support are parallel. If the support makes a 32° angle with the floor, what must $m\angle 1$ be so the top of the step will be parallel to the floor? Explain your reasoning.



27. MP **REASONING** Which rays are parallel? Which rays are not parallel? Explain your reasoning.



28. MP **PRECISION** Which theorems allow you to conclude that $m \parallel n$? Select all that apply. Explain your reasoning.



Ⓐ Corresponding Angles Converse

Ⓑ Alternate Interior Angles Converse

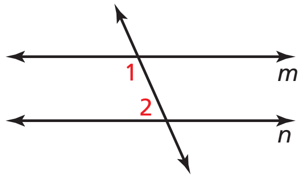
Ⓒ Alternate Exterior Angles Converse

Ⓓ Consecutive Interior Angles Converse

PROOF **In Exercises 29–32, write a proof.**

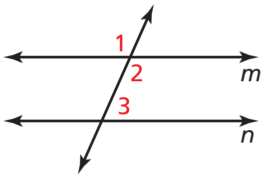
29. **Given** $m\angle 1 = 115°$, $m\angle 2 = 65°$

Prove $m \parallel n$



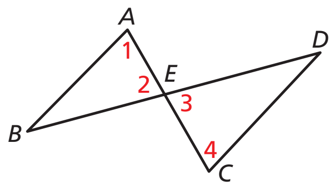
30. **Given** $\angle 1$ and $\angle 3$ are supplementary.

Prove $m \parallel n$



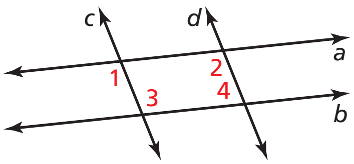
31. **Given** $\angle 1 \cong \angle 2$, $\angle 3 \cong \angle 4$

Prove $\overline{AB} \parallel \overline{CD}$

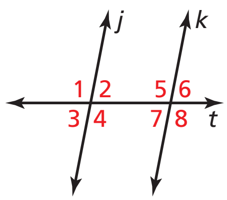


32. **Given** $a \parallel b$, $\angle 2 \cong \angle 3$

Prove $c \parallel d$

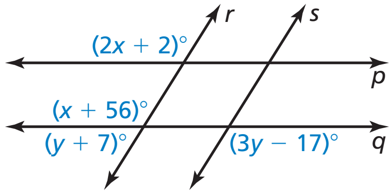


33. **ABSTRACT REASONING** How many angle measures must be given to determine whether $j \parallel k$? Give four examples that would allow you to conclude that $j \parallel k$ using the theorems from this lesson.



34. THOUGHT PROVOKING
Draw a diagram of at least two lines cut by at least one transversal. Mark your diagram so that it cannot be proven that any lines are parallel. Then explain how your diagram needs to change to prove that lines are parallel.

35. CONNECTING CONCEPTS Use the diagram.

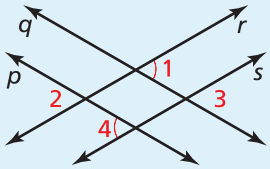


a. Find the value of x that makes $p \parallel q$.

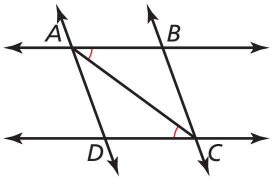
b. Find the value of y that makes $r \parallel s$.

c. Using the values from parts (a) and (b), can r be parallel to s and can p be parallel to q at the same time? Explain your reasoning.

36. HOW DO YOU SEE IT?
Are the markings on the diagram enough to conclude that any lines are parallel? If so, which ones? If not, what information is needed?



37. MAKING AN ARGUMENT
Your friend claims that $\overleftrightarrow{AD} \parallel \overleftrightarrow{BC}$ by the Alternate Interior Angles Converse. Is your friend correct? Explain.



38. PROVING A THEOREM Prove the Transitive Property of Parallel Lines Theorem.

GO DIGITAL

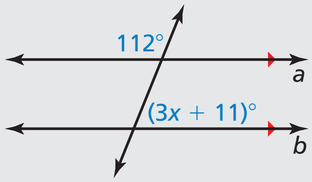
REVIEW & REFRESH

WATCH

In Exercises 39 and 40, find the distance between the two points.

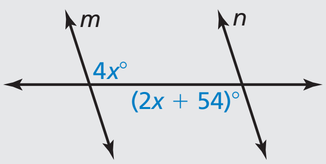
39. (5, −4) and (0, 8) **40.** (13, 1) and (9, −4)

41. Find the value of x.

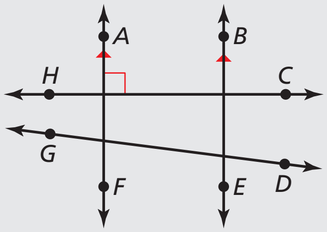


42. MODELING REAL LIFE The height (in feet) of a T-shirt t seconds after it is launched into a crowd can be represented by $h(t) = -16t^2 + 96t + 4$. Estimate and interpret the maximum value of the function.

43. Find the value of x that makes $m \parallel n$. Explain your reasoning.



In Exercises 44 and 45, use the diagram.



44. Name a pair of perpendicular lines.

45. Is $\overleftrightarrow{HC} \parallel \overleftrightarrow{GD}$? Explain.

In Exercises 46 and 47, solve the system using any method. Explain your choice of method.

46. $x = 2y + 5$
$3y - x = -9$

47. $-2x + 3y = 20$
$4x - 2y = -16$

48. Evaluate $f(x) = -3x + 5$ when $x = -2$, 3, and 5.

49. Write a proof using any format.

Given $\angle 1 \cong \angle 3$

Prove $\angle 2 \cong \angle 4$