Solving Multi-Step Equations

Learning Target   Write and solve multi-step linear equations.

Success Criteria
- I can apply more than one property of equality to produce equivalent equations.
- I can solve multi-step linear equations using inverse operations.
- I can write multi-step linear equations that model real-life situations.

EXPLORE IT! Solving a Real-Life Problem

Work with a partner. You earn $9.75 per hour at your part-time job. Your paycheck for last week is shown.

Mobile Deposit

Pay This Amount
$286.65

CONSTRUCTION COMPANY
17999 Anytown Rd.
Anywhere, Ohio 79643

Check
Pay Date

Pay This Amount
$286.65

TWO HUNDRED EIGHTY-SIX DOLLARS AND 65 CENTS

Pay To The
Order Of
JANE DOE
7251 FLOWER TOWN AVE
ANYWHERE, OH 79643

VOID AFTER 90 DAYS

John Smith

a. How many hours did you work last week? Explain how you found your answer.

b. This week you earn the same amount as last week, but that amount includes $39 that you earn babysitting. Without solving, determine whether you work more hours at your part-time job this week than last week. Explain your reasoning.

c. Find the number of hours you work this week at your part-time job. Show two different ways to solve.

d. The equation below represents the amount of money you will earn next week. Let h represent the number of hours you work this week.

$$9.75(h + 6) + 39 = 345.15$$

Explain what each part of the equation represents.

   i. $h + 6$ **ii.** $9.75(h + 6)$ **iii.** 39

e. Solve the equation in part (d) three different ways using each of the following as the first step.

   i. Subtract 39 from each side.

  ii. Subtract 345.15 from each side.

 iii. Divide each side by 9.75.

Compare the solution methods. Is one solution method more efficient than the other solution methods? Explain your reasoning.

Math Practice

Maintain Oversight
Does it matter which step you perform first when solving?

Solving Multi-Step Linear Equations

KEY IDEA

Solving Multi-Step Equations

To solve a multi-step equation, simplify each side of the equation, if necessary. Then use inverse operations to isolate the variable.

EXAMPLE 1   Solving a Two-Step Equation   WATCH

Solve $2.5x - 13 = 2$. Check your solution.

SOLUTION

	$2.5x - 13 = 2$	Write the equation.
Undo the subtraction. →	$\underline{+\ 13 \quad +\ 13}$	Addition Property of Equality
	$2.5x = 15$	Simplify.
Undo the multiplication. →	$\dfrac{2.5x}{2.5} = \dfrac{15}{2.5}$	Division Property of Equality
	$x = 6$	Simplify.

Check

$$2.5x - 13 = 2$$
$$2.5(6) - 13 \overset{?}{=} 2$$
$$2 = 2 \ \checkmark$$

▶ The solution is $x = 6$.

EXAMPLE 2   Combining Like Terms to Solve an Equation   WATCH

Solve $-12 = 9x - 6x + 15$. Check your solution.

SOLUTION

	$-12 = 9x - 6x + 15$	Write the equation.
	$-12 = 3x + 15$	Combine like terms.
Undo the addition. →	$\underline{-15 \qquad\ -15}$	Subtraction Property of Equality
	$-27 = 3x$	Simplify.
Undo the multiplication. →	$\dfrac{-27}{3} = \dfrac{3x}{3}$	Division Property of Equality
	$-9 = x$	Simplify.

Check

$$-12 = 9x - 6x + 15$$
$$-12 \overset{?}{=} 9(-9) - 6(-9) + 15$$
$$-12 = -12 \ \checkmark$$

▶ The solution is $x = -9$.

SELF-ASSESSMENT | **1** I do not understand. | **2** I can do it with help. | **3** I can do it on my own. | **4** I can teach someone else.

Solve the equation. Check your solution.

1. $-2n + 3 = 9$

2. $-21.5 = \frac{1}{2}c - 11$

3. $-2x - 10x + 12 = 18$

4. **COMPLETE THE SENTENCE** To solve the equation $2x + 3x + 5 = 20$, your friend first combines $2x$ and $3x$ because they are _____.

5. **MP REASONING** In Example 2, explain how you know that $-12 = 9x - 6x + 15$ and $-27 = 3x$ have the same solution.

EXAMPLE 3   **Using Structure to Solve a Multi-Step Equation**

Solve $2(1 - x) + 3 = -8$. Check your solution.

SOLUTION

Method 1 One way to solve the equation is by using the Distributive Property.

$2(1 - x) + 3 = -8$	Write the equation.
$2(1) - 2(x) + 3 = -8$	Distributive Property
$2 - 2x + 3 = -8$	Multiply.
$-2x + 5 = -8$	Combine like terms.
$\underline{\quad\quad -5 \quad -5}$	Subtraction Property of Equality
$-2x = -13$	Simplify.
$\dfrac{-2x}{-2} = \dfrac{-13}{-2}$	Division Property of Equality
$x = 6.5$	Simplify.

▶ The solution is $x = 6.5$.

> **REMEMBER**
>
> The Distributive Property states the following for real numbers a, b, and c.
>
> **Sum**
> $a(b + c) = ab + ac$
>
> **Difference**
> $a(b - c) = ab - ac$

Check

$$2(1 - x) + 3 = -8$$
$$2(1 - 6.5) + 3 \overset{?}{=} -8$$
$$-8 = -8 \checkmark$$

Method 2 Another way to solve the equation is by interpreting the expression $1 - x$ as a single quantity.

$2(1 - x) + 3 = -8$	Write the equation.
$\underline{\quad\quad -3 \quad -3}$	Subtraction Property of Equality
$2(1 - x) = -11$	Simplify.
$\dfrac{2(1 - x)}{2} = \dfrac{-11}{2}$	Division Property of Equality
$1 - x = -5.5$	Simplify.
$\underline{\quad -1 \quad\quad -1}$	Subtraction Property of Equality
$-x = -6.5$	Simplify.
$\dfrac{-x}{-1} = \dfrac{-6.5}{-1}$	Division Property of Equality
$x = 6.5$	Simplify.

▶ The solution is $x = 6.5$.

> **Math Practice**
>
> **Look for Structure**
> Explain why it is convenient to first solve for the expression $1 - x$, and then solve for x. How else could you solve the equation?

SELF-ASSESSMENT   1   I do not understand.   2   I can do it with help.   3   I can do it on my own.   4   I can teach someone else.

Solve the equation. Check your solution.

6. $3(x + 1) + 6 = -9$

7. $17 = 7 + 4(2.2d - 8.5)$

8. $13 = -2(y - 4) + 3y$

9. $2x(5 - 3) - 3x = 5$

10. $-4(2m + 5) - \frac{3}{4}m = 22$

11. $5(3 - x) + 2(3 - x) = 14$

12. **MP REASONING** Solve $2(4x - 11) = 10$ in as many ways as you can. Construct a viable argument to justify each of your solution methods.

Solving Real-Life Problems

EXAMPLE 4   **Modeling Real Life**   WATCH

Use the table to find the number of miles you need to bike on Friday so that the mean number of miles biked per day is 5.

Day	Miles
Monday	3.5
Tuesday	5.5
Wednesday	0
Thursday	5

SOLUTION

1. **Understand the Problem** You know how many miles you biked Monday through Thursday. You are asked to find the distance you need to bike on Friday so that the mean number of miles biked per day is 5.

2. **Make a Plan** Use the definition of mean to write an equation that represents the problem. Then solve the equation.

3. **Solve and Check** The mean of a data set is the sum of the data divided by the number of data values. Let x be the number of miles you need to bike on Friday.

$$\frac{3.5 + 5.5 + 0 + 5 + x}{5} = 5 \qquad \text{Write the equation.}$$

$$\frac{14 + x}{5} = 5 \qquad \text{Combine like terms.}$$

$$5 \cdot \frac{14 + x}{5} = 5 \cdot 5 \qquad \text{Multiplication Property of Equality}$$

$$14 + x = 25 \qquad \text{Simplify.}$$

$$\underline{-14 \qquad\qquad -14} \qquad \text{Subtraction Property of Equality}$$

$$x = 11 \qquad \text{Simplify.}$$

▶ You need to bike 11 miles on Friday.

Check Reasonableness Notice that on the days that you did bike, the values are close to the mean. Because you did not bike on Wednesday, you need to bike about twice the mean on Friday. Eleven miles is about twice the mean. So, your solution is reasonable.
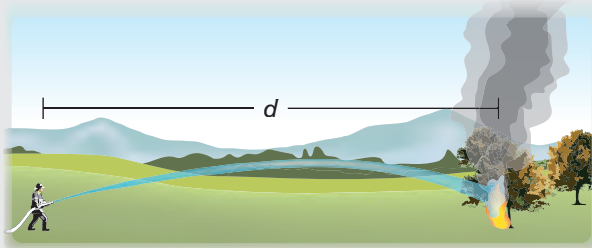

SELF-ASSESSMENT [1] I do not understand. [2] I can do it with help. [3] I can do it on my own. [4] I can teach someone else.

13. The formula $d = \frac{1}{2}n + 26$ relates the nozzle pressure n (in pounds per square inch) of a fire hose and the maximum horizontal distance d (in feet) the water reaches. How much pressure is needed to reach a fire 20 yards away?

EXAMPLE 5   Modeling Real Life

One person buys a used car in Indiana and pays $10,195, including 7% sales tax and $425 in additional fees. Another person buys a used car in Pennsylvania and pays $9995, including 6% sales tax and $420 in additional fees. Compare the list prices of the used cars. (The list price is the price of the car before sales tax and fees.)

SOLUTION

1. **Understand the Problem** You know how much each person pays for a car. You also know the sales tax and additional fees in each state. You are asked to compare the list prices of the cars.

2. **Make a Plan** Use a verbal model to write equations that represent the total amount each person pays. Then solve the equations to find each list price.

3. **Solve and Check**

Verbal Model | List price | $+$ | Sales tax rate (written as a decimal) | $\cdot$ | List price | $+$ | Other fees | $=$ | Total amount paid

Variable Let p be the list price (in dollars) of the used car.

Equations

Indiana:		Pennsylvania:
$p + 0.07p + 425 = 10{,}195$	Write the equation.	$p + 0.06p + 420 = 9995$
$1.07p + 425 = 10{,}195$	Combine like terms.	$1.06p + 420 = 9995$
$\underline{-425 \quad -425}$	Subtraction Property of Equality	$\underline{-420 \quad -420}$
$1.07p = 9770$	Simplify.	$1.06p = 9575$
$\dfrac{1.07p}{1.07} = \dfrac{9770}{1.07}$	Division Property of Equality	$\dfrac{1.06p}{1.06} = \dfrac{9575}{1.06}$
$p \approx 9130.84$	Simplify.	$p \approx 9033.02$

> **REMEMBER**
> To write a percent as a decimal, remove the percent symbol and divide by 100.

▶ So, the list price of the car in Indiana is about $9130.84 − $9033.02 = $97.82 more than the list price of the car in Pennsylvania.

Check

$$p + 0.07p + 425 = 10{,}195 \qquad\qquad p + 0.06p + 420 = 9995$$

$$9130 + 0.07(9130) + 425 \overset{?}{\approx} 10{,}195 \qquad 9030 + 0.06(9030) + 420 \overset{?}{\approx} 9995$$

$$10{,}194.10 \approx 10{,}195 \ ✓ \qquad\qquad 9991.80 \approx 9995 \ ✓$$

SELF-ASSESSMENT | 1 I do not understand. | 2 I can do it with help. | 3 I can do it on my own. | 4 I can teach someone else.

14. You have 96 feet of fencing to enclose a rectangular pen for your dog. To provide sufficient running space for your dog to exercise, the pen should be three times as long as it is wide. Find the dimensions of the pen.

15. You are paid 1.2 times your normal hourly rate for each hour you work over 40 hours in a week. You work 46 hours this week and earn $462.56. What is your normal hourly rate?

In Exercises 1–12, solve the equation. Check your solution. ▶ *Examples 1 and 2*

1. $3w + 7 = 19$

2. $2g - 13 = 3$

3. $11 = 12 - q$

4. $10 = 7 - m$

5. $5 = \dfrac{z}{-4} - 3$

6. $\dfrac{a}{3} + 4 = 6$

7. $\dfrac{h + 6}{5} = 2$

8. $\dfrac{d - 8}{-2} = 12$

9. $12v + 10v + 14 = 80$

10. $24 = 13n - 4n + 9$

11. $3.8y + 5.6y - 2 = 2.7$

12. $\dfrac{7}{10}c - 8 - \dfrac{1}{2}c = -16$

13. MODELING REAL LIFE The altitude a (in feet) of a plane t minutes after liftoff is given by $a = 3400t + 600$. How many minutes after liftoff is the plane at an altitude of 21,000 feet?

14. MODELING REAL LIFE A repair bill for a car is $648.45. The parts cost $265.95. The labor cost is $85 per hour. Write and solve an equation to find the number of hours of labor spent repairing the car.
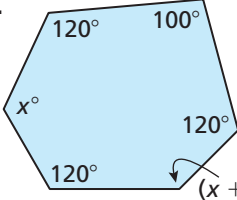
In Exercises 15–22, solve the equation. Check your solution. ▶ *Example 3*

15. $4(z + 5) = 32$

16. $-2(4g - 3) = 30$

17. $6 + 5(m + 1) = 26$

18. $5h + 2(11 - h) = -5$

19. $-15 = -6(3 + x) + 4(x - 6)$

20. $1 = 5(r + 9) - 2(1 - r)$

21. $83.8 = 8.6c - 7.3(6 - 2c)$

22. $3y - 2\dfrac{3}{4}\left(\dfrac{1}{2}y - 4\right) = -2$
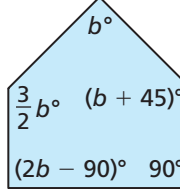
MP **NUMBER SENSE** In Exercises 23–28, write and solve an equation to find the number.

23. The sum of twice a number and 13 is 75.

24. The difference of three times a number and 4 is -19.

25. Eight plus the quotient of a number and 3 is -2.

26. The sum of twice a number and half the number is 10.

27. Six times the sum of a number and 15 is -42.

28. Four times the difference of a number and 7 is 12.

MP **USING TOOLS** In Exercises 29 and 30, find the value of the variable. Then find the angle measures of the polygon. Use a protractor to check the reasonableness of your answer.

29.

120°   100°

$x°$

120°

120°

$(x + 10)°$

Sum of angle measures: 720°

30.

$b°$

$\dfrac{3}{2}b°$ $(b + 45)°$

$(2b - 90)°$ 90°

Sum of angle measures: 540°

ERROR ANALYSIS In Exercises 31 and 32, describe and correct the error in solving the equation.

31.

✗

$-2(7 - y) + 4 = -4$

$-14 - 2y + 4 = -4$

$-10 - 2y = -4$

$-2y = 6$

$y = -3$

32.

✗

$\dfrac{1}{4}(x - 2) + 4 = 12$

$\dfrac{1}{4}(x - 2) = 8$

$x - 2 = 2$

$x = 4$

MODELING REAL LIFE In Exercises 33–36, write and solve an equation to answer the question.

▶ *Examples 4 and 5*

33. During the summer, you work 30 hours per week at a gas station and earn $8.75 per hour. You also work as a landscaper for $11 per hour and can work as many hours as you want. You want to earn a total of $400 per week. How many hours must you work as a landscaper?

34. The area of the surface of the swimming pool is 210 square feet. What is the length of the deep end?
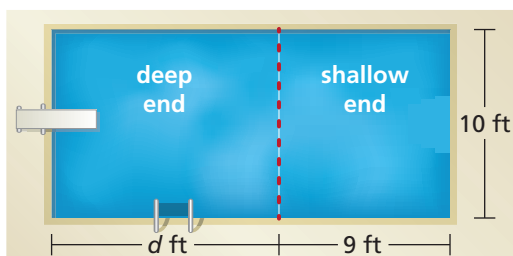

35. Your cell phone has 983.5 MB of free space. You save a 1.4-MB picture and download two songs that are the same size. Your cell phone now has 974.9 MB of free space. What is the size of each song?

36. You order two tacos and a salad. The salad costs $2.50. You pay 8% sales tax and leave a $3 tip. You pay a total of $13.80. How much does one taco cost?

CONNECTING CONCEPTS In Exercises 37 and 38, write and solve an equation to answer the question.

37. The perimeter of the Puerto Rican flag is 150 inches. What are the dimensions of the flag?
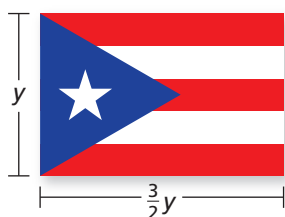
38. The perimeter of the school crossing sign is 102 inches. What is the length of each side?
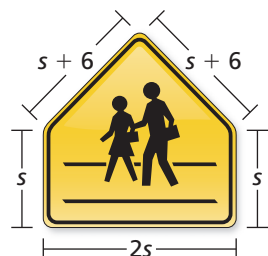
JUSTIFYING STEPS In Exercises 39 and 40, justify each step of the solution.

39. $-\frac{1}{2}(5x - 8) - 1 = 6$ Write the equation.

$-\frac{1}{2}(5x - 8) = 7$ [_____]

$5x - 8 = -14$ [_____]

$5x = -6$ [_____]

$x = -\frac{6}{5}$ [_____]

40. $2(x + 3) + x = -9$ Write the equation.

$2(x) + 2(3) + x = -9$ [_____]

$2x + 6 + x = -9$ [_____]
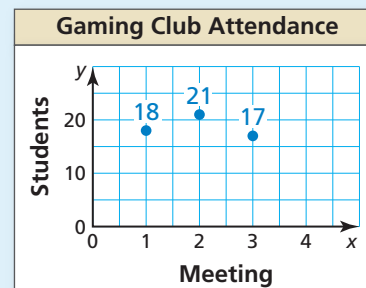
$3x + 6 = -9$ [_____]

$3x = -15$ [_____]

$x = -5$ [_____]

41. **COMPARING METHODS** Solve the equation $2(4 - 8x) + 6 = -1$ using two different methods. Which method do you prefer? Explain.

42. **HOW DO YOU SEE IT?**
The scatter plot shows the attendance for each meeting of a gaming club.

a. The mean attendance for the first four meetings is 20. Is the number of students who attended the fourth meeting greater than or less than 20? Explain.

b. Estimate the number of students who attended the fourth meeting. Describe a way you can check your estimate.

43. **MP** **PROBLEM SOLVING** An online ticket agency charges the amounts shown for basketball tickets. The total cost for an order is $220.70. How many tickets are purchased?

Charge	Amount
Ticket price	$32.50 per ticket
Convenience charge	$3.30 per ticket
Processing charge	$5.90 per order

44. **MAKING AN ARGUMENT** You have quarters and dimes that total $2.80. Your friend says it is possible that the number of quarters is 8 more than the number of dimes. Is your friend correct? Explain.

MP **REASONING** In Exercises 45–48, the letters a, b, and c represent nonzero constants. Solve the equation for x. Then find values of a, b, and c for which the solution is negative.

45. $ax - b = 12.5$

46. $ax + b = c$

47. $2bx - bx = -8$

48. $cx - 4b = 5b$

49. **DIG DEEPER** Find three consecutive even integers that have a sum of 54. Explain your reasoning.

50. **THOUGHT PROVOKING**
Your math teacher assigns a weight to each component of the class. The weight of the final exam is half your grade, and the weights of the remaining components are equal. What is the least possible score you can receive on the final exam to earn an A (90%) in the class? Explain your reasoning.

| Component	Your score	Weight	Score × Weight
Class participation	92%		
Homework	96%		
Quizzes	88%		
Midterm exam	76%		
Final exam			
Total		1	

REVIEW & REFRESH

In Exercises 51–54, find the sum or difference.

51. $-3.37 + 4.135$

52. $1\frac{3}{8} - \frac{7}{8}$

53. $18.36 - (-9.04)$

54. $-\frac{5}{12} + \left(-\frac{7}{3}\right)$

55. **MODELING REAL LIFE** About how many times farther from the Sun is Neptune than Mercury?

Planet	Average distance from the Sun (miles)
Mercury	36,000,000
Neptune	2.795×10^9

56. Order the numbers $\frac{11}{20}$, 49%, and 0.5 from least to greatest.

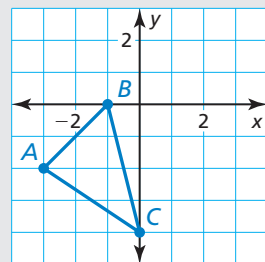**57.** Find the perimeter and the area of the figure.
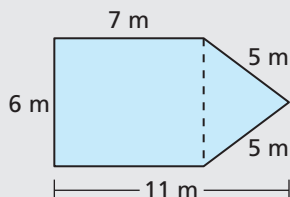
58. **MP** **NUMBER SENSE** The sum of two-thirds a number and eighteen is twenty-three. What is the number?

In Exercises 59–62, solve the equation. Check your solution.

59. $x + 9 = 7$

60. $8.6 = z - 3.8$

61. $3r + 7 = 11$

62. $26 = 9p - 6 - p$

63. Translate the triangle 1 unit right and 3 units up. What are the coordinates of the image?

64. **MODELING REAL LIFE** Your friend borrows $7500 to buy an all-terrain vehicle (ATV). The simple annual interest rate is 6%. She pays off the loan after 5 years of equal monthly payments. How much is each payment?

65. Factor $24x + 32$ using the greatest common factor.