Learning Target   Solve exponential equations.

Success Criteria
- I can solve exponential equations with the same base.
- I can solve exponential equations with unlike bases.
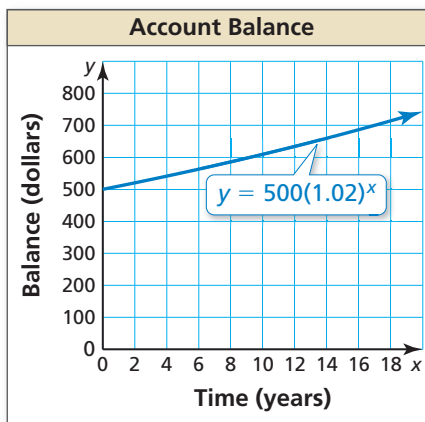- I can solve exponential equations by graphing.

EXPLORE IT!   Solving Exponential Equations

Work with a partner. You open a money market account with $500. The graph shows the balance y (in dollars) after x years.

Math Practice

Use Equations
What does the equation $500(1.02)^x = 600$ represent in this situation? How can you find the solution?

Account Balance

$y = 500(1.02)^x$

a. **MP** **CHOOSE TOOLS** Describe several different ways that you can approximate the amount of time until the balance is $600.

b. You open a savings account with $450. The balance increases by $10 each year. Determine whether the balances of the two accounts are ever equal. Explain your method.

c. Suppose the balance of your savings account increases by $15 each year. Does the balance of your savings account ever equal the balance of your money market account? If so, determine when the balances are equal.

d. The solutions in parts (a)–(c) can be found by writing and solving *exponential equations*. Use technology to solve each exponential equation below. Explain your method.

  i. $2^x = \frac{1}{2}$ ii. $3^{x-1} = 0$

 iii. $2^{x-2} = \frac{3}{2}x - 2$ iv. $4^{x-1} = 2^{x+1}$

e. Can an exponential equation have no solution? more than one solution?

f. Can any of the equations in part (d) be solved algebraically by using properties of exponents? Explain your reasoning.

Solving Exponential Equations with the Same Base

Exponential equations are equations in which variable expressions occur as exponents.

KEY IDEA

Property of Equality for Exponential Equations

Words Two powers with the *same positive base b*, where $b \neq 1$, are equal if and only if their exponents are equal.

Numbers If $2^x = 2^5$, then $x = 5$. If $x = 5$, then $2^x = 2^5$.

Algebra If $b > 0$ and $b \neq 1$, then $b^x = b^y$ if and only if $x = y$.

EXAMPLE 1 Solving Exponential Equations with the Same Base

Solve each equation.

a. $3^{x+1} = 3^5$

b. $6 = 6^{2x-3}$

c. $10^{3x} = 10^{2x+3}$

WATCH

SOLUTION

a. $3^{x+1} = 3^5$ — Write the equation.

$ x + 1 = 5$ — Equate the exponents.

$ x = 4$ — Subtract 1 from each side.

b. $6 = 6^{2x-3}$ — Write the equation.

$ 1 = 2x - 3$ — Equate the exponents.

$ 4 = 2x$ — Add 3 to each side.

$ 2 = x$ — Divide each side by 2.

c. $10^{3x} = 10^{2x+3}$ — Write the equation.

$ 3x = 2x + 3$ — Equate the exponents.

$ x = 3$ — Subtract 2x from each side.

Check

$$10^{3x} = 10^{2x+3}$$

$$10^{3(3)} \stackrel{?}{=} 10^{2(3)+3}$$

$$10^9 = 10^9 \ \checkmark$$

SELF-ASSESSMENT ⬜1 I do not understand. ⬜2 I can do it with help. ⬜3 I can do it on my own. ⬜4 I can teach someone else.

Solve the equation. Check your solution.

1. $2^{2x} = 2^6$

2. $5^{2x} = 5^{x+1}$

3. $7^{3x+5} = 7^{x+1}$

4. WHICH ONE DOESN'T BELONG? Which equation does *not* belong with the other three? Explain your reasoning.

$2^{x+3} = 2^6$ $\quad$ $5^{3x+8} = 5^{2x}$ $\quad$ $3^4 = x + 3^2$ $\quad$ $2^{x-7} = 2^7$

5. 🔴 **MP REASONING** Explain why b cannot equal 1 in the Property of Equality for Exponential Equations.

Solving Exponential Equations with Unlike Bases

GO DIGITAL

To solve some exponential equations, you can first rewrite each side of the equation using the same base.

EXAMPLE 2 Solving Exponential Equations with Unlike Bases

Solve (a) $5^x = 125$, (b) $4^x = 2^{x-3}$, and (c) $9^{x+2} = 27^x$.

SOLUTION

a.

$5^x = 125$	Write the equation.
$5^x = 5^3$	Rewrite 125 as 5^3.
$x = 3$	Equate the exponents.

Check

$4^x = 2^{x-3}$

$4^{-3} \stackrel{?}{=} 2^{-3-3}$

$\frac{1}{64} = \frac{1}{64}$ ✓

b.

$4^x = 2^{x-3}$	Write the equation.
$(2^2)^x = 2^{x-3}$	Rewrite.
$2^{2x} = 2^{x-3}$	Power of a Power Property
$2x = x - 3$	Equate the exponents.
$x = -3$	Simplify.

c.

$9^{x+2} = 27^x$	
$(3^2)^{x+2} = (3^3)^x$	
$3^{2x+4} = 3^{3x}$	
$2x + 4 = 3x$	
$4 = x$	

EXAMPLE 3 Solving Exponential Equations When $0 < b < 1$

Solve (a) $\left(\frac{1}{2}\right)^x = 4$ and (b) $4^{x+1} = \frac{1}{64}$.

SOLUTION

a.

$\left(\frac{1}{2}\right)^x = 4$	Write the equation.
$(2^{-1})^x = 2^2$	Rewrite $\frac{1}{2}$ as 2^{-1} and 4 as 2^2.
$2^{-x} = 2^2$	Power of a Power Property
$-x = 2$	Equate the exponents.
$x = -2$	Divide each side by -1.

Check

$4^{x+1} = \frac{1}{64}$

$4^{-4+1} \stackrel{?}{=} \frac{1}{64}$

$\frac{1}{64} = \frac{1}{64}$ ✓

b.

$4^{x+1} = \dfrac{1}{64}$	Write the equation.
$4^{x+1} = \dfrac{1}{4^3}$	Rewrite 64 as 4^3.
$4^{x+1} = 4^{-3}$	Definition of negative exponent
$x + 1 = -3$	Equate the exponents.
$x = -4$	Subtract 1 from each side.

SELF-ASSESSMENT
1 I do not understand. **2** I can do it with help. **3** I can do it on my own. **4** I can teach someone else.

Solve the equation. Check your solution.

6. $4^x = 256$

7. $9^{2x} = 3^{x-6}$

8. $4^{3x} = 8^{x+1}$

9. $\left(\frac{1}{3}\right)^{x-1} = 27$

6.5   Solving Exponential Equations   333

Solving Exponential Equations by Graphing

Sometimes, it is difficult or impossible to rewrite each side of an exponential equation using the same base. You can solve these types of equations by graphing each side and finding the point(s) of intersection. Exponential equations can have no solution, one solution, or more than one solution.

EXAMPLE 4   Solving Exponential Equations by Graphing

WATCH

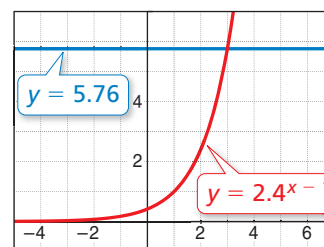Solve (a) $2.4^{x-1} = 5.76$ and (b) $3^{x+2} = x + 1$.

SOLUTION

a. Step 1 Write a function to represent each side of the original equation.
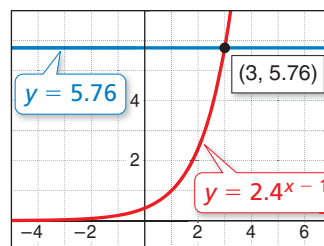$$y = 2.4^{x-1}$$
$$y = 5.76$$

Step 2 Use technology to graph the functions in a viewing window that shows where the graphs could intersect.

$y = 5.76$

$y = 2.4^{x-1}$

ANOTHER WAY

It may be difficult to recognize, but knowing that $24^2 = 576$ helps you reason that $2.4^2 = 5.76$. This can be used to solve part (a) algebraically.

$$2.4^{x-1} = 5.76$$
$$2.4^{x-1} = 2.4^2$$
$$x - 1 = 2$$
$$x = 3$$

Step 3 Find the point of intersection. The graphs intersect at $(3, 5.76)$.
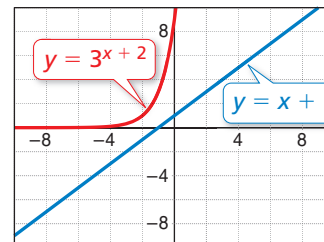

$(3, 5.76)$

$y = 5.76$

$y = 2.4^{x-1}$

▶ So, the solution is $x = 3$.

b. Step 1 Write a function to represent each side of the original equation.
$$y = 3^{x+2}$$
$$y = x + 1$$

Step 2 Use technology to graph the functions in a viewing window that shows where the graphs could intersect.

$y = 3^{x+2}$

$y = x + 1$

▶ The graphs do not intersect. So, the equation has no solution.

SELF-ASSESSMENT   1 I do not understand.   2 I can do it with help.   3 I can do it on my own.   4 I can teach someone else.

Solve the equation.

10. $3.1^{x+2} = 9.61$ **11.** $4^{x-3} = 3x - 8$ **12.** $\left(\frac{1}{4}\right)^x = -2x - 3$

13. MP PRECISION In Example 4(a), explain why the solution is not $(3, 5.76)$.

In Exercises 1–10, solve the equation. Check your solution. ▶ *Examples 1 and 2*

1. $4^{5x} = 4^{10}$

2. $7^{x-4} = 7^8$

3. $3^{9x} = 3^{7x+8}$

4. $2^{4x} = 2^{x+9}$

5. $2^x = 64$

6. $3^x = 243$

7. $7^{x-5} = 49^x$

8. $216^x = 6^{x+10}$

9. $64^{2x+4} = 16^{5x}$

10. $27^x = 81^{x+1}$

In Exercises 11–16, solve the equation. Check your solution. ▶ *Example 3*

11. $\left(\frac{1}{5}\right)^x = 125$

12. $\left(\frac{1}{4}\right)^x = 256$

13. $\frac{1}{128} = 2^{5x+3}$

14. $3^{4x-9} = \frac{1}{243}$

15. $36^{-3x+3} = \left(\frac{1}{216}\right)^{x+1}$
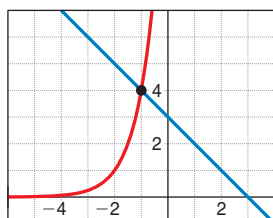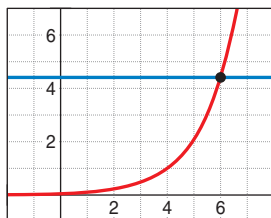
16. $\left(\frac{1}{27}\right)^{4-x} = 9^{2x-1}$

ERROR ANALYSIS In Exercises 17 and 18, describe and correct the error in solving the exponential equation.

17.

> ✗
> $5^{3x+2} = 25^{x-8}$
> $3x + 2 = x - 8$
> $x = -5$

18.

> ✗
> $\left(\frac{1}{8}\right)^{5x} = 32^{x+8}$
> $(2^3)^{5x} = (2^5)^{x+8}$
> $2^{15x} = 2^{5x+40}$
> $15x = 5x + 40$
> $x = 4$

In Exercises 19 and 20, use the graph to solve the equation.

19. $2.1^{x-4} = 4.41$

20. $4^{x+2} = -x + 3$
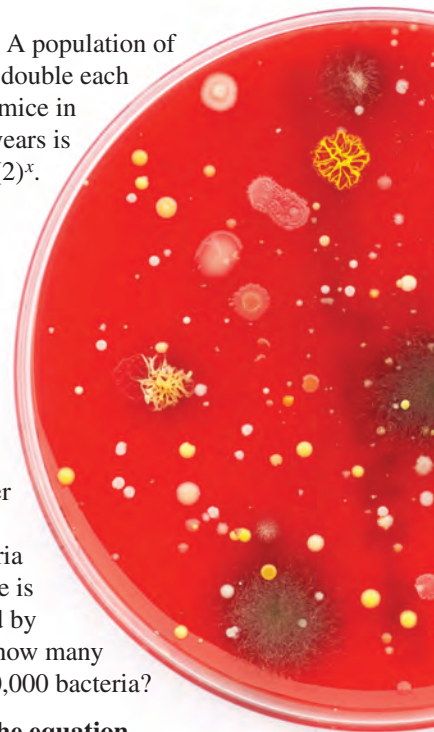

In Exercises 21–30, solve the equation. ▶ *Example 4*

21. $0.25^{x+2} = 16$

22. $1.9^{x-4} = 3.61$

23. $2^{x+3} = 3x + 8$

24. $4x - 3 = 5^{x-1}$

25. $\frac{4}{3}x - 1 = \left(\frac{1}{3}\right)^{2x-1}$

26. $2^{-x+1} = \frac{19 - 15x}{4}$

27. $5^x = -4^{-x+4}$

28. $7^{x-2} = 2^{-x}$

29. $2^{-x-3} = 3^{x+1}$

30. $5^{-2x+3} = -6^{x+5}$

In Exercises 31–34, solve the equation by using the Property of Equality for Exponential Equations.

31. $30 \cdot 5^{x+3} = 150$

32. $12 \cdot 2^{x-7} = 24$

33. $4(3)^{-2x-4} = 36$

34. $2(4)^{2x+1} = 128$

35. **MODELING REAL LIFE** A population of 50 mice is expected to double each year. The number y of mice in the population after x years is represented by $y = 50(2)^x$. After how many years will there be 800 mice in the population?

36. **MODELING REAL LIFE** A bacterial culture quadruples in size every hour. You begin observing the number of bacteria 3 hours after the culture is prepared. The amount y of bacteria x hours after the culture is prepared is represented by $y = 192(4)^{x-3}$. After how many hours will there be 200,000 bacteria?

In Exercises 37–40, solve the equation.

37. $3^{3x+6} = 27^{x+2}$

38. $3^{4x+3} = 81^x$

39. $4^{x+3} = 2^{2(x+1)}$

40. $5^{8(x-1)} = 625^{2x-2}$

**41.** **MP NUMBER SENSE** Explain how you can use mental math to solve the equation $8^{x-4} = 1$.

42. **MP PROBLEM SOLVING** There are a total of 128 teams at the start of a citywide 3-on-3 basketball tournament. Half the teams are eliminated after each round. Write and solve an exponential equation to determine after which round there are 16 teams left.

43. [MP] **PROBLEM SOLVING** You deposit $500 in a savings account that earns 6% annual interest compounded yearly. After how many years will the balance of the account be $750?

44. **HOW DO YOU SEE IT?**
The graph shows the annual attendances at two different events over time.

Event Attendance

$y = 4000(1.25)^x$

$y = 12,000(0.87)^x$

— Event 1
— Event 2

Years after 2010
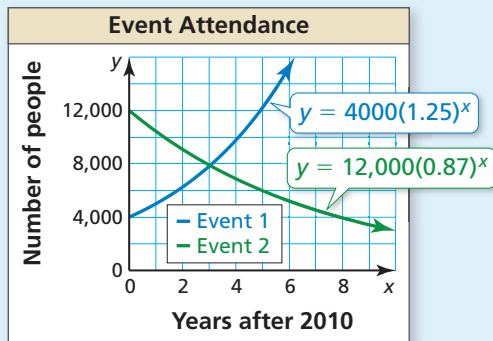
a. Estimate when the events had about the same attendance.

b. Explain how you can verify your answer in part (a).

45. **COLLEGE PREP** What is the value of y when $9^{x+3} = 9^{y-1}$ and $8^{x+4} = 64^{2x+5}$?

Ⓐ -2 Ⓑ -1 Ⓒ 2 Ⓓ 3

46. **THOUGHT PROVOKING**
Write a function g so that the equation $2^{x-1} = g(x)$ has exactly three solutions.

[MP] **STRUCTURE** In Exercises 47–52, solve the equation.

47. $8^{x-2} = \sqrt{8}$

48. $\sqrt{5} = 5^{x+4}$

49. $\left(\sqrt[5]{7}\right)^x = 7^{2x+3}$

50. $12^{2x-1} = \left(\sqrt[4]{12}\right)^x$

51. $\left(\sqrt[3]{6}\right)^{2x} = \left(\sqrt{6}\right)^{x+6}$

52. $\left(\sqrt[5]{3}\right)^{5x-10} = \left(\sqrt[8]{3}\right)^{4x}$

53. **MAKING AN ARGUMENT** Consider the equation $\left(\dfrac{1}{a}\right)^x = b$, where $a > 1$ and $b > 1$. Your friend says the value of x will always be negative. Is your friend correct? Explain.

54. [DIG DEEPER] Solve the equation without graphing.

$$3^x + 3^{x+1} + 3^{x+4} = 61,965$$

REVIEW & REFRESH

In Exercises 55 and 56, determine whether the sequence is arithmetic. If so, find the common difference.

55. $-20, -26, -32, -38, \ldots$

56. $9, 18, 36, 72, \ldots$

57. Determine whether the table represents an *exponential growth function*, an *exponential decay function*, or *neither*. Explain.

x	0	1	2	3
y	7	21	63	189

In Exercises 58 and 59, solve the inequality. Graph the solution, if possible.
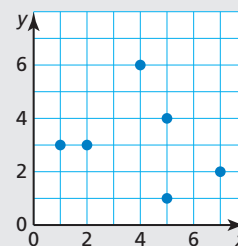
58. $3(2n - 1) < 6n - 4$

59. $|5t + 1| - 10 \le -6$

In Exercises 60 and 61, solve the equation. Check your solution.

60. $11^{x-2} = 11^{2x-7}$

61. $9^{x+3} = \left(\dfrac{1}{81}\right)^x$

62. Graph $y = 6(2)^{x-4} - 1$. Identify the asymptote. Find the domain and range.

63. Determine whether the graph represents a function. Explain.

64. [MP] **REASONING** You are stopped in a line of traffic that is about 1.5 miles long. Estimate the number of vehicles in the line. Explain your reasoning.

65. Rewrite $\left(\sqrt[6]{70}\right)^5$ in rational exponent form.

In Exercises 66 and 67, write an equation in slope-intercept form of the line that passes through the given points.

66. $(1, 7), (3, -3)$

67. $(0, -10), (8, -4)$

68. [MP] **STRUCTURE** Complete the equation so that it has infinitely many solutions.

$$7x - 20 + 8x = -5\left(\boxed{}\,x + \boxed{}\right)$$