**Learning Target**    Write and solve compound inequalities.

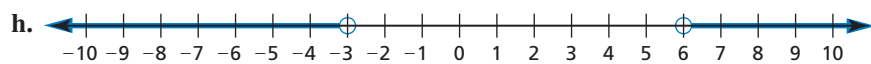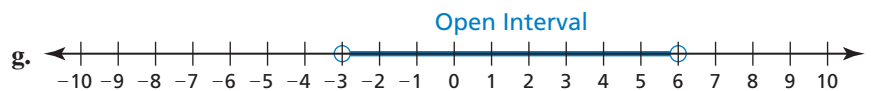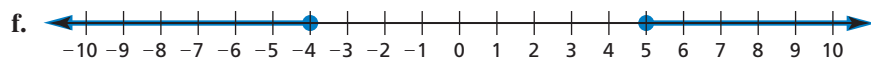**Success Criteria**
- I can write word sentences as compound inequalities.
- I can solve compound inequalities.
- I can graph solutions of compound inequalities.

## EXPLORE IT! Describing Intervals on the Real Number Line

**Work with a partner.** In parts (a)–(h), use two inequalities to describe the interval. Explain your reasoning.

Half-Open Interval

a.

b.

Half-Open Interval

c.

d.

Closed Interval

e.

### Math Practice

**Communicate Precisely**
Describe the difference between a closed interval and an open interval.

f.

Open Interval

g.

h.
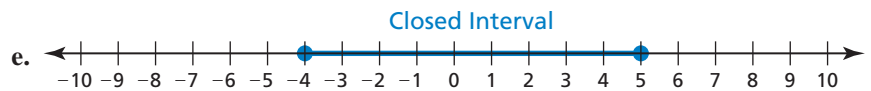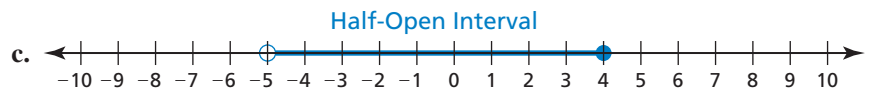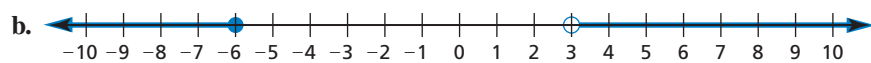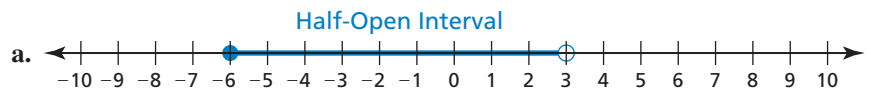
i. Do you use "and" or "or" when writing the inequalities for each graph?
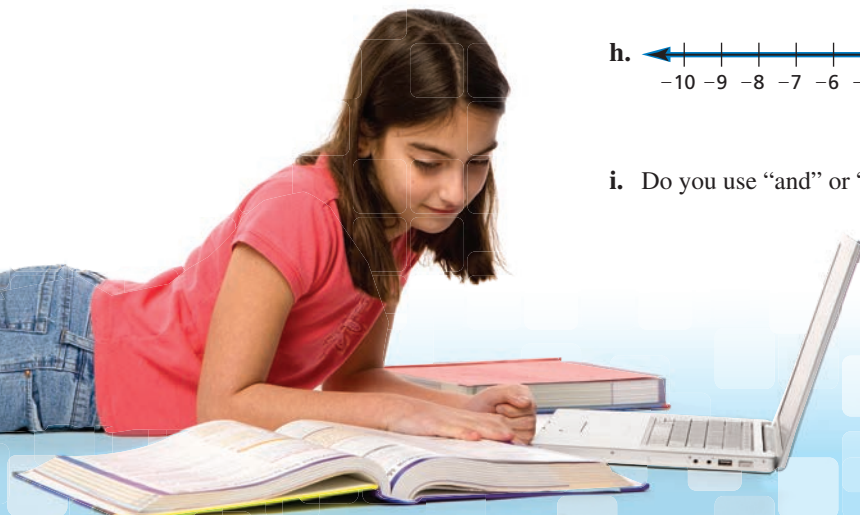
# Writing and Graphing Compound Inequalities

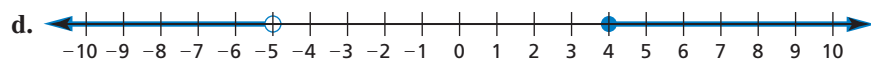The word *compound* can have many meanings, such as a chemical mixture, a group of buildings, or a word made from more than one word. All of these meanings have something in common—they represent something that is made from more than one thing.

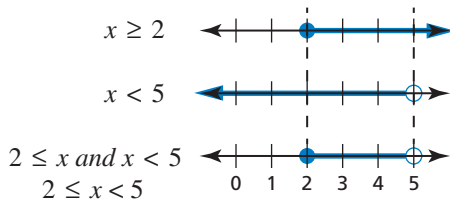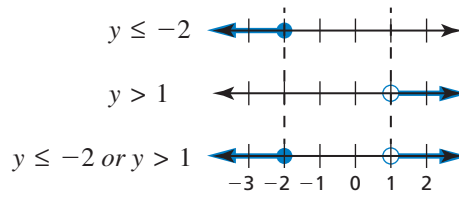A **compound inequality** is an inequality formed by joining two inequalities with the word "and" or the word "or."

The graph of a compound inequality with "and" is the *intersection* of the graphs of the inequalities. The graph shows numbers that are solutions of *both* inequalities.

$x \geq 2$

$x < 5$

$2 \leq x \text{ and } x < 5$
$2 \leq x < 5$

The graph of a compound inequality with "or" is the *union* of the graphs of the inequalities. The graph shows numbers that are solutions of *either* inequality.

$y \leq -2$

$y > 1$

$y \leq -2 \text{ or } y > 1$

**Vocabulary** 🔤 VOCAB

compound inequality, *p. 90*

## EXAMPLE 1    Writing and Graphing Compound Inequalities ▶ WATCH

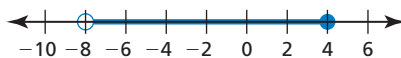Write each sentence as an inequality. Graph each inequality.

**a.** A number $x$ is greater than $-8$ and less than or equal to 4.

**b.** A number $y$ is at most 0 or at least $\frac{3}{2}$.

### SOLUTION

**a.** A number $x$ is greater than $-8$ and less than or equal to 4.

$x > -8$      *and*      $x \leq 4$

▶ An inequality is $-8 < x \leq 4$.

Graph the intersection of the graphs of $x > -8$ and $x \leq 4$.

**b.** A number $y$ is at most 0 or at least $\frac{3}{2}$.

$y \leq 0$      *or*   $y \geq \frac{3}{2}$

▶ An inequality is $y \leq 0 \text{ or } y \geq \frac{3}{2}$.
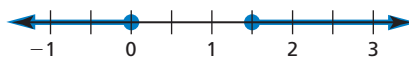
Graph the union of the graphs of $y \leq 0$ and $y \geq \frac{3}{2}$.

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**SELF-ASSESSMENT**  **1** I do not understand.  **2** I can do it with help.  **3** I can do it on my own.  **4** I can teach someone else.

**Write the sentence as an inequality. Graph the inequality.**

1. A number $d$ is more than 0 and less than 10.

2. A number $a$ is fewer than $-6$ or no less than $-3$.

3. **WRITING** Compare the graph of $-6 \leq x \leq -4$ with the graph of $x \leq -6 \text{ or } x \geq -4$.

4. **WHICH ONE DOESN'T BELONG?** Which compound inequality does *not* belong with the other three? Explain your reasoning.

   $a > 4 \text{ or } a < -3$      $a < -2 \text{ or } a > 8$      $a > 7 \text{ or } a < -5$      $a < 6 \text{ or } a > -9$

# Solving Compound Inequalities

You can solve a compound inequality by solving two inequalities separately. When a compound inequality with "and" is written as a single inequality, you can solve the inequality by performing the same operation on each expression.

### EXAMPLE 2 Solving Compound Inequalities with "And" ▶ WATCH

Solve each inequality. Graph each solution.

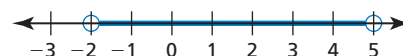**a.** $-4 < x - 2 < 3$        **b.** $-3 < -2x + 1 \le 9$

**SOLUTION**

**a.** Separate the compound inequality into two inequalities, then solve.

| | | | |
|---|---|---|---|
| $-4 < x - 2$ | *and* | $x - 2 < 3$ | Write two inequalities. |
| $\underline{+\ 2 \qquad +\ 2}$ | | $\underline{+\ 2 \quad +\ 2}$ | Addition Property of Inequality |
| $-2 < x$ | *and* | $x < 5$ | Simplify. |

▶ The solution is $-2 < x < 5$.

**Math Practice**

**Look for Structure**
In the inequality $-4 < x - 2 < 3$, what do you know about the quantity $x - 2$? How does this help you begin to solve?

**b.**

| | | |
|---|---|---|
| $-3 < -2x + 1 \le 9$ | | Write the inequality. |
| $\underline{-\ 1 \qquad\quad -\ 1 \quad -\ 1}$ | | Subtraction Property of Inequality |
| $-4 < -2x \quad \le \quad 8$ | | Simplify. |
| $\dfrac{-4}{-2} > \dfrac{-2x}{-2} \quad \ge \quad \dfrac{8}{-2}$ | | Use the Division Property of Inequality. Reverse each inequality symbol. |
| $2 > x \qquad\quad \ge -4$ | | Simplify. |

▶ The solution is $-4 \le x < 2$.

### EXAMPLE 3 Solving a Compound Inequality with "Or" ▶ WATCH

Solve $3y - 5 < -8$ *or* $2y - 1 > 5$. Graph the solution.

**SOLUTION**

| | | | |
|---|---|---|---|
| $3y - 5 < -8$ | *or* | $2y - 1 > 5$ | Write the inequality. |
| $\underline{+\ 5 \quad +\ 5}$ | | $\underline{+\ 1 \quad +\ 1}$ | Addition Property of Inequality |
| $3y < -3$ | | $2y > 6$ | Simplify. |
| $\dfrac{3y}{3} < \dfrac{-3}{3}$ | | $\dfrac{2y}{2} > \dfrac{6}{2}$ | Division Property of Inequality |
| $y < -1$ | *or* | $y > 3$ | Simplify. |

▶ The solution is $y < -1$ *or* $y > 3$.

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**SELF-ASSESSMENT**   **1** I do not understand.   **2** I can do it with help.   **3** I can do it on my own.   **4** I can teach someone else.

Solve the inequality. Graph the solution.

**5.** $5 \le m + 4 < 10$

**6.** $-3 < \frac{2}{3}k - 5 < 0$

**7.** $4c + 3 \le -5$ *or* $c - 8 > -1$

**8.** $2p + 1 < -4$ *or* $3 - 8p \le -1$

**9. OPEN-ENDED** Write a compound inequality that has a solution of all real numbers except $x = 0$.

# Solving Real-Life Problems

**EXAMPLE 4**  **Modeling Real Life**  WATCH  INFO

An electronic device may fail outside of its operating temperature range. Write an inequality that represents the possible operating temperatures (in degrees Fahrenheit) of the smartphone. Then describe a situation in which the phone may be outside of the operating range.

### SOLUTION

1. **Understand the Problem**  You know the operating temperature range in degrees Celsius. You are asked to represent the range in degrees Fahrenheit and to describe a situation outside of this range.

2. **Make a Plan**  Write a compound inequality in degrees Celsius $C$. Use the formula $C = \frac{5}{9}(F - 32)$ to rewrite the inequality in degrees Fahrenheit $F$. Then solve the inequality and describe a situation outside of this range.

3. **Solve and Check**

| | | | |
|---|---|---|---|
| $0 \leq$ | $C$ | $\leq 35$ | Write the inequality using $C$. |
| $0 \leq$ | $\frac{5}{9}(F - 32)$ | $\leq 35$ | Substitute $\frac{5}{9}(F - 32)$ for $C$. |
| $\frac{9}{5} \cdot 0 \leq$ | $\frac{9}{5} \cdot \frac{5}{9}(F - 32) \leq$ | $\frac{9}{5} \cdot 35$ | Multiplication Property of Inequality |
| $0 \leq$ | $F - 32 \leq$ | $63$ | Simplify. |
| $+ 32$ | $+ 32$ | $+ 32$ | Addition Property of Inequality |
| $32 \leq$ | $F$ | $\leq 95$ | Simplify. |

▶ A solution is $32 \leq F \leq 95$. So, the operating temperature range of the smartphone is 32°F to 95°F. Someone might leave the phone in a car on a hot day, where temperatures can exceed 150°F.
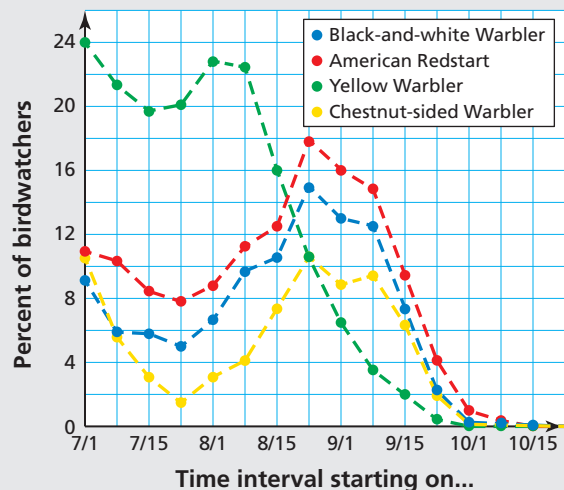
**Check**
You can use the formula $C = \frac{5}{9}(F - 32)$ to check that your answer is correct. Substitute 32 and 95 for $F$ in the formula to verify that 0°C and 35°C are the minimum and maximum operating temperatures in degrees Celsius. ✓

Operating temperature:
0°C to 35°C

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**SELF-ASSESSMENT**  [1] I do not understand.  [2] I can do it with help.  [3] I can do it on my own.  [4] I can teach someone else.

10. A pair of winter boots are rated for temperatures from −40°C to 15°C. Write an inequality that represents the temperature rating (in degrees Fahrenheit) of the boots.

11. Birdwatchers record the types of birds they see or hear. The graph shows results from a location in Canada. Write an inequality that represents the range in the percents of birdwatchers who saw or heard a Black-and-white Warbler from July 1 to September 15.



- Black-and-white Warbler
- American Redstart
- Yellow Warbler
- Chestnut-sided Warbler

Percent of birdwatchers
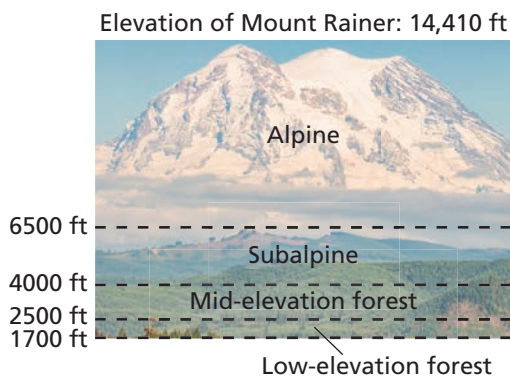
Time interval starting on...

**In Exercises 1–4, write the sentence as an inequality. Graph the inequality.** ▶ *Example 1*

1. A number $p$ is less than 6 and greater than 2.

2. A number $n$ is less than or equal to $-7$ or greater than 12.

3. A number $m$ is more than $-7\frac{2}{3}$ or at most $-10$.

4. A number $r$ is no less than $-1.5$ and fewer than 9.5.

**In Exercises 5–12, solve the inequality. Graph the solution.** ▶ *Examples 2 and 3*

5. $6 < x + 5 \le 11$

6. $24 > -3r \ge -9$

7. $v + 8 < 3 \ or \ -8v < -40$

8. $-14 > w + 3 \ or \ 3w \ge -27$

9. $\frac{1}{2}r + 3 < \frac{7}{4} \ or \ -r + \frac{3}{4} \le \frac{3}{8}$

10. $-6.2 < 2n + 8.6 < 21.4$

11. $-12 < \frac{1}{2}(4x + 16) < 18$

12. $35 < 7(2 - b) \ or \ \frac{1}{3}(15b - 12) \ge 21$

13. **MODELING REAL LIFE** The life zones on Mount Rainier, a mountain in Washington, can be approximately classified by elevation, as follows.

Elevation of Mount Rainer: 14,410 ft

Alpine

6500 ft
Subalpine
4000 ft
Mid-elevation forest
2500 ft
1700 ft
Low-elevation forest

Write an inequality that represents the elevation range for each type of plant life.

a. trees in the low-elevation forest zone

b. flowers in the subalpine and alpine zones

14. **ERROR ANALYSIS** Describe and correct the error in solving the inequality and graphing the solution.

✗
$4 < -2x + 3 < 9$
$4 < -2x < 6$
$-2 > x > -3$

(number line from $-4$ to $0$, open circles at $-3$ and $-2$)

15. **MODELING REAL LIFE** Write an inequality that represents the temperatures (in degrees Fahrenheit) of the interior of the iceberg. ▶ *Example 4*

$-20°C$ to $-15°C$

16. **MODELING REAL LIFE** A melting point is the temperature at which a solid melts to become a liquid. A boiling point is the temperature at which a liquid boils to become a gas. The table shows the melting and boiling points of several elements.

| Element | Melting point (°C) | Boiling point (°C) |
|---|---|---|
| Gold | 1064 | 2807 |
| Silver | 962 | 2212 |
| Copper | 1083 | 2567 |

a. Write an inequality that represents the temperatures (in degrees Fahrenheit) of each element as a liquid.

b. Describe a situation in which someone might need to know the melting point of one of these elements.

**In Exercises 17–22, solve the inequality. Graph the solution, if possible.**

17. $22 < -3c + 4 < 14$

18. $2m - 1 \ge 5 \ or \ 5m > -25$

19. $-y + 3 \le 8 \ and \ y + 2 > 9$

20. $x - 8 \le 4 \ or \ 2x + 3 > 9$

21. $\frac{3}{2}n + 19 \le 10 + \frac{1}{2}n \ or \ -\frac{2}{3}n + 3 < -\frac{1}{3}n + 12$

22. $3.5x - 18 < 4.5x - 23 \ and \ 9.5x - 16 < 22$

**23.** **MP** **PROBLEM SOLVING** A ski shop sells skis with lengths ranging from 150 centimeters to 220 centimeters. The shop says the length of the skis should be about 1.16 times a skier's height (in centimeters). Write an inequality that represents the heights of skiers (in inches) for which the shop does *not* provide skis.

**24.** **HOW DO YOU SEE IT?**
The graph shows the annual profits of a company over 8 years. Write an inequality that represents the annual profits from 2013 to 2020.

**Annual Profit**
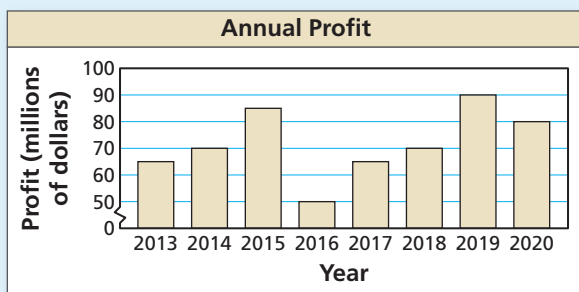


**25.** **DIG DEEPER** Determine the value of $k$ for which the inequality $0.5 < -4x + k \leq 12 - k$ has the solution set $\{x \mid 1.25 \leq x < 2\}$.

**26.** **THOUGHT PROVOKING**
Complete the inequality
$$4(x - 6) \quad \boxed{\phantom{xx}} \quad 2(x - 10)$$
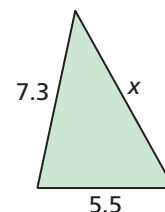*and*
$$5(x + 2) \geq 2(x + 8)$$
with $<$, $\leq$, $>$, or $\geq$ so that the solution is only one value.

**27.** **MAKING AN ARGUMENT**
The sum of the lengths of any two sides of a triangle is greater than the length of the third side. Write three inequalities that represent the possible values of $x$. Your friend claims the value of $x$ can be 1.5. Is your friend correct? Explain.



**28.** **PERFORMANCE TASK** You and your friends want to go on a road trip. You establish a round-trip fuel budget of $100, and plan to use a car with a fuel range of 25–34 miles per gallon. Use current gasoline prices to determine the distances you can travel. Then plan a trip to a city within the allotted distance from your location. Use inequalities to represent how much you will spend on gasoline, the maximum speeds at which you can travel, and how long the trip will take.

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## REVIEW & REFRESH

**In Exercises 29–32, solve the equation. Graph the solutions, if possible.**

**29.** $\left|\dfrac{d}{9}\right| = 6$

**30.** $7|5p - 7| = -21$

**31.** $|r + 2| = 9.4$

**32.** $\left|\dfrac{1}{2}w - 6\right| = |w + 7|$

**33.** The data shows the ages (in months) of children in a daycare group. Find and interpret the mean absolute deviation of the data.

$$24, \ 32, \ 36, \ 30, \ 28, \ 30, \ 34, \ 32, \ 26, \ 28$$

**34.** **MODELING REAL LIFE** You have quarters, nickels, and dimes that total $1.85. You have twice as many nickels as dimes, and 2 more quarters than dimes. How many of each coin do you have?

**In Exercises 35 and 36, write an inequality that represents the graph.**

**35.**



**36.**



**37.** **MODELING REAL LIFE** You need an average exam score of at least 84% to receive a B in a class. For what scores on the fourth exam will you receive a B in the class?

| Exam | Score |
|------|-------|
| 1 | 78% |
| 2 | 93% |
| 3 | 82% |

**In Exercises 38–41, solve the inequality. Graph the solution.**

**38.** $9.4 + q \leq 15.2$

**39.** $z - \left(-\dfrac{3}{5}\right) > \dfrac{7}{20}$

**40.** $2 \leq -\dfrac{2}{9}x$

**41.** $2x \geq \dfrac{3}{4}$

**42.** **MP** **REASONING** Explain how you could predict the number of times you will spin a 4 in 50 spins on the spinner shown.