2.2 Inductive and Deductive Reasoning

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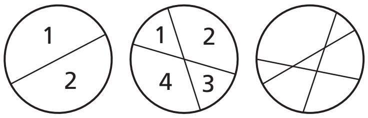
Learning Target Use inductive and deductive reasoning.

Success Criteria
- I can use inductive reasoning to make conjectures.
- I can use deductive reasoning to verify conjectures.
- I can distinguish between inductive and deductive reasoning.

EXPLORE IT! Using Inductive and Deductive Reasoning

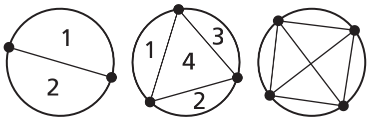
Work with a partner.

a. MP **REPEATED REASONING** A *conjecture* is an unproven statement that is based on observations.



i. You can use line segments to divide a circle into regions. Use technology to complete the table. Then make a conjecture about the maximum number of regions into which a circle can be divided using n line segments. Explain your reasoning.

Number of Segments, n	1	2	3	4	5
Maximum Number of Regions	2	4			



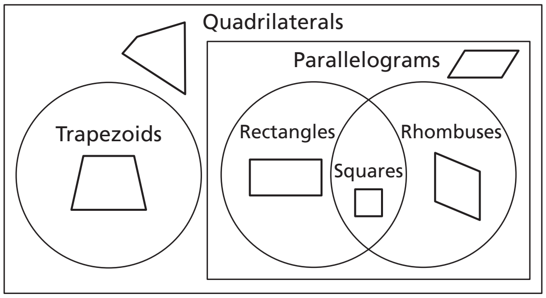
ii. You can connect points on a circle to divide the circle into regions. Use technology to complete the table. Then make a conjecture about the maximum number of regions into which a circle can be divided by connecting n points on the circle. Explain your reasoning.

Number of Points, n	2	3	4	5	6
Maximum Number of Regions	2	4			

Math Practice

Analyze Conjectures
Why might a person incorrectly predict the maximum number of regions for 6 points? What does this tell you about reasoning from examples?

b. The Venn diagram shows the relationships among different types of quadrilaterals. Use the Venn diagram to determine whether each statement is true or false. Justify your answer.



i. If a quadrilateral is a square, then it is a rectangle.

ii. If a quadrilateral is a rhombus, then it is a square.

iii. If a quadrilateral is a rectangle, then it is a parallelogram.

c. *Deductive reasoning* uses facts, definitions, accepted properties, and the laws of logic to form a logical argument. This is different from *inductive reasoning*, which uses specific examples and patterns to form a conjecture. In this Explore It!, how did you use inductive reasoning? deductive reasoning?

Using Inductive Reasoning

Vocabulary
conjecture, *p. 74*
inductive reasoning, *p. 74*
counterexample, *p. 75*
deductive reasoning, *p. 76*

KEY IDEA

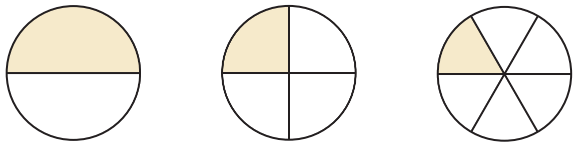
Inductive Reasoning

A **conjecture** is an unproven statement that is based on observations. You use **inductive reasoning** when you find a pattern in specific cases and then write a conjecture for the general case.

EXAMPLE 1 Describing a Visual Pattern

Describe how to sketch the fourth figure in the pattern. Then sketch the fourth figure.

Figure 1 Figure 2 Figure 3



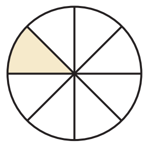
Math Practice

Look for Patterns
Describe the *n*th figure in the pattern.

SOLUTION

Each circle is divided into twice as many equal regions as the figure number. Sketch the fourth figure by dividing a circle into eighths. Shade the section just above the horizontal segment at the left.

Figure 4



SELF-ASSESSMENT

1 I do not understand. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

1. Sketch the fifth figure in the pattern in Example 1.

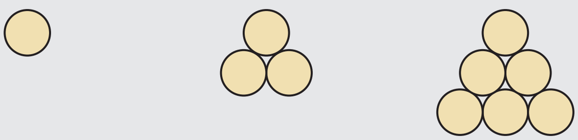
Describe the pattern. Then write or draw the next two numbers, letters, or figures.

2. S, R, Q, P, . . .

3. 1, 2, 6, 24, . . .

4.

5.



EXAMPLE 2 Making and Testing a Conjecture

WATCH

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Consecutive integers follow each other in order, such as 3, 4, and 5.
Make and test a conjecture about the sum of any three consecutive integers.

SOLUTION

Step 1 Find a pattern using a few groups of small numbers.

$3 + 4 + 5 = 12 = 4 \cdot 3$ $\qquad$ $7 + 8 + 9 = 24 = 8 \cdot 3$

$10 + 11 + 12 = 33 = 11 \cdot 3$ $\qquad$ $16 + 17 + 18 = 51 = 17 \cdot 3$

Step 2 Make a conjecture.

Conjecture The sum of any three consecutive integers is three times the second number.

Step 3 Test your conjecture using other numbers. For example, test that it works with the groups $-1, 0, 1$ and $100, 101, 102$.

$-1 + 0 + 1 = 0 = 0 \cdot 3$ ✓

$100 + 101 + 102 = 303 = 101 \cdot 3$ ✓

KEY IDEA

Counterexample

To show that a conjecture is true, you must show that it is true for all cases. You can show that a conjecture is false, however, by finding just one *counterexample*. A **counterexample** is a specific case for which the conjecture is false.

WORDS AND MATH

When you counter something, you are opposing it. A counterexample opposes a statement by demonstrating that it is false.

EXAMPLE 3 Finding a Counterexample

WATCH

A student makes the following conjecture about the sum of two numbers. Find a counterexample to disprove the student's conjecture.

Conjecture The sum of two numbers is always more than the greater number.

SOLUTION

To find a counterexample, you need to find a sum that is less than the greater number.

$-2 + (-3) = -5$

$-5 \ngtr -2$

▶ Because a counterexample exists, the conjecture is false.

SELF-ASSESSMENT

1 I do not understand. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

6. Make and test a conjecture about the sign of the product of any three negative integers.

7. Make and test a conjecture about the sum of any five consecutive integers.

Find a counterexample to show that the conjecture is false.

8. The value of x^2 is always greater than the value of x.

9. The sum of two numbers is always greater than their difference.

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Using Deductive Reasoning

KEY IDEAS

Deductive Reasoning

Deductive reasoning uses facts, definitions, accepted properties, and the laws of logic to form a logical argument. This is different from *inductive reasoning*, which uses specific examples and patterns to form a conjecture.

Laws of Logic

Law of Detachment

If the hypothesis of a true conditional statement is true, then the conclusion is also true.

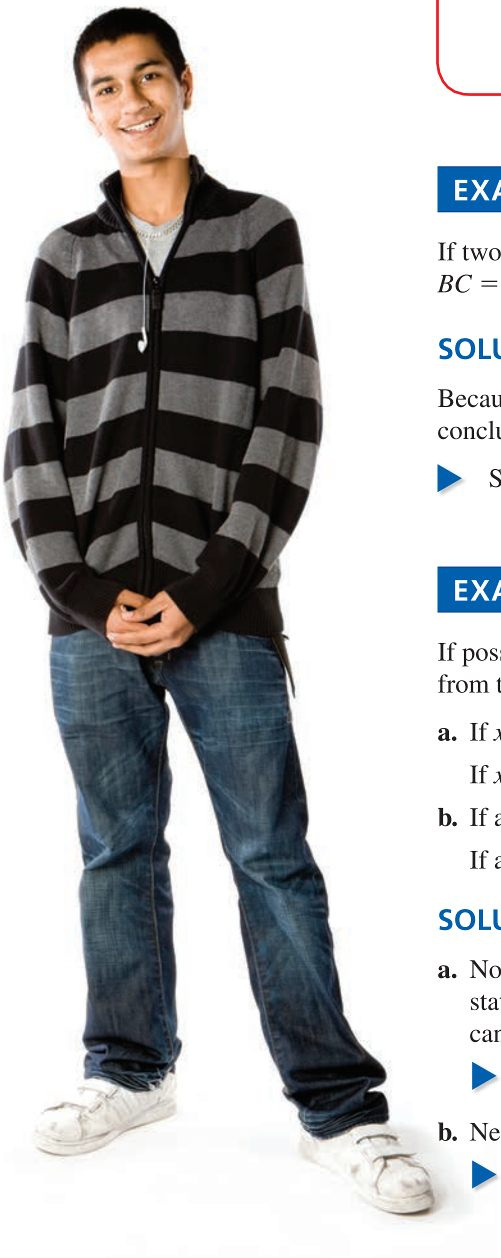
Law of Syllogism

If hypothesis p, then conclusion q.
If hypothesis q, then conclusion r.
} If these statements are true,

If hypothesis p, then conclusion r. ← then this statement is true.

READING

A true conditional statement means that $p \to q$ is true.



EXAMPLE 4 Using the Law of Detachment

WATCH

If two segments have the same length, then they are congruent. You know that $BC = XY$. Using the Law of Detachment, what statement can you make?

SOLUTION

Because $BC = XY$ satisfies the hypothesis of a true conditional statement, the conclusion is also true.

▶ So, $\overline{BC} \cong \overline{XY}$.

EXAMPLE 5 Using the Law of Syllogism

WATCH

If possible, use the Law of Syllogism to write a new conditional statement that follows from the pair of true statements.

a. If $x^2 > 25$, then $x^2 > 20$.

If $x > 5$, then $x^2 > 25$.

b. If a polygon is regular, then all angles in the interior of the polygon are congruent.

If a polygon is regular, then all its sides are congruent.

SOLUTION

a. Notice that the conclusion of the second statement is the hypothesis of the first statement. The order in which the statements are given does not affect whether you can use the Law of Syllogism. So, you can write the following new statement.

▶ If $x > 5$, then $x^2 > 20$.

b. Neither statement's conclusion is the same as the other statement's hypothesis.

▶ You cannot use the Law of Syllogism to write a new conditional statement.

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EXAMPLE 6 Using Inductive and Deductive Reasoning

WATCH

What conclusion can you make about the product of an even integer and any other integer?

SOLUTION

Math Practice

Make Conjectures
Which type of reasoning helps you to make a conjecture? Which type helps you to justify a conjecture? How do you know when to use each type?

Step 1 Look for a pattern in several examples. Use inductive reasoning to make a conjecture.

$(-2)(2) = -4$ $(-1)(2) = -2$ $2(2) = 4$ $3(2) = 6$

$(-2)(-4) = 8$ $(-1)(-4) = 4$ $2(-4) = -8$ $3(-4) = -12$

Conjecture Even integer • Any integer = Even integer

Step 2 Let n and m each be any integer. Use deductive reasoning to show that the conjecture is true.

$2n$ is an even integer because any integer multiplied by 2 is even.

$2nm$ represents the product of an even integer $2n$ and any integer m.

$2nm$ is the product of 2 and an integer nm. So, $2nm$ is an even integer.

▶ The product of an even integer and any integer is an even integer.

EXAMPLE 7 Comparing Inductive and Deductive Reasoning

WATCH

Decide whether inductive reasoning or deductive reasoning is used to reach each conclusion. Explain your reasoning.

a. Each time Monica kicks a ball up in the air, it returns to the ground. So, the next time Monica kicks a ball up in the air, it will return to the ground.

b. All reptiles are cold-blooded. Parrots are not cold-blooded. Sue's pet parrot is not a reptile.

SOLUTION

a. Inductive reasoning, because a pattern is used to reach the conclusion.

b. Deductive reasoning, because facts about animals and the laws of logic are used to reach the conclusion.

SELF-ASSESSMENT

1 I do not understand. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

10. If $90° < m\angle R < 180°$, then $\angle R$ is obtuse. The measure of $\angle R$ is 155°. Using the Law of Detachment, what statement can you make?

11. Use the Law of Syllogism to write a new conditional statement that follows from the pair of true statements.

If you get an A on your math test, then you can go to the movies.
If you go to the movies, then you can watch your favorite actor.

12. Use inductive reasoning to make a conjecture about the sum of a number and itself. Then use deductive reasoning to show that the conjecture is true.

13. Decide whether inductive reasoning or deductive reasoning is used to reach the conclusion. Explain your reasoning.

All multiples of 8 are divisible by 4.
64 is a multiple of 8.
So, 64 is divisible by 4.

2.2 Practice with CalcChat® and CalcView®

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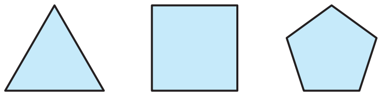
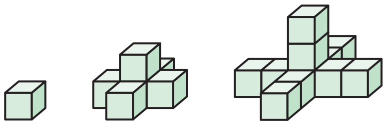
In Exercises 1–6, describe the pattern. Then write or draw the next two numbers, letters, or figures. *Example 1*

1. 1, −2, 3, −4, 5, . . .
2. 0, 2, 6, 12, 20, . . .
3. Z, Y, X, W, V, . . .
4. A, D, G, J, M, . . .
5. 
6. 

In Exercises 7–10, make and test a conjecture about the given quantity. *Example 2*

7. the sum of an even integer and an odd integer
8. the product of any two even integers
9. the quotient of a number and its reciprocal
10. the quotient of two negative integers

In Exercises 11–14, find a counterexample to show that the conjecture is false. *Example 3*

11. The product of two positive numbers is always greater than either number.
12. If n is a nonzero integer, then $\frac{n+1}{n}$ is always greater than 1.
13. If two angles are supplements of each other, then one of the angles must be acute.
14. A line s divides $\overline{MN}$ into two line segments. So, the line s is a segment bisector of $\overline{MN}$.

In Exercises 15–18, use the Law of Detachment to determine what you can conclude from the given information, if possible. *Example 4*

15. If you download a GIF, then your device crashes. You download a GIF.
16. If your cousin lets you borrow a car, then you will go to the mountains with your friend. You will go to the mountains with your friend.
17. If a quadrilateral is a square, then it has four right angles. Quadrilateral $QRST$ has four right angles.
18. If a point divides a line segment into two congruent line segments, then the point is a midpoint. Point P divides $\overline{LH}$ into two congruent line segments.

In Exercises 19–22, use the Law of Syllogism to write a new conditional statement that follows from the pair of true statements, if possible. *Example 5*

19. If $x < -2$, then $|x| > 2$. If $x > 2$, then $|x| > 2$.
20. If $a = 3$, then $5a = 15$. If $\frac{1}{2}a = 1\frac{1}{2}$, then $a = 3$.
21. If a figure is a rhombus, then the figure is a parallelogram. If a figure is a parallelogram, then the figure has two pairs of opposite sides that are parallel.
22. If a figure is a square, then the figure has four congruent sides. If a figure is a square, then the figure has four right angles.

In Exercises 23–26, state the law of logic that is illustrated.

23. If you do your homework, then you can watch TV. If you watch TV, then you can watch your favorite show.

    If you do your homework, then you can watch your favorite show.
24. If you miss practice the day before a game, then you will not be a starting player in the game.

    You miss practice on Tuesday. You will not start the game Wednesday.
25. If $x > 12$, then $x + 9 > 20$. The value of x is 14.

    So, $x + 9 > 20$.
26. If $\angle 1$ and $\angle 2$ are vertical angles, then $\angle 1 \cong \angle 2$. If $\angle 1 \cong \angle 2$, then $m\angle 1 = m\angle 2$.

    If $\angle 1$ and $\angle 2$ are vertical angles, then $m\angle 1 = m\angle 2$.

In Exercises 27 and 28, use inductive reasoning to make a conjecture about the given quantity. Then use deductive reasoning to show that the conjecture is true. *Example 6*

27. the sum of two odd integers
28. the product of two odd integers

In Exercises 29–32, decide whether inductive reasoning or deductive reasoning is used to reach the conclusion. Explain your reasoning. *Example 7*

29. Each time you go to bed, you charge your electronic devices. So, the next time you go to bed, you will charge your electronic devices.

30. Even numbers are divisible by 2. Odd numbers are not divisible by 2. So, 4 is an even number.

31. All photosynthetic organisms produce oxygen. Phytoplankton are photosynthetic organisms. So, phytoplankton produce oxygen.

32. Each time you clean your room, you are allowed to go out with your friends. So, the next time you clean your room, you will be allowed to go out with your friends.

ERROR ANALYSIS In Exercises 33 and 34, describe and correct the error in interpreting the statement.

33. If a figure is a rectangle, then the figure has four sides. A trapezoid has four sides.

> ✗ Using the Law of Detachment, you can conclude that a trapezoid is a rectangle.

34. Each day, you get to school before your friend.

> ✗ Using deductive reasoning, you can conclude that you will arrive at school before your friend tomorrow.

35. **MODELING REAL LIFE** The table shows the average weights of several subspecies of tigers. What conjecture can you make about the relation between the weights of female tigers and the weights of male tigers? Explain your reasoning.

STEM

	Weight of female (pounds)	Weight of male (pounds)
Siberian	370	660
Bengal	300	480
South China	240	330
Sumatran	200	270
Indo-Chinese	250	400

36. **HOW DO YOU SEE IT?** The graph shows the total mass loss of the Greenland ice sheet since 2004. Write a conjecture using the graph.

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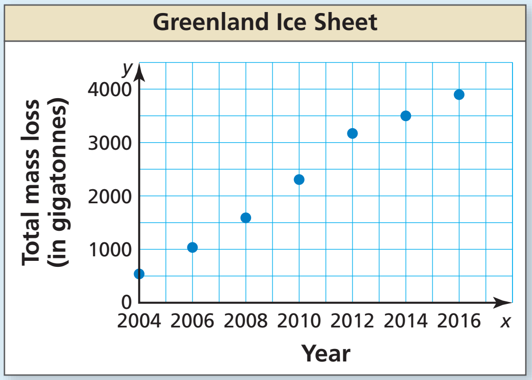


37. **CONNECTING CONCEPTS** Use the table to make a conjecture about the relationship between x and y. Then write an equation for y in terms of x. Use the equation to test your conjecture for other values of x.

x	0	1	2	3	4
y	2	5	8	11	14

38. **THOUGHT PROVOKING** The first two terms of a sequence are $\frac{1}{4}$ and $\frac{1}{2}$. Describe three different possible patterns for the sequence. List the first five terms for each sequence.

39. **MP PATTERNS** The following are the first nine *Fibonacci numbers*.

1, 1, 2, 3, 5, 8, 13, 21, 34, . . .

a. Make a conjecture about each of the Fibonacci numbers after the first two.

b. Write the next three numbers in the pattern.

c. Research to find and explain a real-world example of this pattern.

40. **MAKING AN ARGUMENT** Which argument is correct? Explain your reasoning.

Argument 1: If two angles measure 30° and 60°, then the angles are complementary. $\angle 1$ and $\angle 2$ are complementary. So, $m\angle 1 = 30°$ and $m\angle 2 = 60°$.

Argument 2: If two angles measure 30° and 60°, then the angles are complementary. The measure of $\angle 1$ is 30° and the measure of $\angle 2$ is 60°. So, $\angle 1$ and $\angle 2$ are complementary.

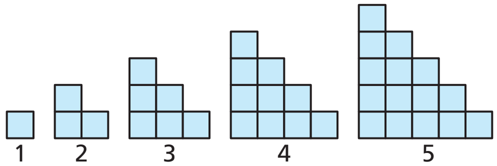
41. DRAWING CONCLUSIONS Decide whether each conclusion is valid. Explain your reasoning.

- You and your friend went camping at Yellowstone National Park.
- When you go camping, you go canoeing.
- If you go on a hike, your friend goes with you.
- You go on a hike.
- There is a 3-mile-long trail near your campsite.

a. Your friend went canoeing.

b. You and your friend went on a hike on a 3-mile-long trail.

42. MP REPEATED REASONING Each figure is made of squares that are 1 unit by 1 unit.



Predict the perimeter of the 20th figure.

43. CRITICAL THINKING Geologists use the Mohs scale to determine a mineral's hardness. Using the scale, a mineral with a higher rating will leave a scratch on a mineral with a lower rating. Testing a mineral's hardness can help identify the mineral.

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Mineral	 Talc	 Gypsum	 Calcite	 Fluorite
Mohs rating	1	2	3	4

a. The four minerals are randomly labeled *A*, *B*, *C*, and *D*. Mineral *A* is scratched by Mineral *B*. Mineral *C* is scratched by all three of the other minerals. What can you conclude? Explain.

b. How can you identify *all* the minerals in part (a)?

44. CONNECTING CONCEPTS Use inductive reasoning to write a formula for the sum of the first n positive even integers.

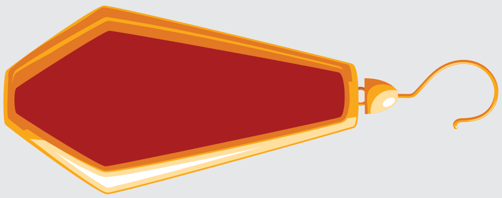
REVIEW & REFRESH

WATCH

45. Identify the hypothesis and the conclusion. Then rewrite the conditional statement in if-then form.

Storm surge causes the erosion of coastline.

46. MODELING REAL LIFE Classify the polygon by the number of sides. Tell whether it is *convex* or *concave*.



47. Write a recursive rule for the sequence.

n	1	2	3	4
a_n	4	11	18	25

48. Write an equation of the line that passes through the points (3, 2) and (0, 8).

49. $\angle 3$ is a complement of $\angle 4$, and $m\angle 3 = 19^\circ$. Find $m\angle 4$.

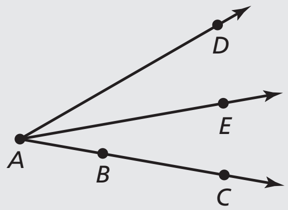
50. Determine whether the equation $y = x^2 + 5$ represents a *linear* or *nonlinear* function.

51. Solve the equation $-6x + 5 = x - 9$. Check your solution.

52. Graph $g(x) = 2x^2$. Compare the graph to the graph of $f(x) = x^2$.

53. Approximate $\sqrt{70}$ to the nearest (a) integer and (b) tenth.

54. CRITICAL THINKING Determine which postulate is illustrated by the statement $m\angle DAC = m\angle DAE + m\angle EAB$.



55. Find $(2n^2 - n - 6) + (-5n^2 - 4n + 8)$.

56. Solve $3^{x+4} = 3^8$.

57. Find the next two numbers in the pattern $2, -2, 3, -3, 4, \ldots$.