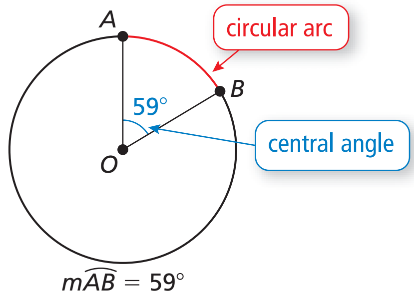
10.2 Finding Arc Measures

Essential Question

How are circular arcs measured?

A **central angle** of a circle is an angle whose vertex is the center of the circle. A *circular arc* is a portion of a circle, as shown below. The measure of a circular arc is the measure of its central angle.

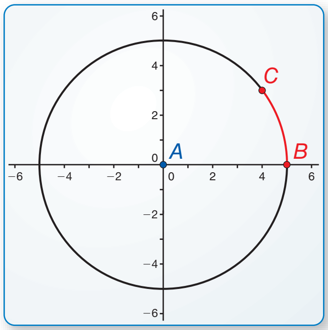
If $m\angle AOB < 180°$, then the circular arc is called a **minor arc** and is denoted by $\overset{\frown}{AB}$.



EXPLORATION 1 Measuring Circular Arcs

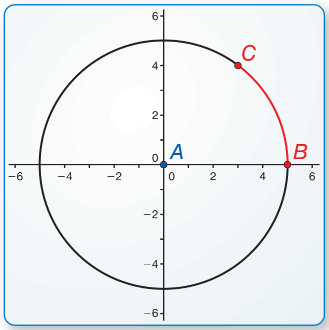
Work with a partner. Use dynamic geometry software to find the measure of $\overset{\frown}{BC}$. Verify your answers using trigonometry.

a.



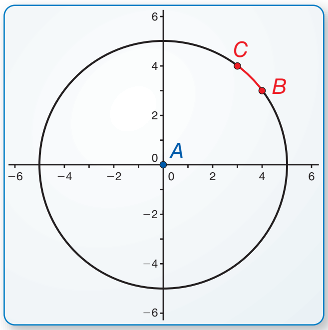
Points
$A(0, 0)$
$B(5, 0)$
$C(4, 3)$

b.



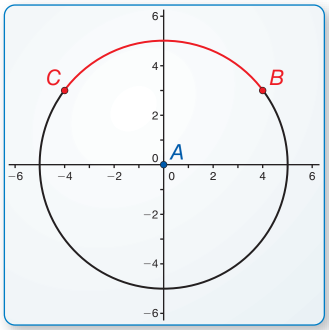
Points
$A(0, 0)$
$B(5, 0)$
$C(3, 4)$

c.



Points
$A(0, 0)$
$B(4, 3)$
$C(3, 4)$

d.



Points
$A(0, 0)$
$B(4, 3)$
$C(-4, 3)$

USING TOOLS STRATEGICALLY

To be proficient in math, you need to use technological tools to explore and deepen your understanding of concepts.

Communicate Your Answer

2. How are circular arcs measured?

3. Use dynamic geometry software to draw a circular arc with the given measure.

a. 30° **b.** 45°
c. 60° **d.** 90°

10.2 Lesson

Core Vocabulary

central angle, *p. 538*
minor arc, *p. 538*
major arc, *p. 538*
semicircle, *p. 538*
measure of a minor arc, *p. 538*
measure of a major arc, *p. 538*
adjacent arcs, *p. 539*
congruent circles, *p. 540*
congruent arcs, *p. 540*
similar arcs, *p. 541*

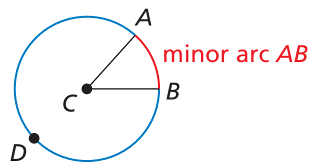
What You Will Learn

- Find arc measures.
- Identify congruent arcs.
- Prove circles are similar.

Finding Arc Measures

A **central angle** of a circle is an angle whose vertex is the center of the circle. In the diagram, $\angle ACB$ is a central angle of $\odot C$.

If $m\angle ACB$ is less than 180°, then the points on $\odot C$ that lie in the interior of $\angle ACB$ form a **minor arc** with endpoints A and B. The points on $\odot C$ that do not lie on the minor arc AB form a **major arc** with endpoints A and B. A **semicircle** is an arc with endpoints that are the endpoints of a diameter.



Minor arcs are named by their endpoints. The minor arc associated with $\angle ACB$ is named $\overset{\frown}{AB}$. Major arcs and semicircles are named by their endpoints and a point on the arc. The major arc associated with $\angle ACB$ can be named $\overset{\frown}{ADB}$.

STUDY TIP

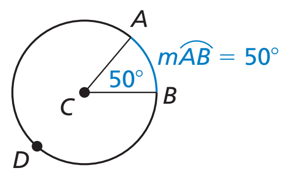
The measure of a minor arc is less than 180°. The measure of a major arc is greater than 180°.

Core Concept

Measuring Arcs

The **measure of a minor arc** is the measure of its central angle. The expression $m\overset{\frown}{AB}$ is read as "the measure of arc AB."

The measure of the entire circle is 360°. The **measure of a major arc** is the difference of 360° and the measure of the related minor arc. The measure of a semicircle is 180°.



$m\overset{\frown}{ADB} = 360° - 50° = 310°$

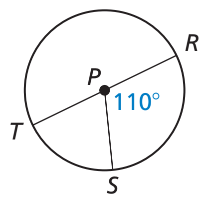
EXAMPLE 1 Finding Measures of Arcs

Find the measure of each arc of $\odot P$, where $\overline{RT}$ is a diameter.

a. $\overset{\frown}{RS}$

b. $\overset{\frown}{RTS}$

c. $\overset{\frown}{RST}$



SOLUTION

a. $\overset{\frown}{RS}$ is a minor arc, so $m\overset{\frown}{RS} = m\angle RPS = 110°$.

b. $\overset{\frown}{RTS}$ is a major arc, so $m\overset{\frown}{RTS} = 360° - 110° = 250°$.

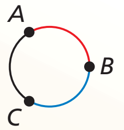
c. $\overline{RT}$ is a diameter, so $\overset{\frown}{RST}$ is a semicircle, and $m\overset{\frown}{RST} = 180°$.

Two arcs of the same circle are **adjacent arcs** when they intersect at exactly one point. You can add the measures of two adjacent arcs.

Postulate

Postulate 10.1 Arc Addition Postulate

The measure of an arc formed by two adjacent arcs is the sum of the measures of the two arcs.

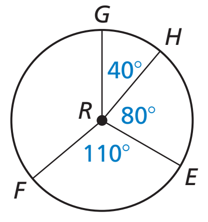


$m\overset{\frown}{ABC} = m\overset{\frown}{AB} + m\overset{\frown}{BC}$

EXAMPLE 2 Using the Arc Addition Postulate

Find the measure of each arc.

a. $\overset{\frown}{GE}$ **b.** $\overset{\frown}{GEF}$ **c.** $\overset{\frown}{GF}$



SOLUTION

a. $m\overset{\frown}{GE} = m\overset{\frown}{GH} + m\overset{\frown}{HE} = 40° + 80° = 120°$

b. $m\overset{\frown}{GEF} = m\overset{\frown}{GE} + m\overset{\frown}{EF} = 120° + 110° = 230°$

c. $m\overset{\frown}{GF} = 360° - m\overset{\frown}{GEF} = 360° - 230° = 130°$

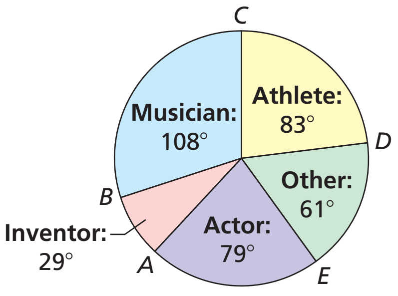
EXAMPLE 3 Finding Measures of Arcs

A recent survey asked teenagers whether they would rather meet a famous musician, athlete, actor, inventor, or other person. The circle graph shows the results. Find the indicated arc measures.

a. $m\overset{\frown}{AC}$ **b.** $m\overset{\frown}{ACD}$

c. $m\overset{\frown}{ADC}$ **d.** $m\overset{\frown}{EBD}$

Whom Would You Rather Meet?



SOLUTION

a. $m\overset{\frown}{AC} = m\overset{\frown}{AB} + m\overset{\frown}{BC}$
$= 29° + 108°$
$= 137°$

b. $m\overset{\frown}{ACD} = m\overset{\frown}{AC} + m\overset{\frown}{CD}$
$= 137° + 83°$
$= 220°$

c. $m\overset{\frown}{ADC} = 360° - m\overset{\frown}{AC}$
$= 360° - 137°$
$= 223°$

d. $m\overset{\frown}{EBD} = 360° - m\overset{\frown}{ED}$
$= 360° - 61°$
$= 299°$

Monitoring Progress

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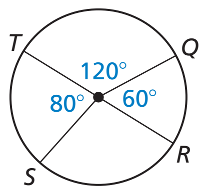
Identify the given arc as a *major arc*, *minor arc*, or *semicircle*. Then find the measure of the arc.

1. $\overset{\frown}{TQ}$ **2.** $\overset{\frown}{QRT}$ **3.** $\overset{\frown}{TQR}$

4. $\overset{\frown}{QS}$ **5.** $\overset{\frown}{TS}$ **6.** $\overset{\frown}{RST}$



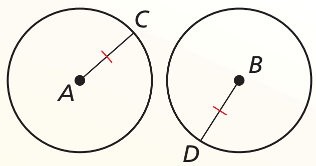
Identifying Congruent Arcs

Two circles are **congruent circles** if and only if a rigid motion or a composition of rigid motions maps one circle onto the other. This statement is equivalent to the Congruent Circles Theorem below.

Theorem

Theorem 10.3 Congruent Circles Theorem

Two circles are congruent circles if and only if they have the same radius.



Proof Ex. 35, p. 544

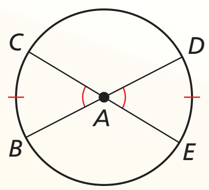
$\odot A \cong \odot B$ if and only if $\overline{AC} \cong \overline{BD}$.

Two arcs are **congruent arcs** if and only if they have the same measure and they are arcs of the same circle or of congruent circles.

Theorem

Theorem 10.4 Congruent Central Angles Theorem

In the same circle, or in congruent circles, two minor arcs are congruent if and only if their corresponding central angles are congruent.



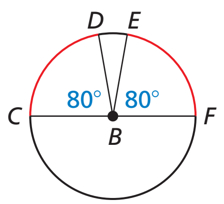
$\overset{\frown}{BC} \cong \overset{\frown}{DE}$ if and only if $\angle BAC \cong \angle DAE$.

Proof Ex. 37, p. 544

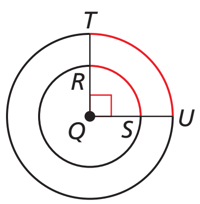
EXAMPLE 4 Identifying Congruent Arcs

Tell whether the red arcs are congruent. Explain why or why not.

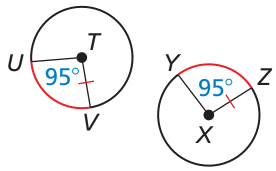
a.



b.



c.



STUDY TIP

The two circles in part (c) are congruent by the Congruent Circles Theorem because they have the same radius.

SOLUTION

a. $\overset{\frown}{CD} \cong \overset{\frown}{EF}$ by the Congruent Central Angles Theorem because they are arcs of the same circle and they have congruent central angles, $\angle CBD \cong \angle FBE$.

b. $\overset{\frown}{RS}$ and $\overset{\frown}{TU}$ have the same measure, but are not congruent because they are arcs of circles that are not congruent.

c. $\overset{\frown}{UV} \cong \overset{\frown}{YZ}$ by the Congruent Central Angles Theorem because they are arcs of congruent circles and they have congruent central angles, $\angle UTV \cong \angle YXZ$.

Monitoring Progress

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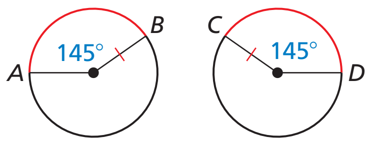
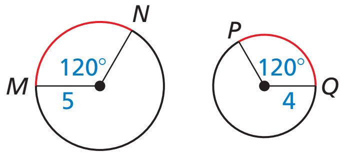
Tell whether the red arcs are congruent. Explain why or why not.

7.



8.



Proving Circles Are Similar

Theorem

Theorem 10.5 Similar Circles Theorem

All circles are similar.

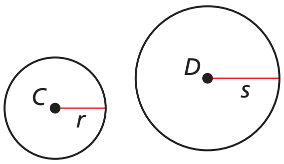
Proof p. 541; Ex. 33, p. 544

PROOF **Similar Circles Theorem**

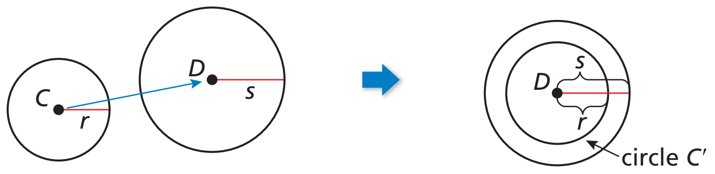
All circles are similar.

Given $\odot C$ with center C and radius r, $\odot D$ with center D and radius s

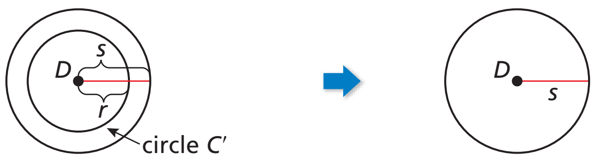
Prove $\odot C \sim \odot D$



First, translate $\odot C$ so that point C maps to point D. The image of $\odot C$ is $\odot C'$ with center D. So, $\odot C'$ and $\odot D$ are concentric circles.



$\odot C'$ is the set of all points that are r units from point D. Dilate $\odot C'$ using center of dilation D and scale factor $\frac{s}{r}$.



This dilation maps the set of all the points that are r units from point D to the set of all points that are $\frac{s}{r}(r) = s$ units from point D. $\odot D$ is the set of all points that are s units from point D. So, this dilation maps $\odot C'$ to $\odot D$.

Because a similarity transformation maps $\odot C$ to $\odot D$, $\odot C \sim \odot D$.

Two arcs are **similar arcs** if and only if they have the same measure. All congruent arcs are similar, but not all similar arcs are congruent. For instance, in Example 4, the pairs of arcs in parts (a), (b), and (c) are similar but only the pairs of arcs in parts (a) and (c) are congruent.

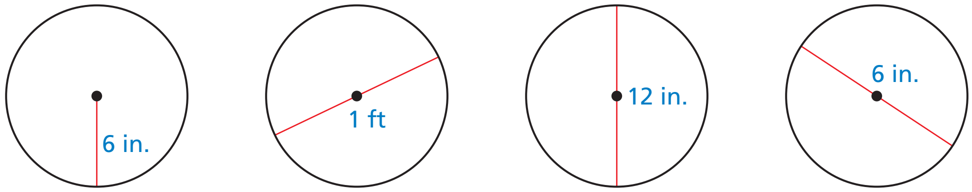
10.2 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

Vocabulary and Core Concept Check

1. **VOCABULARY** Copy and complete: If $\angle ACB$ and $\angle DCE$ are congruent central angles of $\odot C$, then $\overset{\frown}{AB}$ and $\overset{\frown}{DE}$ are ______________.

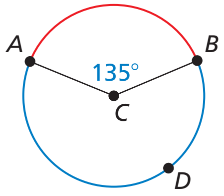
2. **WHICH ONE DOESN'T BELONG?** Which circle does *not* belong with the other three? Explain your reasoning.



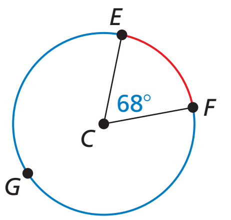
Monitoring Progress and Modeling with Mathematics

In Exercises 3–6, name the red minor arc and find its measure. Then name the blue major arc and find its measure.

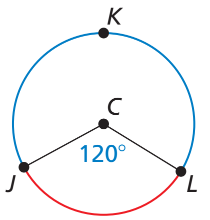
3.



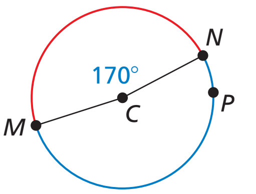
4.



5.



6.



In Exercises 7–14, identify the given arc as a *major arc*, *minor arc*, or *semicircle*. Then find the measure of the arc. *(See Example 1.)*

7. $\overset{\frown}{BC}$

8. $\overset{\frown}{DC}$

9. $\overset{\frown}{ED}$

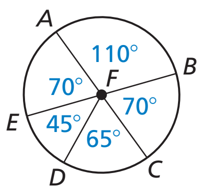
10. $\overset{\frown}{AE}$

11. $\overset{\frown}{EAB}$

12. $\overset{\frown}{ABC}$

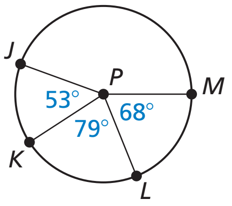
13. $\overset{\frown}{BAC}$

14. $\overset{\frown}{EBD}$

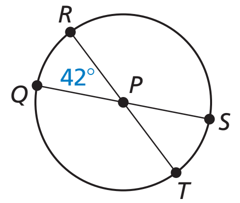


In Exercises 15 and 16, find the measure of each arc. *(See Example 2.)*

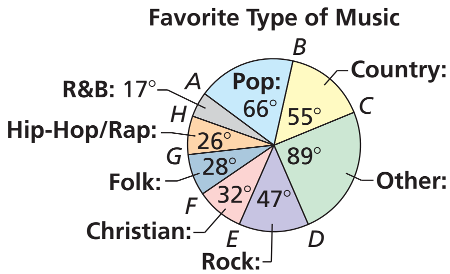
15. a. $\overset{\frown}{JL}$
 b. $\overset{\frown}{KM}$
 c. $\overset{\frown}{JLM}$
 d. $\overset{\frown}{JM}$



16. a. $\overset{\frown}{RS}$
 b. $\overset{\frown}{QRS}$
 c. $\overset{\frown}{QST}$
 d. $\overset{\frown}{QT}$

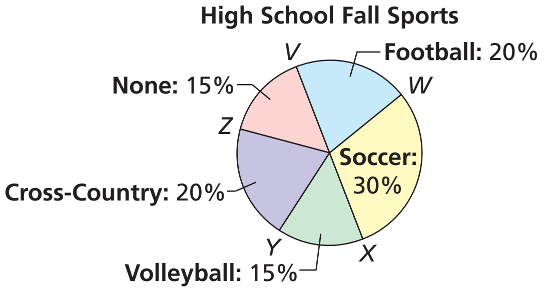


17. **MODELING WITH MATHEMATICS** A recent survey asked high school students their favorite type of music. The results are shown in the circle graph. Find each indicated arc measure. *(See Example 3.)*



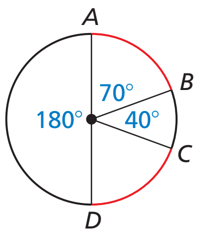
 a. $m\overset{\frown}{AE}$
 b. $m\overset{\frown}{ACE}$
 c. $m\overset{\frown}{GDC}$
 d. $m\overset{\frown}{BHC}$
 e. $m\overset{\frown}{FD}$
 f. $m\overset{\frown}{FBD}$

18. ABSTRACT REASONING The circle graph shows the percentages of students enrolled in fall sports at a high school. Is it possible to find the measure of each minor arc? If so, find the measure of the arc for each category shown. If not, explain why it is not possible.

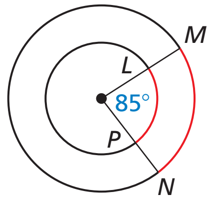


In Exercises 19–22, tell whether the red arcs are congruent. Explain why or why not. *(See Example 4.)*

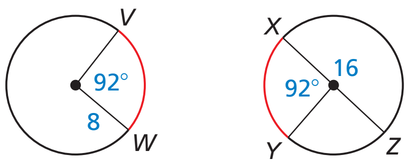
19.



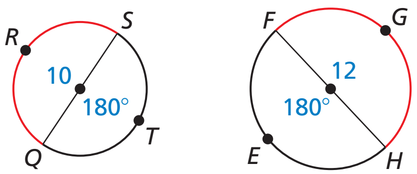
20.



21.

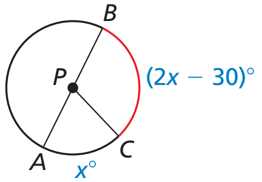


22.

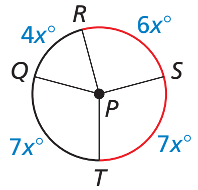


MATHEMATICAL CONNECTIONS **In Exercises 23 and 24, find the value of x. Then find the measure of the red arc.**

23.

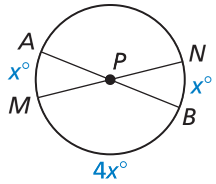


24.

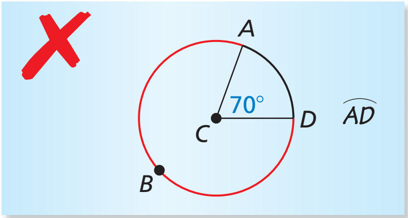


25. MAKING AN ARGUMENT Your friend claims that any two arcs with the same measure are similar. Your cousin claims that any two arcs with the same measure are congruent. Who is correct? Explain.

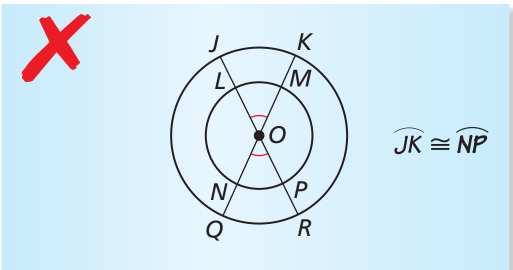
26. MAKING AN ARGUMENT Your friend claims that there is not enough information given to find the value of x. Is your friend correct? Explain your reasoning.



27. ERROR ANALYSIS Describe and correct the error in naming the red arc.



28. ERROR ANALYSIS Describe and correct the error in naming congruent arcs.

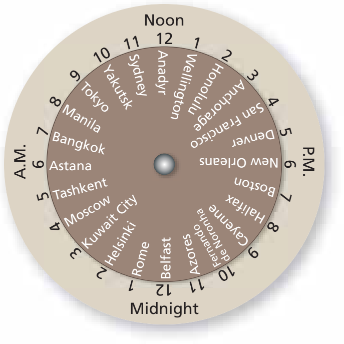


29. ATTENDING TO PRECISION Two diameters of $\odot P$ are $\overline{AB}$ and $\overline{CD}$. Find $m\overset{\frown}{ACD}$ and $m\overset{\frown}{AC}$ when $m\overset{\frown}{AD} = 20°$.

30. REASONING In $\odot R$, $m\overset{\frown}{AB} = 60°$, $m\overset{\frown}{BC} = 25°$, $m\overset{\frown}{CD} = 70°$, and $m\overset{\frown}{DE} = 20°$. Find two possible measures of $\overset{\frown}{AE}$.

31. MODELING WITH MATHEMATICS On a regulation dartboard, the outermost circle is divided into twenty congruent sections. What is the measure of each arc in this circle?

32. **MODELING WITH MATHEMATICS** You can use the time zone wheel to find the time in different locations across the world. For example, to find the time in Tokyo when it is 4 P.M. in San Francisco, rotate the small wheel until 4 P.M. and San Francisco line up, as shown. Then look at Tokyo to see that it is 9 A.M. there.



a. What is the arc measure between each time zone on the wheel?

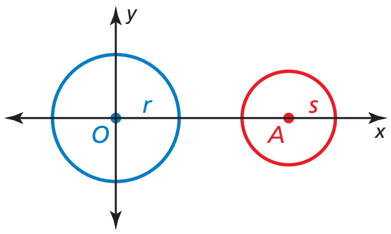
b. What is the measure of the minor arc from the Tokyo zone to the Anchorage zone?

c. If two locations differ by 180° on the wheel, then it is 3 P.M. at one location when it is _____ at the other location.

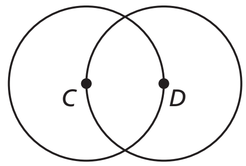
33. **PROVING A THEOREM** Write a coordinate proof of the Similar Circles Theorem (Theorem 10.5).

Given $\odot O$ with center $O(0, 0)$ and radius r, $\odot A$ with center $A(a, 0)$ and radius s

Prove $\odot O \sim \odot A$



34. **ABSTRACT REASONING** Is there enough information to tell whether $\odot C \cong \odot D$? Explain your reasoning.

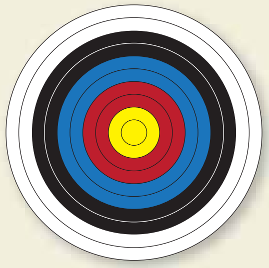


35. **PROVING A THEOREM** Use the diagram on page 540 to prove each part of the biconditional in the Congruent Circles Theorem (Theorem 10.3).

a. **Given** $\overline{AC} \cong \overline{BD}$
Prove $\odot A \cong \odot B$

b. **Given** $\odot A \cong \odot B$
Prove $\overline{AC} \cong \overline{BD}$

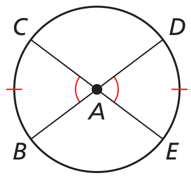
36. **HOW DO YOU SEE IT?** Are the circles on the target *similar* or *congruent*? Explain your reasoning.



37. **PROVING A THEOREM** Use the diagram to prove each part of the biconditional in the Congruent Central Angles Theorem (Theorem 10.4).

a. **Given** $\angle BAC \cong \angle DAE$
Prove $\overset{\frown}{BC} \cong \overset{\frown}{DE}$

b. **Given** $\overset{\frown}{BC} \cong \overset{\frown}{DE}$
Prove $\angle BAC \cong \angle DAE$



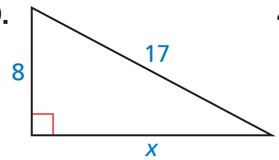
38. **THOUGHT PROVOKING** Write a formula for the length of a circular arc. Justify your answer.

Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Find the value of x. Tell whether the side lengths form a Pythagorean triple. *(Section 9.1)*

39.



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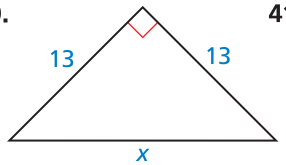

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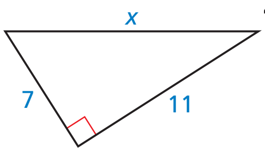

42.