2.1 Conditional Statements

GO DIGITAL

Learning Target Understand and write conditional statements.

Success Criteria

- I can write conditional statements.
- I can write biconditional statements.
- I can determine if conditional statements are true by using truth tables.

A *conditional statement*, symbolized by $p \rightarrow q$, can be written as an “if-then statement” that contains a *hypothesis* p and a *conclusion* q. Here is an example.

$$\textit{If } \underbrace{\textit{a polygon is a triangle}}_{\text{hypothesis, } p}\textit{, then } \underbrace{\textit{the sum of its angle measures is } 180^\circ}_{\text{conclusion, } q}.$$

EXPLORE IT! Determining Whether Statements Are True or False

Work with a partner. A hypothesis can be either true or false. The same is true of a conclusion. When a conditional statement is true, the hypothesis and conclusion do not necessarily both have to be true.

a. Determine whether each conditional statement is true or false. Justify your answer.

i. If yesterday was Wednesday, then today is Thursday.

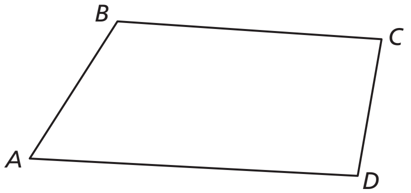
ii. If an angle is acute, then it has a measure of 30°.

S	M	T	W	T	F	S
1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30					

iii. If a month has 30 days, then it is June.

iv. If $\triangle ADC$ is a right triangle, then the Pythagorean Theorem is valid for $\triangle ADC$.

v. If a polygon is a quadrilateral, then the sum of its angle measures is 180°.



vi. If points A, B, and C are collinear, then they lie on the same line.

b. Write one true conditional statement and one false conditional statement that are different from those given in part (a). Justify your answer.

c. Conditional statements do not have to be written in if-then form. Determine whether each conditional statement is true or false. Justify your answer.

i. Two angles are complementary if the sum of their measures is 90°.

ii. The product of two numbers is negative when both numbers are negative.

Math Practice

View as Components
Which parts of the statements in part (c) are the hypotheses? the conclusions?

Writing Conditional Statements

Vocabulary

- conditional statement, *p. 64*
- if-then form, *p. 64*
- hypothesis, *p. 64*
- conclusion, *p. 64*
- negation, *p. 64*
- converse, *p. 65*
- inverse, *p. 65*
- contrapositive, *p. 65*
- equivalent statements, *p. 65*
- perpendicular lines, *p. 66*
- biconditional statement, *p. 67*
- truth value, *p. 68*
- truth table, *p. 68*

KEY IDEA

Conditional Statement

A **conditional statement** is a logical statement that has two parts, a *hypothesis p* and a *conclusion q*. When a conditional statement is written in **if-then form**, the "if" part contains the **hypothesis** and the "then" part contains the **conclusion**.

Words If *p*, then *q*. **Symbols** $p \rightarrow q$ (read as "*p* implies *q*")

EXAMPLE 1 Rewriting a Statement in If-Then Form

Identify the hypothesis and the conclusion. Then rewrite the conditional statement in if-then form.

a. All birds have feathers.

b. You are in Texas if you are in Houston.

SOLUTION

a. $\underbrace{\text{All birds}}_{\text{hypothesis}}$ have $\underbrace{\text{feathers}}_{\text{conclusion}}$.

▶ If an animal is a bird, then it has feathers.

b. $\underbrace{\text{You are in Texas}}_{\text{conclusion}}$ if $\underbrace{\text{you are in Houston}}_{\text{hypothesis}}$.

▶ If you are in Houston, then you are in Texas.

KEY IDEA

Negation

The **negation** of a statement is the *opposite* of the original statement. To write the negation of a statement *p*, you write the symbol for negation ($\sim$) before the letter.

Words not *p* **Symbols** $\sim p$ (read as "not *p*")

EXAMPLE 2 Writing a Negation

Write the negation of each statement.

a. The ball is red.

b. The cat is not black.

SOLUTION

a. The ball is not red.

b. The cat is black.

SELF-ASSESSMENT

1 I do not understand. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

Identify the hypothesis and the conclusion. Then rewrite the conditional statement in if-then form.

1. All 30° angles are acute angles.

2. $2x + 7 = 1$, because $x = -3$.

Write the negation of the statement.

3. The shirt is green.

4. The shoes are not red.

GO DIGITAL

KEY IDEA

Related Conditionals

Consider the conditional statement below.

Words If p, then q. **Symbols** $p \rightarrow q$

Converse To write the **converse** of a conditional statement, exchange the hypothesis and the conclusion.

Words If q, then p. **Symbols** $q \rightarrow p$

Inverse To write the **inverse** of a conditional statement, negate both the hypothesis and the conclusion.

Words If not p, then not q. **Symbols** $\sim p \rightarrow \sim q$

Contrapositive To write the **contrapositive** of a conditional statement, first write the converse. Then negate both the hypothesis and the conclusion.

Words If not q, then not p. **Symbols** $\sim q \rightarrow \sim p$

A conditional statement and its contrapositive are either both true or both false. Similarly, the converse and inverse of a conditional statement are either both true or both false. In general, when two statements are both true or both false, they are called **equivalent statements**.

COMMON ERROR

Just because a conditional statement and its contrapositive are both true does not mean that its converse and inverse are both false. The converse and inverse can also both be true.

EXAMPLE 3 Writing Related Conditional Statements

WATCH

Let p be "you are a guitar player" and let q be "you are a musician." Write each statement in words. Then decide whether it is *true* or *false*.

a. the conditional statement $p \rightarrow q$

b. the converse $q \rightarrow p$

c. the inverse $\sim p \rightarrow \sim q$

d. the contrapositive $\sim q \rightarrow \sim p$

SOLUTION

a. Conditional: If you are a guitar player, then you are a musician.
true; Guitar players are musicians.

b. Converse: If you are a musician, then you are a guitar player.
false; Not all musicians play the guitar.

c. Inverse: If you are not a guitar player, then you are not a musician.
false; Even if you do not play the guitar, you can still be a musician.

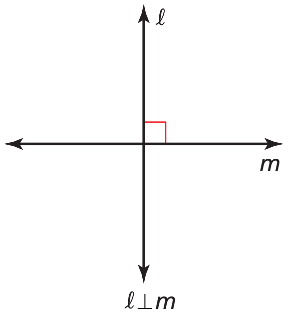
d. Contrapositive: If you are not a musician, then you are not a guitar player.
true; A person who is not a musician cannot be a guitar player.

SELF-ASSESSMENT

1	I do not understand.	2	I can do it with help.	3	I can do it on my own.	4	I can teach someone else.

5. Repeat Example 3 when p is "the stars are visible" and q is "it is night."

GO DIGITAL



You can write a definition as a conditional statement in if-then form or as its converse. Both the conditional statement and its converse are true for definitions. For example, consider the definition of *perpendicular lines*.

If two lines intersect to form a right angle, then they are **perpendicular lines**.

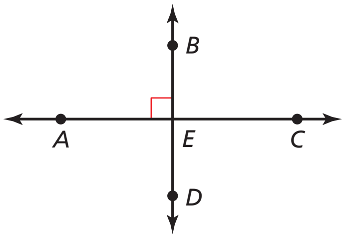
You can also write the definition using the converse: If two lines are perpendicular lines, then they intersect to form a right angle.

You can write "line ℓ is perpendicular to line m" as $\ell \perp m$.

EXAMPLE 4 Using Definitions

WATCH

Decide whether each statement about the diagram is true. Explain your answer using the definitions you have learned.



a. $\overleftrightarrow{AC} \perp \overleftrightarrow{BD}$

b. $\angle AEB$ and $\angle CEB$ are a linear pair.

c. $\overrightarrow{EA}$ and $\overrightarrow{EB}$ are opposite rays.

SOLUTION

a. This statement is *true*. The right angle symbol in the diagram indicates that the lines intersect to form a right angle. So, you can say the lines are perpendicular.

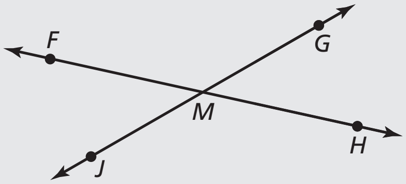
b. This statement is *true*. By definition, if the noncommon sides of adjacent angles are opposite rays, then the angles are a linear pair. Because $\overrightarrow{EA}$ and $\overrightarrow{EC}$ are opposite rays, $\angle AEB$ and $\angle CEB$ are a linear pair.

c. This statement is *false*. The rays have the same endpoint, but they do not form a line. So, the rays are not opposite rays.

SELF-ASSESSMENT

1 I do not understand. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

Use the diagram. Decide whether the statement is true. Explain your answer using the definitions you have learned.



6. $\angle JMF$ and $\angle FMG$ are supplementary.

7. Point M is the midpoint of $\overline{FH}$.

8. $\angle JMF$ and $\angle HMG$ are vertical angles.

9. $\overleftrightarrow{FH} \perp \overleftrightarrow{JG}$

Writing Biconditional Statements

GO DIGITAL

KEY IDEA

Biconditional Statement

When a conditional statement and its converse are both true, you can write them as a single *biconditional statement*. A **biconditional statement** is a statement that contains the phrase "if and only if."

Words p if and only if q **Symbols** $p \leftrightarrow q$

Any definition can be written as a biconditional statement.

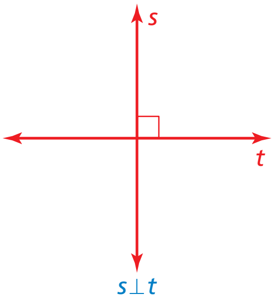
EXAMPLE 5 Writing a Biconditional Statement

WATCH

Rewrite the definition of perpendicular lines as a biconditional statement.

Definition If two lines intersect to form a right angle, then they are perpendicular lines.

SOLUTION



Let p be "two lines intersect to form a right angle" and let q be "they are perpendicular lines." Use red to identify p and blue to identify q. Write the definition $p \rightarrow q$.

Definition If two lines intersect to form a right angle, then they are perpendicular lines.

Write the converse $q \rightarrow p$.

Converse If two lines are perpendicular lines, then they intersect to form a right angle.

Use the definition and its converse to write the biconditional statement $p \leftrightarrow q$.

▶ **Biconditional** Two lines intersect to form a right angle if and only if they are perpendicular lines.

SELF-ASSESSMENT

1 I do not understand. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

10. Rewrite the definition of a right angle as a single biconditional statement.

Definition If an angle is a right angle, then its measure is 90°.

11. Rewrite the definition of congruent segments as a single biconditional statement.

Definition If two line segments have the same length, then they are congruent segments.

Rewrite the statements as a biconditional statement.

12. If Mary is taking theater class, then she will be in the fall play. If Mary is in the fall play, then she must be taking theater class.

13. If you can run for president, then you are at least 35 years old. If you are at least 35 years old, then you can run for president.

GO DIGITAL

Making Truth Tables

The **truth value** of a statement is either true (T) or false (F). You can determine the conditions under which a conditional statement is true by using a **truth table**. The truth table below shows the truth values for hypothesis p and conclusion q.

Conditional

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

The conditional statement $p \to q$ is false only when a true hypothesis produces a false conclusion.

Two statements are *logically equivalent* when they have the same truth table.

EXAMPLE 6 Making a Truth Table

WATCH

Use the truth table above to make truth tables for the converse, inverse, and contrapositive of a conditional statement $p \to q$.

SOLUTION

The truth tables for the converse and the inverse are shown below. Notice that the converse and the inverse are logically equivalent because they have the same truth table.

Converse

p	q	$q \to p$
T	T	T
T	F	T
F	T	F
F	F	T

Inverse

p	q	$\sim p$	$\sim q$	$\sim p \to \sim q$
T	T	F	F	T
T	F	F	T	T
F	T	T	F	F
F	F	T	T	T

The truth table for the contrapositive is shown below. Notice that a conditional statement and its contrapositive are logically equivalent because they have the same truth table.

Contrapositive

p	q	$\sim q$	$\sim p$	$\sim q \to \sim p$
T	T	F	F	T
T	F	T	F	F
F	T	F	T	T
F	F	T	T	T

SELF-ASSESSMENT

1 I do not understand. **2** I can do it with help. **3** I can do it on my own. **4** I can teach someone else.

Create a truth table for the logical statement.

14. $p \to \sim q$

15. $\sim(p \to q)$

2.1 Practice WITH CalcChat® AND CalcView®

In Exercises 1–4, identify the hypothesis and the conclusion.

1. If a polygon is a pentagon, then it has five sides.

2. If two lines form vertical angles, then they intersect.

3. If you run, then you are fast.

4. If you like math, then you like science.

In Exercises 5–10, rewrite the conditional statement in if-then form. *Example 1*

5. $9x + 5 = 23$, because $x = 2$.

6. Today is Friday, so tomorrow is the weekend.

7. When a glacier melts, the sea level rises.

8. Two right angles are supplementary angles.

9. People who are registered are allowed to vote.

10. The measures of complementary angles sum to 90°.

In Exercises 11–14, write the negation of the statement. *Example 2*

11. The sky is blue.

12. The lake is cold.

13. The ball is not pink.

14. The dog is not a Labrador retriever.

In Exercises 15–22, write the conditional statement $p \rightarrow q$, the converse $q \rightarrow p$, the inverse $\sim p \rightarrow \sim q$, and the contrapositive $\sim q \rightarrow \sim p$ in words. Then decide whether each statement is *true* or *false*. *Example 3*

15. Let p be "two angles are supplementary" and let q be "the measures of the angles sum to 180°."

16. Let p be "you are in math class" and let q be "you are in Geometry."

17. Let p be "you do your math homework" and let q be "you will do well on the test."

18. Let p be "you are not an only child" and let q be "you have a sibling."

19. Let p be "it does not snow" and let q be "I will run outside."

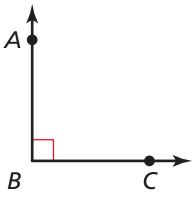
20. Let p be "the Sun is out" and let q be "it is daytime."

21. Let p be "$3x - 7 = 20$" and let q be "$x = 9$."

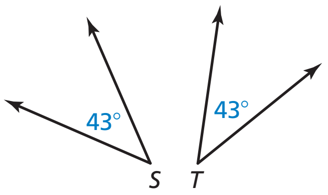
22. Let p be "it is Valentine's Day" and let q be "it is February."

In Exercises 23–26, decide whether the statement about the diagram is true. Explain your answer using the definitions you have learned. *Example 4*

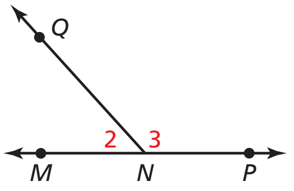
23. $m\angle ABC = 90°$



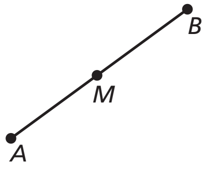
24. $\angle S \cong \angle T$



25. $m\angle 2 + m\angle 3 = 180°$



26. M is the midpoint of $\overline{AB}$.



In Exercises 27–30, rewrite the definition as a biconditional statement. *Example 5*

27. The *midpoint* of a segment is the point that divides the segment into two congruent segments.

28. Two angles are *vertical angles* when their sides form two pairs of opposite rays.

29. *Adjacent angles* are two angles that share a common vertex and side but have no common interior points.

30. Two angles are *supplementary angles* when the sum of their measures is 180°.

In Exercises 31–34, rewrite the statements as a biconditional statement.

31. If a polygon has three sides, then it is a triangle.
If a polygon is a triangle, then it has three sides.

32. If a polygon has four sides, then it is a quadrilateral.
If a polygon is a quadrilateral, then it has four sides.

33. If an angle is a right angle, then it measures 90°.
If an angle measures 90°, then it is a right angle.

34. If an angle is obtuse, then it has a measure between 90° and 180°.
If an angle has a measure between 90° and 180°, then it is obtuse.

In Exercises 35–40, create a truth table for the logical statement. ▶ *Example 6*

35. $\sim p \rightarrow q$

36. $\sim q \rightarrow p$

37. $\sim(\sim p \rightarrow \sim q)$

38. $\sim(p \rightarrow \sim q)$

39. $q \rightarrow \sim p$

40. $\sim(q \rightarrow p)$

41. **ERROR ANALYSIS** Describe and correct the error in writing the converse of the conditional statement.

> ✗
> *Conditional statement*
> If it is raining, then I will bring an umbrella.
>
> *Converse*
> If it is not raining, then I will not bring an umbrella.

42. **ERROR ANALYSIS** Describe and correct the error in determining the truth value of the statement.

> ✗
> *Conditional statement*
> If a triangle is concave, then a square is a quadrilateral.
>
> The hypothesis is false and the conclusion is true, so the conditional statement is false.

43. **MP REASONING** You know that the contrapositive of a statement is true. Does that help you determine whether the statement can be rewritten as a true biconditional statement? Explain your reasoning.

GO DIGITAL

44. **MAKING AN ARGUMENT** Can the statement "If I bought a shirt, then I went to the mall" be rewritten as a true biconditional statement? Explain your reasoning.

In Exercises 45–48, rewrite the conditional statement in if-then form. Then identify the hypothesis and the conclusion.

45.

> *If you tell the truth, you don't have to remember anything.*
>
> *Mark Twain*

46.

> *You have to expect things of yourself before you can do them.*
>
> *Michael Jordan*

47.

> *If one is lucky, a solitary fantasy can totally transform one million realities.*
>
> *Maya Angelou*

48.

> *Whoever is happy will make others happy too.*
>
> *Anne Frank*

49. **MP REASONING** The statements below describe three ways that rocks are formed.

Igneous rock is formed from the cooling of molten rock.

Sedimentary rock is formed from pieces of other rocks.

Metamorphic rock is formed by changing temperature, pressure, or chemistry.

a. Write each statement in if-then form.

b. Write the converse of each of the statements in part (a). Is the converse of each statement true? Explain your reasoning.

c. Write a true if-then statement about rocks that is different from the ones in parts (a) and (b). Is the converse of your statement true or false? Explain your reasoning.

50. **MP STRUCTURE** Use the conditional statement to identify the if-then statement as the converse, inverse, or contrapositive of the conditional statement. Then use the symbols to represent both statements.

Conditional statement

If I rode my bike to school, then I did not walk to school.

If-then statement

If I did not ride my bike to school, then I walked to school.

p q $\sim$ $\rightarrow$ $\leftrightarrow$

51. **COLLEGE PREP** The given statement is true. Which of the following statements must be true? Select all that apply. Explain your reasoning.

Given statement

If I go to the movie theater, then I will eat popcorn.

Ⓐ If I do not eat popcorn, then I did not go to the movie theater.

Ⓑ I will go to the movie theater if and only if I eat popcorn.

Ⓒ If I eat popcorn, then I went to the movie theater.

Ⓓ If I do not go to the movie theater, then I will not eat popcorn.

52. **CONNECTING CONCEPTS** Can the statement "If $x = 4$, then $x^2 - 10 = x + 2$" be combined with its converse to form a true biconditional statement? Explain your reasoning.

53. **CONNECTING CONCEPTS** A data set consists of the heights of the students in your class.

a. Tell whether the following statement is true: If x and y are the least and greatest values in the data set, then the mean of the data is between x and y.

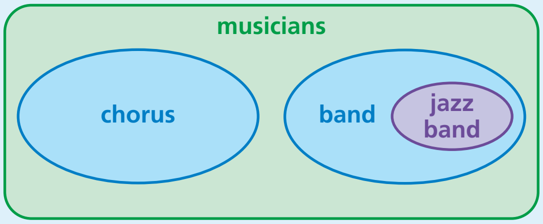
b. Write the converse of the statement in part (a). Is the converse true? Explain your reasoning.

c. Complete the following statement using *mean*, *median*, or *mode* to make a conditional statement that is true for any data set. Explain your reasoning.

If a data set has a mean, a median, and a mode, then the _______ of the data set will always be a data value.

GO DIGITAL

54. **HOW DO YOU SEE IT?** The Venn diagram represents all the musicians at a high school. Write three conditional statements in if-then form describing the relationships between the various groups of musicians.



55. **MULTIPLE REPRESENTATIONS** Create a Venn diagram representing each conditional statement. Write the converse of each conditional statement. Then determine whether each conditional statement and its converse are true or false. Explain your reasoning.

a. If you go to the zoo to see a lion, then you will see a cat.

b. If you play a sport, then you wear a helmet.

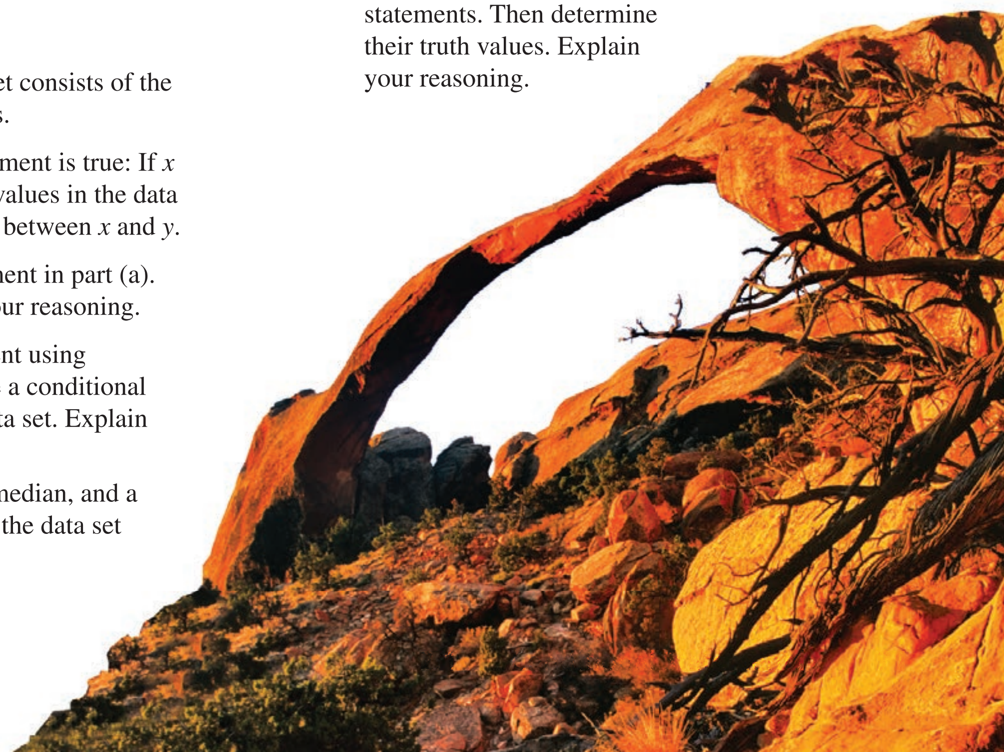
c. If this month has 31 days, then it is not February.

56. **MODELING REAL LIFE** The largest natural arch in the United States is Landscape Arch, located in Thompson, Utah. It spans 290 feet.

a. Use the information to write at least two true conditional statements.

b. Which type of related conditional statement must also be true? Write the related conditional statements.

c. What are the other two types of related conditional statements? Write the related conditional statements. Then determine their truth values. Explain your reasoning.



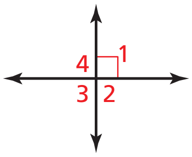
57. **CRITICAL THINKING** One example of a true conditional statement involving dates is “If today is August 31, then tomorrow is September 1.” Write a conditional statement using dates so that the truth value depends on when the statement is read.

58. **THOUGHT PROVOKING**
Write three conditional statements, where one is always true, one is always false, and one depends on the person interpreting the statement.

59. **OPEN-ENDED** Advertising slogans such as “Buy these shoes! They will make you a better athlete!” often imply conditional statements. Find an advertisement or write your own slogan. Then write it as a conditional statement.

60. **OPEN-ENDED** Write a conditional statement that is true, but its converse is false.

61. **CRITICAL THINKING** Write a series of if-then statements that allow you to find the measure of each angle, given that $m\angle 1 = 90^\circ$.



62. **DIG DEEPER** Can the converse and the contrapositive of a conditional statement both have truth values that are false? Justify your answer.

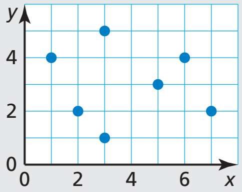
REVIEW & REFRESH

In Exercises 63 and 64, write the next three terms of the arithmetic sequence.

63. 7, 5, 3, 1, . . .

64. 12, 23, 34, 45, . . .

65. Determine whether the graph represents a function. Explain.

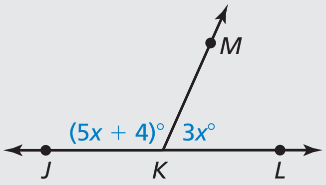


In Exercises 66 and 67, use the graphs of f and g to describe the transformation from the graph of f to the graph of g.

66. $f(x) = 2x + 1$, $g(x) = f(x) + 5$

67. $f(x) = \frac{1}{3}x - 6$, $g(x) = 3f(x)$

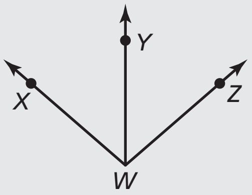
68. Find the measure of each angle.



69. Find the perimeter and the area of $\triangle QRS$ with vertices $Q(-3, 4)$, $R(5, 4)$, and $S(1, -2)$.

70. **MODELING REAL LIFE** The average distance from Earth to the moon is 3.844×10^5 kilometers. Write this number in standard form.

71. In the diagram, $\overrightarrow{WY}$ bisects $\angle XWZ$, and $m\angle YWZ = 49^\circ$. Find $m\angle XWY$ and $m\angle XWZ$.



In Exercises 72–75, perform the operation.

72. $3x^2(-x + 7)$

73. $(z - 1)(z + 8)$

74. $(5b^2 - 6b + 3) - (4b - 2)$

75. $(-4n^3 - n^2 + 8) + (6n^2 + 5n - 9)$

76. Write an inequality that represents the graph.

77. Let p be “you play a video game” and let q be “you beat the video game.” Write the conditional statement $p \rightarrow q$, the converse $q \rightarrow p$, the inverse $\sim p \rightarrow \sim q$, and the contrapositive $\sim q \rightarrow \sim p$ in words. Then decide whether each statement is *true* or *false*.