An **absolute value equation** is an equation that contains an absolute value expression. Here are three examples:

\[ |x| = 5 \quad |x + 1| = 2 \quad |2x - 2| = 5 \]

You can solve these types of equations by solving two related linear equations.

**Key Idea**

**Solving Absolute Value Equations**

To solve \(|ax + b| = c| \) for \(c \geq 0\), solve the related linear equations

\[ ax + b = c \quad \text{or} \quad ax + b = -c \]

**EXAMPLE 1**

**Solving Absolute Value Equations**

a. Solve \(|x - 3| = 5\). Graph the solutions.

   Write two related linear equations for \(|x - 3| = 5\). Then solve each equation.

   \[ x - 3 = 5 \quad \text{or} \quad x - 3 = -5 \]
   
   +3 +3 +3 +3
   
   \[ x = 8 \quad \text{or} \quad x = -2 \]

   Write related linear equations. Add 3 to each side. Simplify.

   The solutions are -2 and 8.

b. Solve \(4x + 7| = -4\).

   The absolute value of an expression must be greater than or equal to 0. The expression \(|4x + 7|\) cannot equal -4.

   So, the equation has no solution.

**On Your Own**

Solve the equation. Graph the solutions.

1. \(|x| = 4\)
2. \(|x - 8| = -1\)
3. \(|x + 1| = 2\)
4. \(|3x - 2| = 10\)
EXAMPLE 2  Solving an Absolute Value Equation

Solve \(|2x + 4| - 8 = -2\).

\[
|2x + 4| - 8 = -2 \\
\hspace{1cm} + 8 \hspace{1cm} + 8 \\
\hspace{1cm} |2x + 4| = 6
\]

Write the equation.
Add 8 to each side.
Simplify.

Write two related linear equations for \(|2x + 4| = 6\). Then solve each equation.

\[
\begin{align*}
2x + 4 &= 6 \\
&\hspace{1cm} - 4 \hspace{1cm} - 4 \\
2x &= 2 \\
&\hspace{1cm} \div 2 \hspace{1cm} \div 2 \\
x &= 1
\end{align*}
\]

or

\[
\begin{align*}
2x + 4 &= -6 \\
&\hspace{1cm} - 4 \hspace{1cm} - 4 \\
2x &= -10 \\
&\hspace{1cm} \div 2 \hspace{1cm} \div 2 \\
x &= -5
\end{align*}
\]

The solutions are \(-5\) and 1.

EXAMPLE 3  Real-Life Application

In a cheerleading competition, the minimum length of a routine is 4 minutes. The maximum length of a routine is 5 minutes. Write an absolute value equation that has these minimum and maximum lengths as its solutions.

Step 1: Graph the minimum and maximum lengths on a number line. Then find the point that is halfway between the lengths.

4.5 is halfway between 4 and 5.

Step 2: Write the equation. Each solution is 0.5 unit from 4.5.

\[
|x - 4.5| = 0.5
\]

The equation is \(|x - 4.5| = 0.5\).

On Your Own

Solve the equation. Check your solutions.

5. \(|x + 1| + 4 = 5\)  
6. \(4|x - 8| = 8\)  
7. \(2|3x - 2| + 5 = 7\)

8. For a poem contest, the minimum length of a poem is 16 lines. The maximum length is 32 lines. Write an absolute value equation that has these minimum and maximum lengths as its solutions.
Vocabulary and Concept Check

1. **Writing** How do you solve absolute value equations?

2. **Which One Doesn’t Belong** Which equation does not belong with the other three? Explain your reasoning.

   - $|x| = 6$
   - $|2x + 1| = 4$
   - $x - |9| = 7$
   - $3|x + 4| = 11$

Practice and Problem Solving

Solve the equation. Graph the solutions, if possible.

1. $|x| = 5$
2. $|x + 3| = -15$
3. $|x + 2.1| = -1.9$
4. $|x| = -4$
5. $|x - 6| = 14$
6. $|x - 6| = 14$
7. $|2x + 1| = 5$
8. $|x - 1.1| = 12.4$
9. $|3x + 3| = 24$
10. $|4x - 3| = -3$
11. $|6x - 2| = 20$

13. **Error Analysis** Describe and correct the error in solving the absolute value equation.

   \[
   \begin{align*}
   |x + 8| &= -2 \\
   x + 8 &= -2 \quad \text{or} \quad x + 8 &= 2 \\
   x &= -10 \quad \text{or} \quad x &= -6
   \end{align*}
   \]

Solve the equation. Check your solutions, if possible.

14. $|x + 1| - 2 = 14$
15. $|x - 3| + 4 = 6$
16. $|4x - 10| + 7 = 18$
17. $|-3x + 1| - 19 = -15$
18. $|3| 2x - 2| - 1 = 8$
19. $-2|x + 1| + 4 = -4$
20. **Cell Phone** A new cell phone design allows for a minimum length of 10.75 centimeters and a maximum length of 11.25 centimeters. Write an absolute value equation that has these minimum and maximum lengths as its solutions.

21. **Talent Show** Your school is holding a talent show. The minimum length of an act is 2.5 minutes. The maximum length is 3.5 minutes. Write an absolute value equation that has these minimum and maximum lengths as its solutions.
Match the absolute value equation with its graph.

22. $|x - 4| = 3$  
23. $|x + 4| = 3$  
24. $|x - 3| = 4$

A. 

B. 

C. 

25. **EXERCISE PROGRAM** Your uncle is starting a new exercise program. To maintain his current weight, the program suggests a daily calorie intake of 2800 calories.

a. To lose or gain weight, the program recommends adjusting the daily calorie intake by 18%. Write an absolute value equation to find the minimum and maximum calorie intake per day for this program.

b. A second exercise program also recommends a daily calorie intake of 2800 calories. To lose or gain weight with this program, the daily calorie intake should be adjusted by 20%. Write an absolute value equation to find the minimum and maximum calorie intake per day for this program.

26. **CRITICAL THINKING** Explain graphically and in words what $x$, $b$, and $c$ represent in the absolute value equation below.

$$|x - b| = c$$

**Fair Game Review** What you learned in previous grades & lessons

**Solve the equation. Check your solution.** (Section 1.3)

27. $2x - 4 = 6x + 32$  
28. $-16 + 7y = -2(y - 1)$  
29. $-(9z - 12) = -3(2z + 2)$

30. **MULTIPLE CHOICE** What is the surface area of the rectangular prism? (Skills Review Handbook)

- **A** 95.5 m²  
- **B** 156 m²  
- **C** 191 m²  
- **D** 200 m²