

Section 10.1**Lesson 1: Lines and Segments That Intersect Circles**

In this lesson, students will work in pairs to sort *Circle Vocabulary Cards* to match vocabulary words with their corresponding definitions and diagrams. Each student pair will check their answers with another pair and create their own diagram including as many vocabulary words as possible. Groups will participate in a gallery walk to give feedback to other groups about their diagrams, and answer the questions provided on the diagrams.

Section 10.2**Lesson 2: Finding Arc Measures**

In this lesson, students will participate in a *Circle-Storm* in pairs to brainstorm definitions for given circle vocabulary words. Students will share their understandings through a class discussion and formalize the definitions of each vocabulary word onto foldables. Students will apply their understandings by completing practice problems in pairs.

Section 10.3**Lesson 3: Using Chords**

In this lesson, students will rotate through *Circle Stations* in groups to explore the Congruent Corresponding Chords Theorem (Theorem 10.6), the Perpendicular Chord Bisector Theorem (Theorem 10.7), and the Perpendicular Chord Bisector Converse (Theorem 10.8) using compasses, rulers, protractors, and tracing paper. Students will participate in a discussion to summarize their learning, and then complete culminating constructions to demonstrate their knowledge.

Section 10.4**Lesson 4: Inscribed Angles and Polygons**

In this lesson, students will explore relationships of inscribed angles and polygons in pairs using dynamic geometry software or by hand. Students will then form groups of four to share their findings and discuss the strengths and weaknesses of each method. Students will summarize their understandings through a class discussion.

Section 10.5**Lesson 5: Angle Relationships in Circles**

In this lesson, students will complete Explorations 1 and 2 in their textbooks. Students will apply their understandings of the Tangent and Intersected Chord Theorem (Theorem 10.14) and the Angles Inside the Circle Theorem (Theorem 10.15) by playing the *Quiz-Quiz-Take* game.

Section 10.6

Lesson 6: Segment Relationships in Circles

In this lesson, students will work in pairs to complete a class investigation of segment relationships in circles. Students will use the class data to informally prove the Segments of Chords Theorem (Theorem 10.18) and the Segments of Secants Theorem (Theorem 10.19). Students will demonstrate their knowledge by applying the theorems to complete practice problems.

Section 10.7

Lesson 7: Circles in the Coordinate Plane

In this lesson, students will work in groups to derive the formula for circles in the coordinate plane. Students will apply their knowledge through participating in a matching activity called *What's My Equation?*

Lesson Preparation

Materials: Resource Sheet 1: Circle Vocabulary Cards, compasses, straightedges, envelopes, sticky notes

Beforehand: Make copies of Resource Sheet 1 (one per pair of students). Cut each sheet into a set of cards, shuffle them, and place each set of cards in an envelope.

Classroom Management: Strategically arrange students into pairs. Pairs will eventually combine and form groups of four students, so consider determining these groups ahead of time to maximize learning.

Lesson Procedure

Arrange students into pairs and distribute an envelope that contains a set of *Circle Vocabulary Cards* from Resource Sheet 1. Say, “Today, we are going to review circle vocabulary that we will be using throughout Geometry—Chapter 10.” Direct each pair to sort their cards from their envelope by matching each correct trio (vocabulary word, definition, and diagram).

Instruct each pair to combine with another pair to form a group of four students. Have students share their matches and discuss any differences. Allow time for students to make any corrections to their matches.

Lead a class discussion about which vocabulary words were most challenging and why. Allow students time to ask questions and address any misconceptions.

Provide groups with plain paper, compasses, and straightedges. Challenge each group to create a diagram that uses as many of the circle vocabulary words as possible. Have each group write a series of questions about their diagram and create an answer key to accompany their work. Remind students to be precise in their notation and to use correct vocabulary. Circulate as groups work to provide support to struggling students.

Extend students’ understanding by having groups participate in a gallery walk to examine other groups’ diagrams and answer the questions provided. Distribute sticky notes to allow students to leave feedback for each group, including any errors in the answer key. Allow time for students to return to their diagrams to examine their own feedback. Circulate as groups work and address any misconceptions as a class.

Assessment

Monitor students’ responses as they rotate to each diagram and answer the questions provided. Use this information to assess whether students are prepared to complete the exercises in the textbook (Geometry—Chapter 10, Lesson 1).

Lesson Connections

Connections to the Standards for Mathematical Practice:

MP.4 Model with mathematics.

MP.5 Use appropriate tools strategically.

MP.6 Attend to precision.

Connections to Universal Design for Learning:

Checkpoint 1.1 Offer ways of customizing the display of information.

Checkpoint 1.3 Offer alternatives for visual information.

Checkpoint 2.1 Clarify vocabulary and symbols.

Checkpoint 3.2 Highlight patterns, critical features, big ideas, and relationships.

Checkpoint 4.1 Vary the methods for response and navigation.

Checkpoint 5.2 Use multiple tools for construction and composition.

Checkpoint 6.3 Facilitate managing information and resources.

Checkpoint 7.3 Minimize threats and distractions.

Checkpoint 8.3 Foster collaboration and community.

Connections to the Depths of Knowledge:

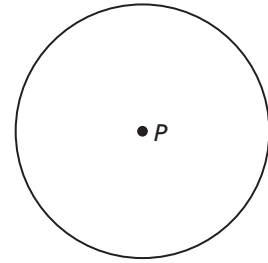
Level 2 Working With Skills and Concepts

Level 3 Short-Term Strategic Thinking

Circle Vocabulary Cards

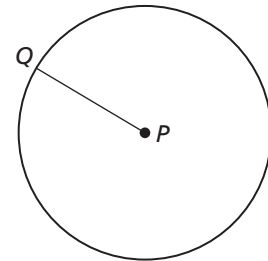
circle

the set of all points equidistant from a given point called the center



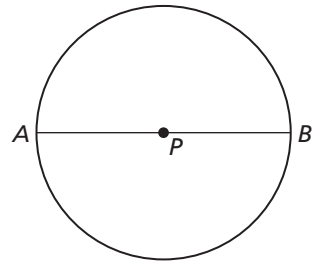
radius

a segment whose endpoints are the center and any point on a circle



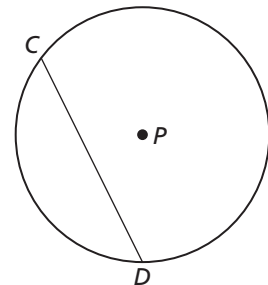
diameter

a chord that contains the center of the circle



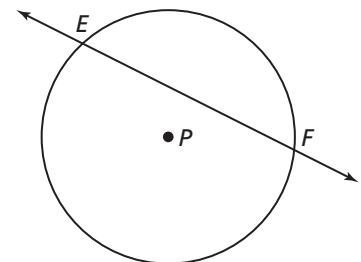
chord

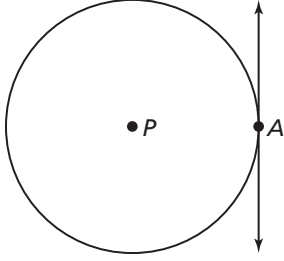
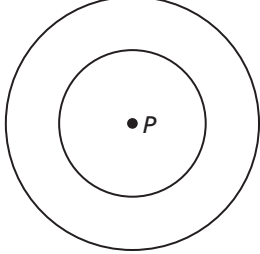
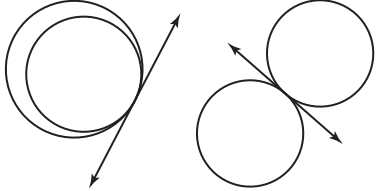
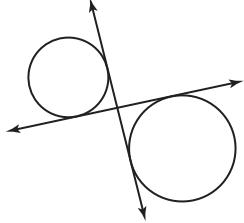
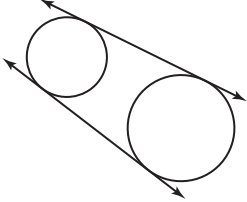
a segment whose endpoints are on a circle



secant

a line that intersects a circle in two points



<p>tangent</p>	<p>a line in the plane of a circle that intersects the circle in exactly one point</p>	
<p>concentric circles</p>	<p>coplanar circles that have a common center</p>	
<p>tangent circles</p>	<p>coplanar circles that intersect in one point</p>	
<p>common internal tangent</p>	<p>a line or segment that is tangent to two circles and intersects the segment that joins the centers of the two circles</p>	
<p>common external tangent</p>	<p>a line or segment that is tangent to two circles that does not intersect the segment that joins the centers of the two circles</p>	

Lesson 2

Finding Arc Measures

For use with Section 10.2

Lesson Preparation

Materials: Resource Sheet 2: Circle-Storm, Worksheet 2: Applying Circle Vocabulary, plain paper/construction paper, compasses, document camera

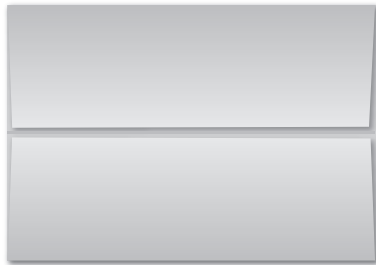
Beforehand: Make a copy of Resource Sheet 2 to display at the front of the room and make copies of Worksheet 2 (one per pair of students). Consider creating an example foldable for students to refer to throughout the activity.

Classroom Management: Strategically arrange students into pairs.

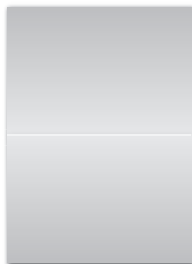
Lesson Procedure

Arrange students into pairs and display Resource Sheet 2 using a document camera. Ask students to participate in a *Circle-Storm*, or a brainstorm of the definitions of the given circle vocabulary. Provide students with compasses to construct circles if necessary.

Say, “Now you are going to formalize the definitions of the vocabulary words in your own foldable to refer to throughout the rest of the unit.” Distribute a sheet of plain paper or construction paper to each student. Have students fold their papers in half vertically. Then have students open up their papers and fold the ends toward the middle crease to divide their papers into fourths, as shown.



Ask students to fold their papers in half the opposite direction (horizontally), as shown.



Ask students to fold their papers in half again (horizontally) to divide their papers into fourths, as shown.



Ask students to unfold their papers to reveal eight sections. Ask students to cut along the eight folded creases of their papers to create the eight sections, as shown



Ask students to label each tab of their papers with a vocabulary word, similar to the picture shown.

circular arc	central angle
minor arc	semicircle
major arc	congruent circles
congruent arcs	adjacent arcs

Lead a class discussion about the definition of each vocabulary word. Have students share their definitions and diagrams before writing the formal definitions of the vocabulary words onto their foldable under the appropriate tab. Address any misconceptions as students share their brainstormed definitions. [Refer to the diagrams and definitions found in Geometry—Chapter 10, Lesson 2.] Ensure students understand that the measure of a minor arc is equal to the measure of its central angle, and ensure they include the correct notation.

2 Finding Arc Measures (continued)

Explain to students that they will work with their partners to apply the circle vocabulary. Distribute a copy of Worksheet 2 to each pair of students, and instruct each pair of students to use the diagram provided to answer the questions. Remind students they can use the foldables they created to help them. Circulate to provide support to struggling students.

Summarize understandings by reviewing the answers as a class and address any misconceptions.

Assessment

After completing the activity, students will be prepared to complete the exercises in the textbook (Geometry—Chapter 10, Lesson 2).

Lesson Connections

Connections to the Standards for Mathematical Practice:

- MP.4 Model with mathematics.
- MP.5 Use appropriate tools strategically.
- MP.6 Attend to precision.

Connections to Universal Design for Learning:

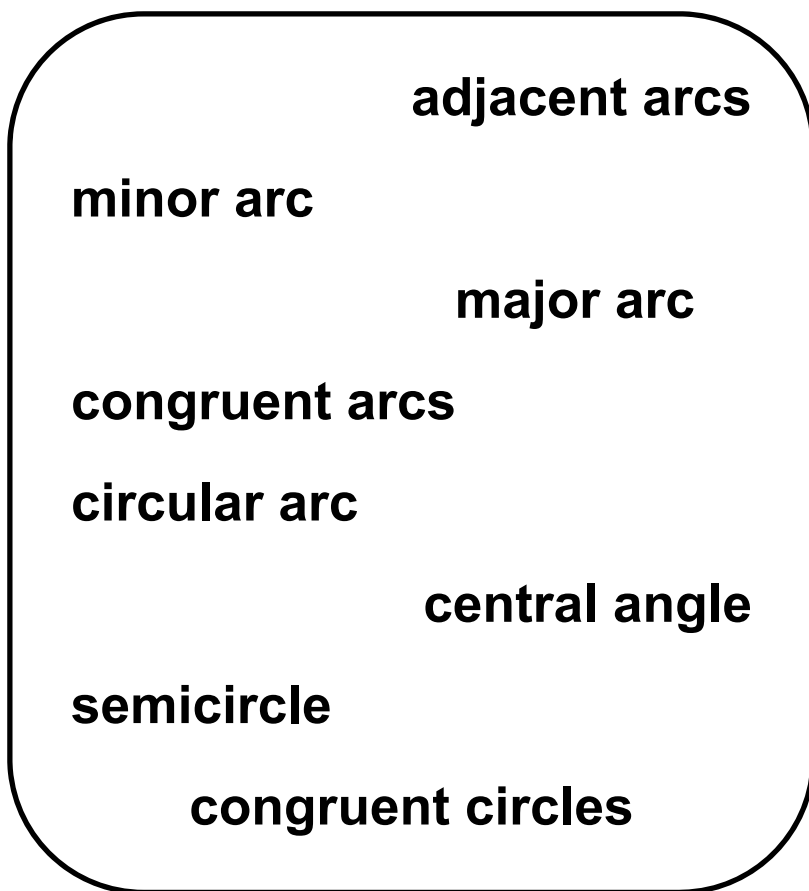
- Checkpoint 1.3 Offer alternatives for visual information.
- Checkpoint 2.1 Clarify vocabulary and symbols.
- Checkpoint 3.1 Activate or supply background knowledge.
- Checkpoint 3.2 Highlight patterns, critical features, big ideas, and relationships.
- Checkpoint 3.4 Maximize transfer and generalization.
- Checkpoint 4.1 Vary the methods for response and navigation.
- Checkpoint 7.3 Minimize threats and distractions.
- Checkpoint 8.3 Foster collaboration and community.

Connections to the Depths of Knowledge:

- Level 1 Recall and Reproduction
- Level 2 Working With Skills and Concepts

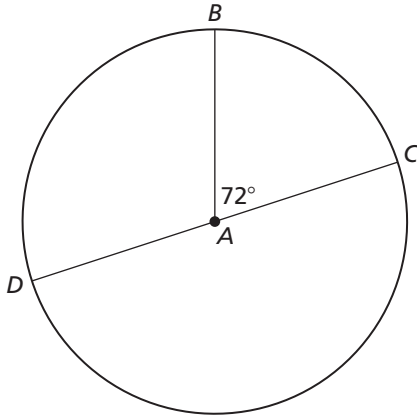
Circle-Storm

With your partner, brainstorm the definitions of the circle vocabulary words shown. If needed, draw a diagram to accompany your definitions.



Worksheet
2**Applying Circle Vocabulary**

Use the diagram. \overline{DC} is the diameter of $\odot A$.



1. Find $m\widehat{BC}$. Is \widehat{BC} a major arc or a minor arc? Explain your reasoning.
2. Find $m\widehat{BD}$. Explain how you found your answer.
3. Give an example of a major arc and determine its measure.
4. How can you describe \widehat{DBC} ?
5. Find $m\widehat{BCD}$. Explain how you found your answer.

Lesson Preparation

Materials: Resource Sheet 3A: Circle Stations, Resource Sheet 3B: Circle Stations Record Sheet, tracing paper (or wax paper/patty paper), rulers, compasses, protractors

Beforehand: Make a copy of Resource Sheet 3A (or two copies for larger class sizes), and copies of Resource Sheet 3B (one per student). Arrange the stations around the room with the appropriate materials. Pre-assess students' understanding of constructing a perpendicular bisector using tracing paper.

Classroom Management: Strategically arrange the class into groups of three-to-four students based on pre-assessment information. For a class larger than 12 students, consider making two sets of each station to avoid overcrowding.

Lesson Procedure

Arrange the class into groups of three-to-four students and distribute Resource Sheet 3B. Explain to students that they will be rotating through three different stations to explore various properties of chords and each student will summarize his/her understandings on a *Circle Stations Record Sheet* (Resource Sheet 3B). Remind students to have each group member complete the station activity to see whether their conjectures will hold true for a variety of cases. Remind students to be precise in their constructions, because there will be a slight margin of error due to completing the constructions by hand, instead of using dynamic geometry software.

Assign each group to a station and set a timer for 8–10 minutes. When the time is up, have students rotate to their next assigned stations and reset the timer. Repeat the process until students have visited all three stations and have captured their ideas in their record sheets. Circulate as students work and provide support as needed.

Lead a discussion about the different station activities and have students share their knowledge. Address any misconceptions and formalize student knowledge by introducing the theorems that were proved at each station. [Station 1: Congruent Corresponding Chords Theorem (Theorem 10.6), Station 2: Perpendicular Chord Bisector Theorem (Theorem 10.7), Station 3: Perpendicular Chord Bisector Converse (Theorem 10.8)]

Assessment

Provide each student with tracing paper, a compass, and a ruler. Ask each student to construct a circle on a piece of tracing paper without marking the center. Say, "Using what you have learned during the *Circle Stations* activity, determine how to construct the center of your circle." Circulate as students work to assess whether students are prepared to complete the exercises in the textbook (Geometry—Chapter 10, Lesson 3). Lead a discussion about the various methods students used to construct the centers of their circles.

Lesson Connections**Connections to the Standards for Mathematical Practice:**

- MP.1 Make sense of problems and persevere in solving them.
- MP.2 Reason abstractly and quantitatively.
- MP.5 Use appropriate tools strategically.
- MP.6 Attend to precision.

Connections to Universal Design for Learning:

- Checkpoint 1.3 Offer alternatives for visual information.
- Checkpoint 2.1 Clarify vocabulary and symbols.
- Checkpoint 3.2 Highlight patterns, critical features, big ideas, and relationships.
- Checkpoint 3.4 Maximize transfer and generalization.
- Checkpoint 4.1 Vary the methods for response and navigation.
- Checkpoint 5.2 Use multiple tools for construction and composition.
- Checkpoint 6.3 Facilitate managing information and resources.
- Checkpoint 7.3 Minimize threats and distractions.
- Checkpoint 8.3 Foster collaboration and community.

Connections to the Depths of Knowledge:

- Level 2 Working With Skills and Concepts
- Level 3 Short-Term Strategic Thinking

Circle Stations

Circle Station 1

Materials: plain paper, compasses, rulers, protractors

Directions:

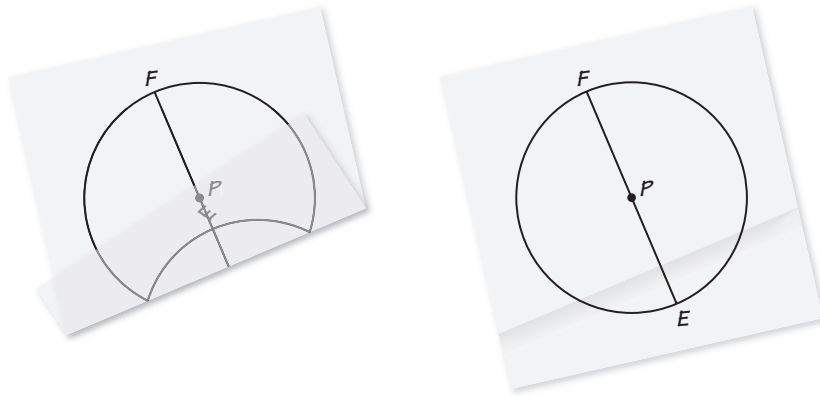
1. Construct a circle and label it $\odot P$.
2. Draw two congruent chords that are not diameters, and label the segments AB and CD .
3. Use a ruler to draw segments AP , BP , CP , and DP .
4. Measure $\angle APB$ and $\angle CPD$. What do you notice?
5. Compare your results to your group members' results. Are there additional conclusions you can draw from your diagram?
6. Summarize your understandings on your record sheet.

Circle Station 2

Materials: tracing paper, compasses, rulers

Directions:

1. Construct a circle on tracing paper and label it $\odot P$.
2. Draw a diameter and label it segment EF .
3. Fold the diameter onto itself to construct a chord perpendicular to the diameter, as shown below.



4. Repeat this construction three more times.
5. Investigate the chords using any of the tools provided. What do you notice about a chord when it is perpendicular to the diameter?
6. Compare your results to your group members' results.
7. Summarize your understandings on your record sheet.

Circle Station 3

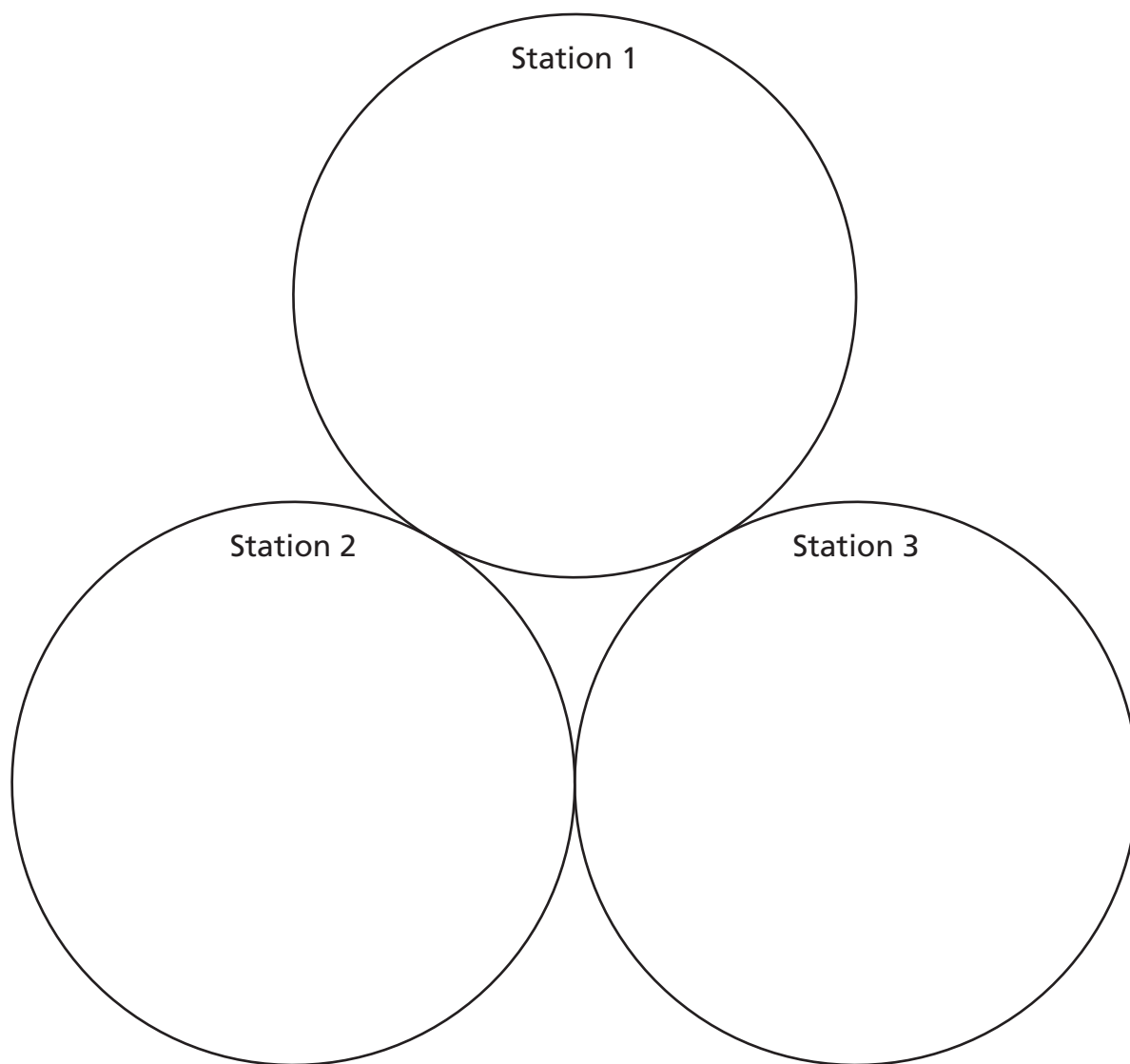
Materials: tracing paper, compasses, rulers

Directions:

1. Construct a circle on tracing paper and label it $\odot P$.
2. Draw a chord and label it segment GH .
3. Construct the perpendicular bisector of the chord. What do you notice?
4. Compare your results to your group members' results. What conclusions can you draw?
5. Summarize your understandings on your record sheet.

Circle Stations Record Sheet

As you travel to each station, summarize your knowledge in the graphic organizer.



Lesson Preparation

Materials: Resource Sheet 4: Examples and Non-Examples of Inscribed Angles and Polygons, Worksheet 4: Exploring Inscribed Angles and Polygons, textbooks, compasses, rulers, protractors, dynamic geometry software, document camera

Beforehand: Make a copy of Resource Sheet 4 to display at the front of the room. Make copies of Worksheet 4 (one per pair of students). Arrange the room for two groups and set out the necessary materials.

Classroom Management: Strategically arrange students into pairs and assign pairs to complete the exploration either by hand or by using dynamic geometry software based on the strengths of the students. Consider allowing students to choose which tools they would like to use.

Lesson Procedure

Display Resource Sheet 4 using a document camera. Ask students to independently develop their own definitions of inscribed angles and polygons based on the examples and non-examples.

Arrange students into pairs and ask partners to share their definitions with each other. Have several groups share their definitions with the class and address any misconceptions. Present students with the formal definitions of *inscribed angle* [an angle whose vertex is on the circle and whose sides contain chords of the circle] and *inscribed polygon* [when all vertices of a polygon lie on the circle] from the textbook (Geometry—Chapter 10, Lesson 4), so students can compare their definitions.

Explain to students that today they will use different tools to explore the relationship between inscribed angles and polygons. Explain that half the class will be completing Explorations 1 and 2 from the textbook (Geometry—Chapter 10, Lesson 4) using dynamic geometry software, and the other half of the class will be completing Worksheet 4 using compasses, protractors, and rulers. At the end of class, a pair from each activity will form a group to share their findings and discuss knowledge.

Assign each pair either Explorations 1 and 2 from the textbook (Geometry—Chapter 10, Lesson 4) or the exploration from Worksheet 4. Distribute the appropriate materials and remind students of the time they will have to complete the assignment. Circulate to help struggling students.

4 Inscribed Angles and Polygons (continued)

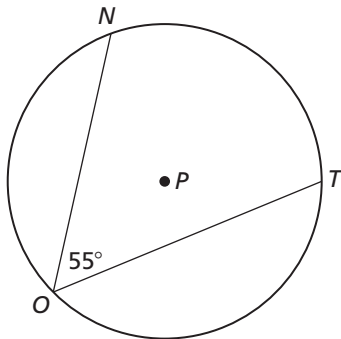
Instruct each pair that completed Explorations 1 and 2 from the textbook (Geometry—Chapter 10, Lesson 4) to form a group with one pair that completed Worksheet 4 to share findings once they have finished. Have groups discuss the strengths and weaknesses of each activity and decide which method(s) they would choose if they had to conduct the exploration again. [*Sample response:* Drawing the inscribed angles and quadrilaterals by hand only allows you to explore one case, but using dynamic geometry software allows you to test several different cases by sliding the vertices or changing the radius of the circle. By constructing the inscribed angles by hand, you reinforce the vocabulary and practice using tools, but when using dynamic geometry software, the program performs all the operations for you.]

Lead a class discussion about the strengths and weaknesses each group identified for the two activities. Have students share their findings and address any misconceptions.

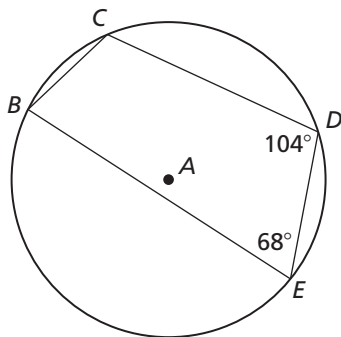
Assessment

Have students complete the problems below to assess whether they are prepared to complete the exercises in the textbook (Geometry—Chapter 10, Lesson 4).

1. Use the diagram to determine $m\widehat{NT}$.



2. Use the diagram to find the missing angles of the inscribed quadrilateral.



Lesson Connections**Connections to the Standards for Mathematical Practice:**

- MP.3 Construct viable arguments and critique the reasoning of others.
- MP.4 Model with mathematics.
- MP.5 Use appropriate tools strategically.
- MP.6 Attend to precision.

Connections to Universal Design for Learning:

- Checkpoint 1.3 Offer alternatives for visual information.
- Checkpoint 2.1 Clarify vocabulary and symbols.
- Checkpoint 3.2 Highlight patterns, critical features, big ideas, and relationships.
- Checkpoint 3.4 Maximize transfer and generalization.
- Checkpoint 4.1 Vary the methods for response and navigation.
- Checkpoint 5.2 Use multiple tools for construction and composition.
- Checkpoint 6.3 Facilitate managing information and resources.
- Checkpoint 7.3 Minimize threats and distractions.
- Checkpoint 8.3 Foster collaboration and community.

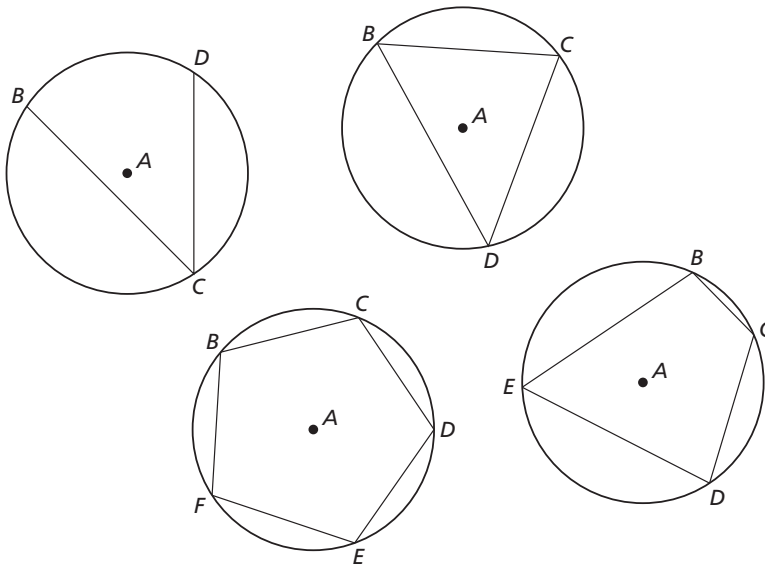
Connections to the Depths of Knowledge:

- Level 2 Working With Skills and Concepts
- Level 3 Short-Term Strategic Thinking

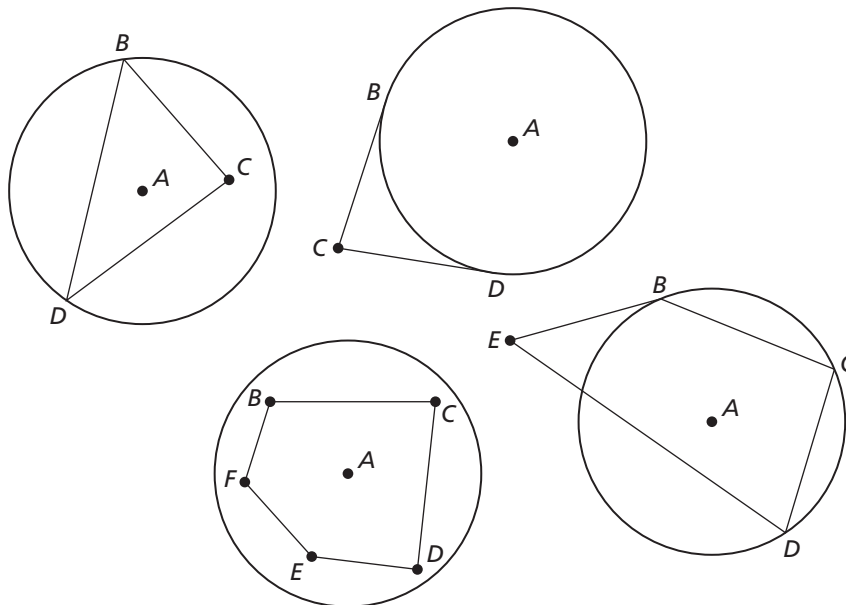
Examples and Non-Examples of Inscribed Angles and Polygons

Use the examples and non-examples to develop definitions of *inscribed angles* and *inscribed polygons*.

Examples:



Non-Examples:



Lesson 5

Angle Relationships in Circles

For use with Section 10.5

Lesson Preparation

Materials: Resource Sheet 5: Quiz-Quiz-Take Cards, dynamic geometry software, textbooks

Beforehand: Have students complete Explorations 1 and 2 from the textbook (Geometry—Chapter 10, Lesson 5) before beginning the lesson. Make copies of Resource Sheet 5, cut out each row, and fold each row in half to make cards. (**Note:** Each row represents one question and the corresponding answer.) Make enough copies so that each student will get one card. Consider laminating the cards, so students can write on the cards with dry erase markers and then erase when they finish each round.

Classroom Management: Each student will randomly be assigned a partner, and students will rotate their partners throughout the activity. If there are more than 12 students in the class, consider forming 2 groups. If there are an odd number of students, assign a group of three students.

Lesson Procedure

Explain to students that they will be playing the game *Quiz-Quiz-Take*, where they will apply the two theorems they proved in Explorations 1 and 2 from the textbook, the Tangent and Intersected Chord Theorem (Theorem 10.14) and the Angles Inside the Circle Theorem (Theorem 10.15), to solve problems.

Arrange the class into two rows facing each other and distribute a card from Resource Sheet 5 to each student. Explain to students that they will hold up their cards so they can *quiz* each other. When they both finish, each partner will reveal the answer on the back of the card and discuss both correct and incorrect solutions. The pairs will have 3–5 minutes to solve both problems. Use a timer to indicate the end of the round. When time is up, have each student in one of the rows move one seat to the right. The student on the end will move to the opposite end of the row. The new partners will repeat the process by *quizzing* each other, and after 3–5 minutes, the students in the same row will move one more seat to the right. The process will repeat one more time, except this time each student will *take* his/her partner's card at the end of the round and become responsible for the new card. The game should continue until students reunite with their original partners. (**Note:** Provide students with the opportunity to bring pieces of paper and pencils with them if they need to show their work.)

Circulate as students solve the problems and provide support as needed. Adjust the time for each round, based on the needs of the students.

Lead a discussion about the problems students found most difficult and why. Have students pair up one last time to share the two theorems that they were applying and discuss one question each of them still has.

5 Angle Relationships in Circles (continued)

Assessment

After completing the lesson activity, students will be prepared to complete the exercises in the textbook (Geometry—Chapter 10, Lesson 5).

Lesson Connections

Connections to the Standards for Mathematical Practice:

MP.2 Reason abstractly and quantitatively.

MP.6 Attend to precision.

MP.8 Look for and express regularity in repeated reasoning.

Connections to Universal Design for Learning:

Checkpoint 1.3 Offer alternatives for visual information.

Checkpoint 2.1 Clarify vocabulary and symbols.

Checkpoint 3.2 Highlight patterns, critical features, big ideas, and relationships.

Checkpoint 4.1 Vary the methods for response and navigation.

Checkpoint 6.3 Facilitate managing information and resources.

Checkpoint 7.3 Minimize threats and distractions.

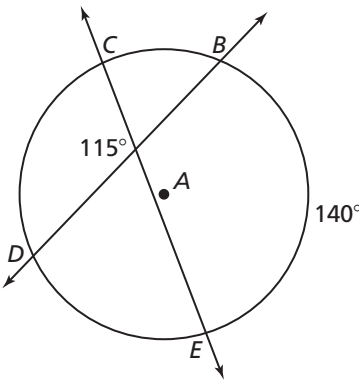
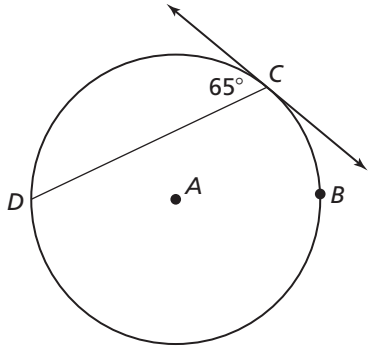
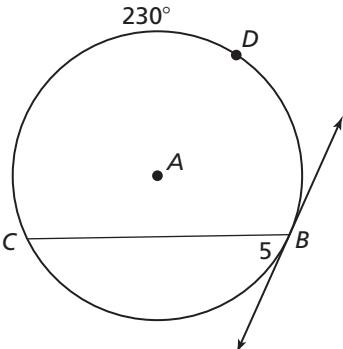
Checkpoint 8.3 Foster collaboration and community.

Connections to the Depths of Knowledge:

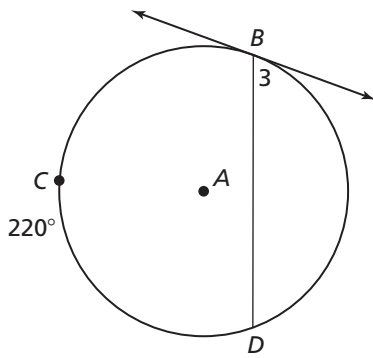
Level 1 Recall and Reproduction

Level 2 Working With Skills and Concepts

Quiz-Quiz-Take Cards

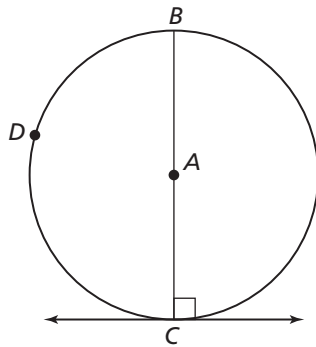
 <p>Find $m\widehat{DC}$.</p>	$m\widehat{DC} = 90^\circ$
 <p>Find $m\widehat{CD}$.</p>	$m\widehat{CD} = 130^\circ$
 <p>Find $m\angle 5$.</p>	$m\angle 5 = 65^\circ$

5 Quiz-Quiz-Take Cards (continued)



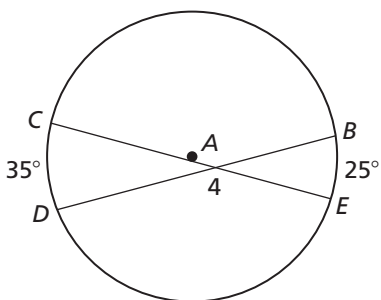
Find $m\angle 3$.

$$m\angle 3 = 70^\circ$$



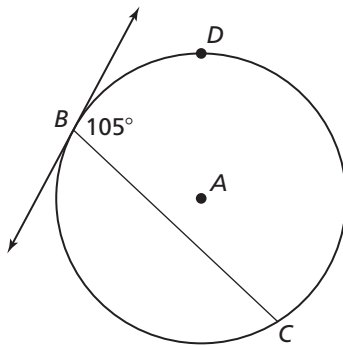
Find $m\widehat{BDC}$.

$$m\widehat{BDC} = 180^\circ$$



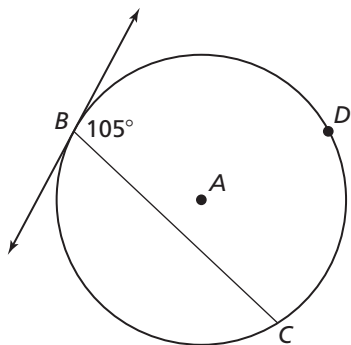
Find $m\angle 4$.

$$m\angle 4 = 150^\circ$$



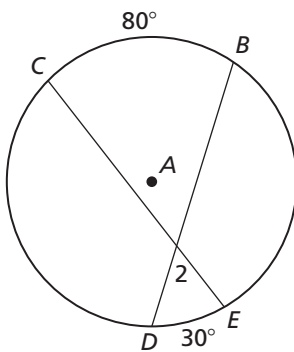
Find $m\widehat{BC}$.

$$m\widehat{BC} = 150^\circ$$



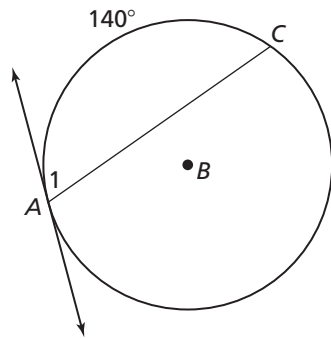
Find $m\widehat{BDC}$.

$$m\widehat{BDC} = 210^\circ$$



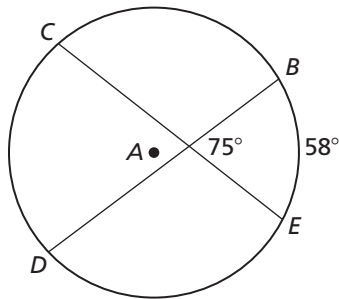
Find $m\angle 2$.

$$m\angle 2 = 55^\circ$$



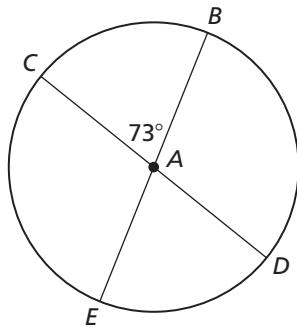
Find $m\angle 1$.

$$m\angle 1 = 70^\circ$$



Find $m\widehat{CD}$.

$$m\widehat{CD} = 92^\circ$$



Find $m\widehat{DE}$ and $m\widehat{CB}$.

$$m\widehat{DE} = 73^\circ$$

$$m\widehat{CB} = 73^\circ$$

Lesson
6

Segment Relationships in Circles

For use with Section 10.6

Lesson Preparation

Materials: compasses, rulers, plain paper, calculators

Beforehand: Display the intersecting chords chart and the intersecting secants chart on the board. Pre-assess students' accuracy with measuring using a ruler.

Classroom Management: Strategically arrange students into cooperative pairs.

Lesson Procedure

Say, "Today, we are going to explore segment relationships in circles by forming and testing our own class conjectures." Arrange students into pairs and distribute compasses, rulers, and calculators.

Ask each pair to construct a circle. Say, "First, we are going to investigate the relationship between two intersecting chords." Have each pair work together to draw two chords in their circle. Have students label the chords AB and CD and the intersection E .

Display the following chart on the board (include one row for each pair). Direct each pair of students to measure each of the segments and record the measurements and calculations for their circle. Remind students to look for patterns as other groups add their data to the board. (**Note:** Students may ask what unit of measurement to use. Remind students as long as each pair is consistent in using the same measurement for each of their segments, the relationships will still be observable.)

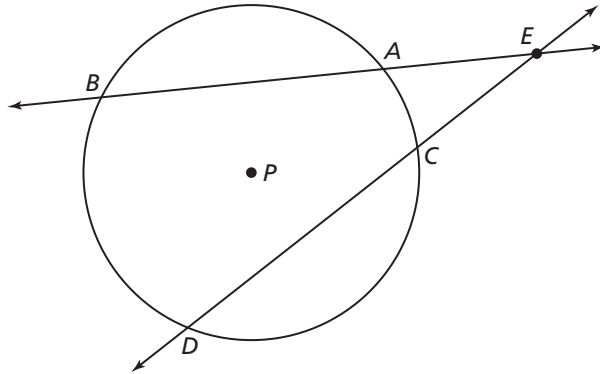
Intersecting Chords Chart

Pair	EA	EB	$EA \cdot EB$	EC	ED	$EC \cdot ED$

Lead a discussion about the relationships students noticed between the columns. Ask students to develop conjectures about the relationship between intersecting chords in the same circle. [Students should be able to informally describe the Segments of Chords Theorem (Theorem 10.18), which states that if two chords intersect in the interior of a circle, then the product of the lengths of the segments of one chord is equal to the product of the lengths of the segments of the other chord.]

6 Segment Relationships in Circles (continued)

Ask pairs to construct another circle. Say, “Now we will be investigating the relationships between two secants that intersect outside a circle.” Have each pair draw two secants that intersect outside of a circle and label as shown.



Display the following chart on the board (include one row for each pair). Tell students to measure their segments and record the measurements and calculations. Remind students to look for patterns as other groups add their data to the board.

Intersecting Secants Chart

Pair	EA	EB	$EA \cdot EB$	EC	ED	$EC \cdot ED$

Lead a discussion about the relationships students noticed between the columns.

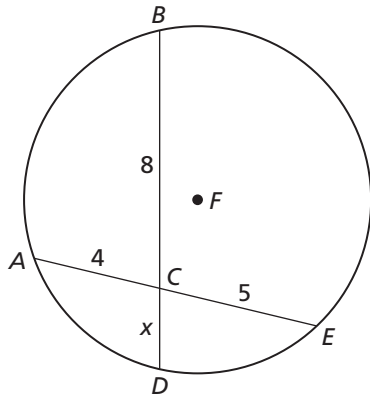
Ask students to develop conjectures about the relationship among secants that intersect outside the same circle. [Students should be able to informally describe the Segments of Secants Theorem (Theorem 10.19) which states that if two secant segments share the same endpoint outside a circle, then the product of the lengths of one secant segment and its external segment equals the product of the lengths of the other secant segment and its external segment.]

Assessment

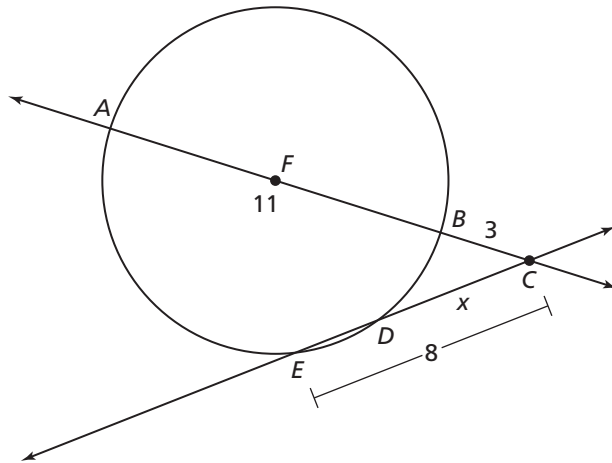
Have students complete the following problems to assess their knowledge of segment relationships in circles.

Use the diagram to find the value of x .

1.



2.



Lesson Connections

Connections to the Standards for Mathematical Practice:

MP.2 Reason abstractly and quantitatively.

MP.5 Use appropriate tools strategically.

MP.6 Attend to precision.

Connections to Universal Design for Learning:

Checkpoint 1.3 Offer alternatives for visual information.

Checkpoint 2.1 Clarify vocabulary and symbols.

Checkpoint 3.2 Highlight patterns, critical features, big ideas, and relationships.

Checkpoint 3.4 Maximize transfer and generalization.

Checkpoint 4.1 Vary the methods for response and navigation.

Checkpoint 6.3 Facilitate managing information and resources.

Checkpoint 7.3 Minimize threats and distractions.

Checkpoint 8.3 Foster collaboration and community.

Connections to the Depths of Knowledge:

Level 2 Working With Skills and Concepts

Level 3 Short-Term Strategic Thinking

Lesson 7

Circles in the Coordinate Plane

For use with Section 10.7

Lesson Preparation

Materials: Resource Sheet 7A: Pi County Campgrounds, Resource Sheet 7B: Exploring the Equation of a Circle, Resource Sheet 7C: What's My Equation? Matching Cards, rulers, envelopes

Beforehand: Make copies of Resource Sheets 7A and 7B (one of each per student). Make copies of Resource Sheet 7C (one per group of students), cut the cards apart, shuffle them, and place each set in an envelope. Pre-assess students' understanding of the Pythagorean Theorem.

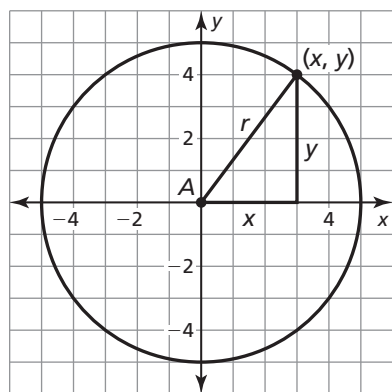
Classroom Management: Use pre-assessment information to strategically arrange the class into groups of three-to-four students to maximize learning.

Lesson Procedure

Arrange the class into groups of three-to-four students and distribute Resource Sheet 7A to each student. Explain to students that they are going to be investigating how to determine the distance between the two campsites given the information provided. Circulate to assist struggling students as needed.

Ask students to share their methods for finding the distance between the two points. [Students may choose to calculate the distance between the two points by drawing a right triangle and applying the Pythagorean Theorem, and others may choose to use the Distance Formula.] If groups do not choose to use the Pythagorean Theorem to find the distance between the two points, demonstrate this method, because they will be using it to find the equation of a circle.

Say, "Today, we will be using the Pythagorean Theorem to help us write the equation for a circle." Distribute Resource Sheet 7B to each student. Have students place a point on the circle and label it (x, y) . Then have students draw a right triangle, where the hypotenuse goes from the center of the circle to the point (x, y) . Ask students, "What information do we know about this triangle?" [*Sample response:* The hypotenuse is equal to the radius of the circle, which is 5. The horizontal leg represents x and the vertical leg represents y .] Have students label these values on their graphs, as shown.



Ask students, “How can we use the Pythagorean Theorem to write the equation of this circle?” Lead students to discover that the equation of the circle is $x^2 + y^2 = 5^2$, or $x^2 + y^2 = 25$. Have students compare the graph in the second example to the first example. Ask students to describe the differences. [*Sample response:* The center of the circle in the second example is not at the origin, it is at (5, 2).] Have students work with their groups to determine the equation of the circle. Circulate to help struggling students.

Lead a discussion about groups’ methods for finding the equation of the circle. $[(x - 5)^2 + (y - 2)^2 = 9]$ Allow students to ask questions and address any misconceptions.

Distribute an envelope to each group with the cut up matching cards from Resource Sheet 7C. Explain to students that they are going to play an activity called *What’s My Equation?* Tell students they will be matching each graph of a circle with the correct equation. Allow students time to sort the graphs and circulate to provide support. Have students share their answers and address any misconceptions.

Direct students to develop a formula for finding the equation of a circle given the center (h, k) and radius r . Allow groups time to find the formula and circulate to provide support to struggling students. Have groups share their formulas and make sure by the end of the discussion, groups understand the formula for finding the equation of a circle. $[(x - h)^2 + (y - k)^2 = r^2]$

Assessment

After completing this activity, students will be prepared to complete the exercises in the textbook (Geometry—Chapter 10, Lesson 7).

Lesson Connections

Connections to the Standards for Mathematical Practice:

- MP.2 Reason abstractly and quantitatively.
- MP.5 Use appropriate tools strategically.
- MP.6 Attend to precision.
- MP.8 Look for and express regularity in repeated reasoning.

Connections to Universal Design for Learning:

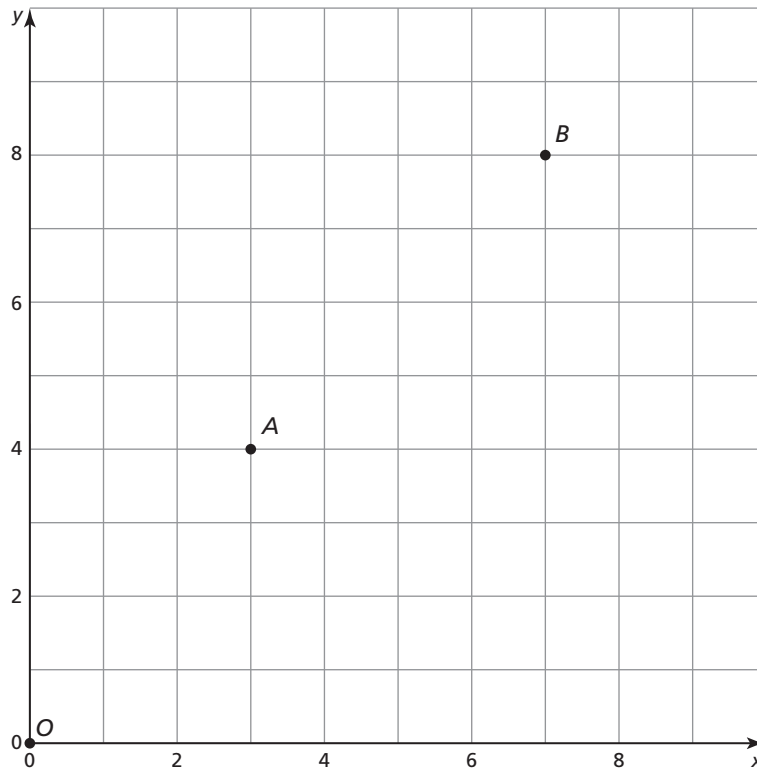
- Checkpoint 1.3 Offer alternatives for visual information.
- Checkpoint 2.1 Clarify vocabulary and symbols.
- Checkpoint 3.2 Highlight patterns, critical features, big ideas, and relationships.
- Checkpoint 3.4 Maximize transfer and generalization.
- Checkpoint 4.1 Vary the methods for response and navigation.
- Checkpoint 5.2 Use multiple tools for construction and composition.
- Checkpoint 6.3 Facilitate managing information and resources.
- Checkpoint 7.3 Minimize threats and distractions.
- Checkpoint 8.3 Foster collaboration and community.

Connections to the Depths of Knowledge:

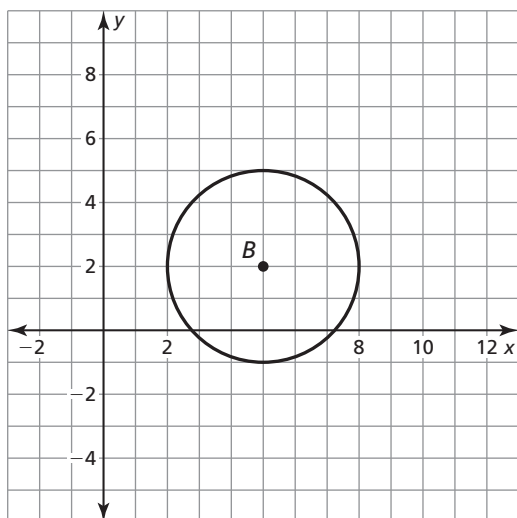
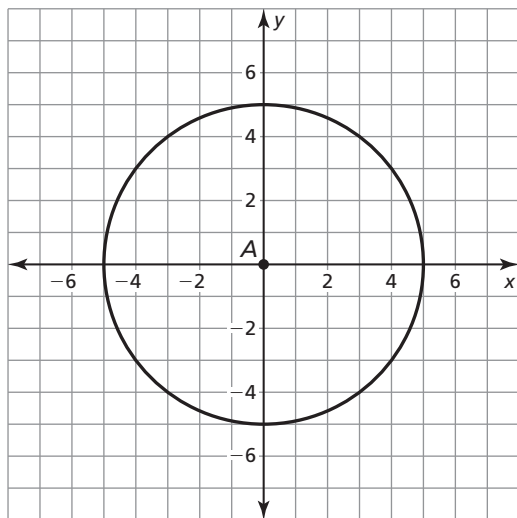
- Level 2 Working With Skills and Concepts
- Level 3 Short-Term Strategic Thinking

Pi County Campgrounds

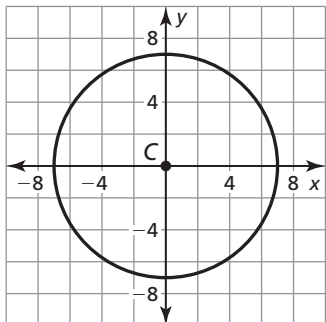
Below is a map of the Pi County campgrounds. Point O represents the park ranger station. The campground is designed in a 10-mile by 10-mile grid, as shown. If your family sets up their campsite at point A and your friend's family sets up a campsite at point B , how far are your campsites from each other? Show all necessary work.



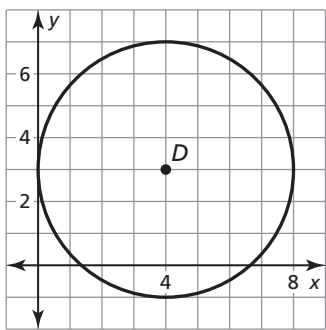
Exploring the Equation of a Circle



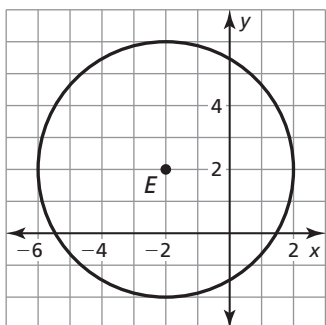
What's My Equation? Matching Cards



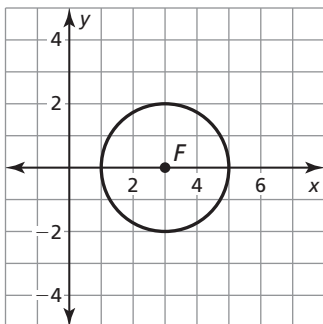
$$x^2 + y^2 = 7^2$$



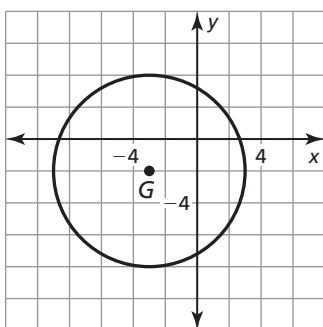
$$(x - 4)^2 + (y - 3)^2 = 4^2$$



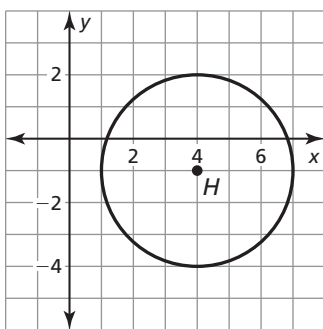
$$(x + 2)^2 + (y - 2)^2 = 4^2$$



$$(x - 3)^2 + y^2 = 2^2$$



$$(x + 3)^2 + (y + 2)^2 = 6^2$$



$$(x - 4)^2 + (y + 1)^2 = 3^2$$