

10.1 Sample Spaces and Probability

Essential Question How can you list the possible outcomes in the sample space of an experiment?

The **sample space** of an experiment is the set of all possible outcomes for that experiment.

EXPLORATION 1 Finding the Sample Space of an Experiment

Work with a partner. In an experiment, three coins are flipped. List the possible outcomes in the sample space of the experiment.



EXPLORATION 2 Finding the Sample Space of an Experiment

Work with a partner. List the possible outcomes in the sample space of the experiment.

a. One six-sided die is rolled.

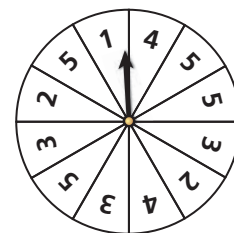
b. Two six-sided dice are rolled.



EXPLORATION 3 Finding the Sample Space of an Experiment

Work with a partner. In an experiment, a spinner is spun.

- How many ways can you spin a 1? 2? 3? 4? 5?
- List the sample space.
- What is the total number of outcomes?



EXPLORATION 4 Finding the Sample Space of an Experiment

Work with a partner. In an experiment, a bag contains 2 blue marbles and 5 red marbles. Two marbles are drawn from the bag.

- How many ways can you choose two blue? a red then blue? a blue then red? two red?
- List the sample space.
- What is the total number of outcomes?



LOOKING FOR A PATTERN

To be proficient in math, you need to look closely to discern a pattern or structure.

Communicate Your Answer

- How can you list the possible outcomes in the sample space of an experiment?
- For Exploration 3, find the ratio of the number of each possible outcome to the total number of outcomes. Then find the sum of these ratios. Repeat for Exploration 4. What do you observe?

10.1 Lesson

Core Vocabulary

probability experiment, p. 538
 outcome, p. 538
 event, p. 538
 sample space, p. 538
 probability of an event, p. 538
 theoretical probability, p. 539
 geometric probability, p. 540
 experimental probability, p. 541

Previous

tree diagram

What You Will Learn

- ▶ Find sample spaces.
- ▶ Find theoretical probabilities.
- ▶ Find experimental probabilities.

Sample Spaces

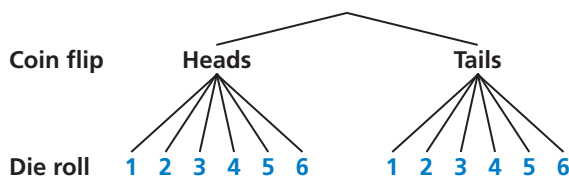
A **probability experiment** is an action, or trial, that has varying results. The possible results of a probability experiment are **outcomes**. For instance, when you roll a six-sided die, there are 6 possible outcomes: 1, 2, 3, 4, 5, or 6. A collection of one or more outcomes is an **event**, such as rolling an odd number. The set of all possible outcomes is called a **sample space**.

EXAMPLE 1 Finding a Sample Space

You flip a coin and roll a six-sided die. How many possible outcomes are in the sample space? List the possible outcomes.

SOLUTION

Use a tree diagram to find the outcomes in the sample space.



▶ The sample space has 12 possible outcomes. They are listed below.

Heads, 1 Heads, 2 Heads, 3 Heads, 4 Heads, 5 Heads, 6
 Tails, 1 Tails, 2 Tails, 3 Tails, 4 Tails, 5 Tails, 6

ANOTHER WAY

Using H for “heads” and T for “tails,” you can list the outcomes as shown below.

H1 H2 H3 H4 H5 H6
 T1 T2 T3 T4 T5 T6

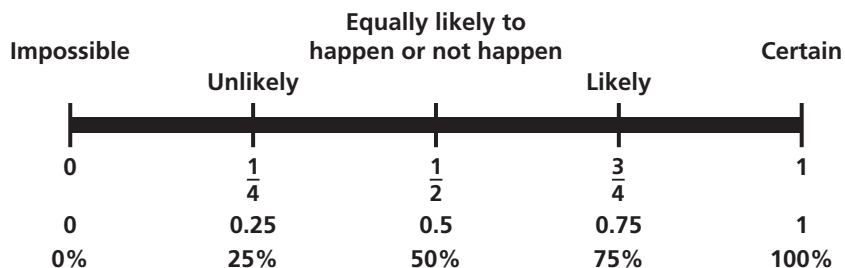
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Find the number of possible outcomes in the sample space. Then list the possible outcomes.

1. You flip two coins.
2. You flip two coins and roll a six-sided die.

Theoretical Probabilities

The **probability of an event** is a measure of the likelihood, or chance, that the event will occur. Probability is a number from 0 to 1, including 0 and 1, and can be expressed as a decimal, fraction, or percent.



ATTENDING TO PRECISION

Notice that the question uses the phrase “exactly two answers.” This phrase is more precise than saying “two answers,” which may be interpreted as “at least two” or as “exactly two.”

The outcomes for a specified event are called *favorable outcomes*. When all outcomes are equally likely, the **theoretical probability** of the event can be found using the following.

$$\text{Theoretical probability} = \frac{\text{Number of favorable outcomes}}{\text{Total number of outcomes}}$$

The probability of event A is written as $P(A)$.

EXAMPLE 2 Finding a Theoretical Probability

A student taking a quiz randomly guesses the answers to four true-false questions. What is the probability of the student guessing exactly two correct answers?

SOLUTION

Step 1 Find the outcomes in the sample space. Let C represent a correct answer and I represent an incorrect answer. The possible outcomes are:

Number correct	Outcome
0	IIII
1	CIII ICII IICI IIIC
2	IICC ICIC ICCI CIIC CICI CCII
3	ICCC CICC CCIC CCCI
4	CCCC

exactly two correct

Step 2 Identify the number of favorable outcomes and the total number of outcomes. There are 6 favorable outcomes with exactly two correct answers and the total number of outcomes is 16.

Step 3 Find the probability of the student guessing exactly two correct answers. Because the student is randomly guessing, the outcomes should be equally likely. So, use the theoretical probability formula.

$$\begin{aligned} P(\text{exactly two correct answers}) &= \frac{\text{Number of favorable outcomes}}{\text{Total number of outcomes}} \\ &= \frac{6}{16} \\ &= \frac{3}{8} \end{aligned}$$

► The probability of the student guessing exactly two correct answers is $\frac{3}{8}$, or 37.5%.

The sum of the probabilities of all outcomes in a sample space is 1. So, when you know the probability of event A , you can find the probability of the *complement* of event A . The *complement* of event A consists of all outcomes that are not in A and is denoted by \bar{A} . The notation \bar{A} is read as “ A bar.” You can use the following formula to find $P(\bar{A})$.

Core Concept

Probability of the Complement of an Event

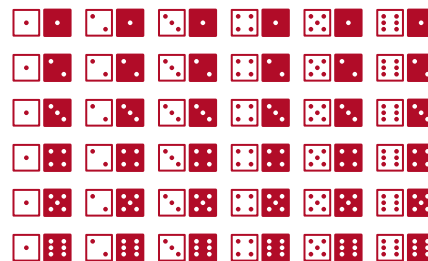
The probability of the complement of event A is

$$P(\bar{A}) = 1 - P(A).$$

EXAMPLE 3 Finding Probabilities of Complements

When two six-sided dice are rolled, there are 36 possible outcomes, as shown. Find the probability of each event.

- The sum is not 6.
- The sum is less than or equal to 9.



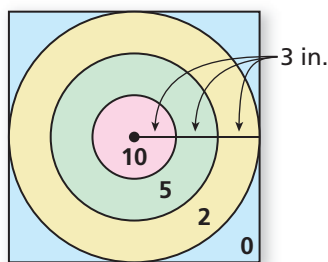
SOLUTION

- $P(\text{sum is not } 6) = 1 - P(\text{sum is } 6) = 1 - \frac{5}{36} = \frac{31}{36} \approx 0.861$
- $P(\text{sum} \leq 9) = 1 - P(\text{sum} > 9) = 1 - \frac{6}{36} = \frac{30}{36} = \frac{5}{6} \approx 0.833$

Some probabilities are found by calculating a ratio of two lengths, areas, or volumes. Such probabilities are called **geometric probabilities**.

EXAMPLE 4 Using Area to Find Probability

You throw a dart at the board shown. Your dart is equally likely to hit any point inside the square board. Are you more likely to get 10 points or 0 points?



SOLUTION

The probability of getting 10 points is

$$P(10 \text{ points}) = \frac{\text{Area of smallest circle}}{\text{Area of entire board}} = \frac{\pi \cdot 3^2}{18^2} = \frac{9\pi}{324} = \frac{\pi}{36} \approx 0.0873.$$

The probability of getting 0 points is

$$\begin{aligned} P(0 \text{ points}) &= \frac{\text{Area outside largest circle}}{\text{Area of entire board}} \\ &= \frac{18^2 - (\pi \cdot 9^2)}{18^2} \\ &= \frac{324 - 81\pi}{324} \\ &= \frac{4 - \pi}{4} \\ &\approx 0.215. \end{aligned}$$

► You are more likely to get 0 points.

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- You flip a coin and roll a six-sided die. What is the probability that the coin shows tails and the die shows 4?

Find $P(\bar{A})$.

- $P(A) = 0.45$
- $P(A) = 1$
- In Example 4, are you more likely to get 10 points or 5 points?
- In Example 4, are you more likely to score points (10, 5, or 2) or get 0 points?
- $P(A) = \frac{1}{4}$
- $P(A) = 0.03$

Experimental Probabilities

An **experimental probability** is based on repeated *trials* of a probability experiment. The number of trials is the number of times the probability experiment is performed. Each trial in which a favorable outcome occurs is called a *success*. The experimental probability can be found using the following.

$$\text{Experimental probability} = \frac{\text{Number of successes}}{\text{Number of trials}}$$

EXAMPLE 5 Finding an Experimental Probability

Spinner Results			
red	green	blue	yellow
5	9	3	3

Each section of the spinner shown has the same area. The spinner was spun 20 times. The table shows the results. For which color is the experimental probability of stopping on the color the same as the theoretical probability?



SOLUTION

The theoretical probability of stopping on each of the four colors is $\frac{1}{4}$. Use the outcomes in the table to find the experimental probabilities.

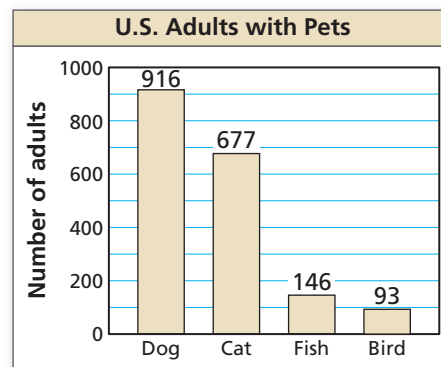
$$P(\text{red}) = \frac{5}{20} = \frac{1}{4} \qquad P(\text{green}) = \frac{9}{20}$$

$$P(\text{blue}) = \frac{3}{20} \qquad P(\text{yellow}) = \frac{3}{20}$$

- The experimental probability of stopping on red is the same as the theoretical probability.

EXAMPLE 6 Solving a Real-Life Problem

In the United States, a survey of 2184 adults ages 18 and over found that 1328 of them have at least one pet. The types of pets these adults have are shown in the figure. What is the probability that a pet-owning adult chosen at random has a dog?



SOLUTION

The number of trials is the number of pet-owning adults, 1328. A success is a pet-owning adult who has a dog. From the graph, there are 916 adults who said that they have a dog.

$$P(\text{pet-owning adult has a dog}) = \frac{916}{1328} = \frac{229}{332} \approx 0.690$$

- The probability that a pet-owning adult chosen at random has a dog is about 69%.

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- In Example 5, for which color is the experimental probability of stopping on the color greater than the theoretical probability?
- In Example 6, what is the probability that a pet-owning adult chosen at random owns a fish?

Vocabulary and Core Concept Check

- COMPLETE THE SENTENCE** A number that describes the likelihood of an event is the _____ of the event.
- WRITING** Describe the difference between theoretical probability and experimental probability.

Monitoring Progress and Modeling with Mathematics

In Exercises 3–6, find the number of possible outcomes in the sample space. Then list the possible outcomes.

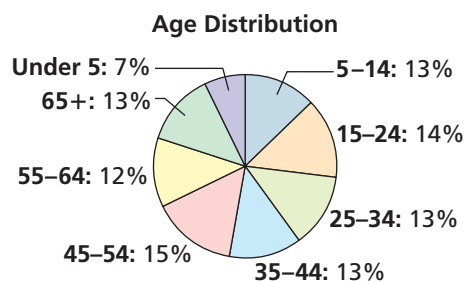
(See Example 1.)

- You roll a die and flip three coins.
- You flip a coin and draw a marble at random from a bag containing two purple marbles and one white marble.
- A bag contains four red cards numbered 1 through 4, four white cards numbered 1 through 4, and four black cards numbered 1 through 4. You choose a card at random.
- You draw two marbles without replacement from a bag containing three green marbles and four black marbles.
- PROBLEM SOLVING** A game show airs on television five days per week. Each day, a prize is randomly placed behind one of two doors. The contestant wins the prize by selecting the correct door. What is the probability that exactly two of the five contestants win a prize during a week? (See Example 2.)



- PROBLEM SOLVING** Your friend has two standard decks of 52 playing cards and asks you to randomly draw one card from each deck. What is the probability that you will draw two spades?
- PROBLEM SOLVING** When two six-sided dice are rolled, there are 36 possible outcomes. Find the probability that (a) the sum is not 4 and (b) the sum is greater than 5. (See Example 3.)

- PROBLEM SOLVING** The age distribution of a population is shown. Find the probability of each event.



- A person chosen at random is at least 15 years old.
 - A person chosen at random is from 25 to 44 years old.
- ERROR ANALYSIS** A student randomly guesses the answers to two true-false questions. Describe and correct the error in finding the probability of the student guessing both answers correctly.



The student can either guess two incorrect answers, two correct answers, or one of each. So the probability of guessing both answers correctly is $\frac{1}{3}$.

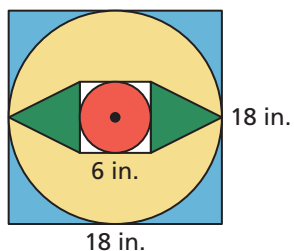
- ERROR ANALYSIS** A student randomly draws a number between 1 and 30. Describe and correct the error in finding the probability that the number drawn is greater than 4.



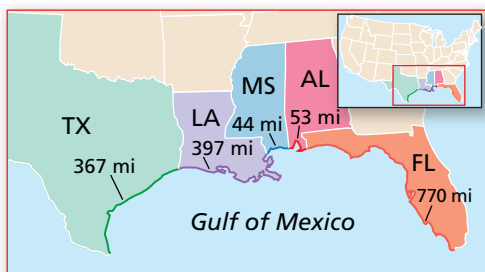
The probability that the number is less than 4 is $\frac{3}{30}$, or $\frac{1}{10}$. So, the probability that the number is greater than 4 is $1 - \frac{1}{10}$, or $\frac{9}{10}$.

13. MATHEMATICAL CONNECTIONS

You throw a dart at the board shown. Your dart is equally likely to hit any point inside the square board. What is the probability your dart lands in the yellow region? (See Example 4.)



14. MATHEMATICAL CONNECTIONS The map shows the length (in miles) of shoreline along the Gulf of Mexico for each state that borders the body of water. What is the probability that a ship coming ashore at a random point in the Gulf of Mexico lands in the given state?



- a. Texas
- b. Alabama
- c. Florida
- d. Louisiana

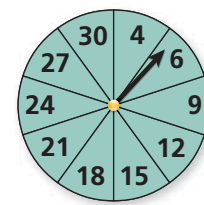
15. DRAWING CONCLUSIONS You roll a six-sided die 60 times. The table shows the results. For which number is the experimental probability of rolling the number the same as the theoretical probability? (See Example 5.)

Six-sided Die Results					
11	14	7	10	6	12

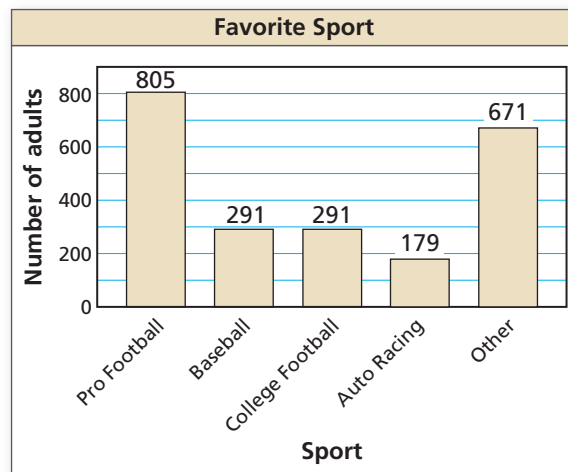
16. DRAWING CONCLUSIONS A bag contains 5 marbles that are each a different color. A marble is drawn, its color is recorded, and then the marble is placed back in the bag. This process is repeated until 30 marbles have been drawn. The table shows the results. For which marble is the experimental probability of drawing the marble the same as the theoretical probability?

Drawing Results				
white	black	red	green	blue
5	6	8	2	9

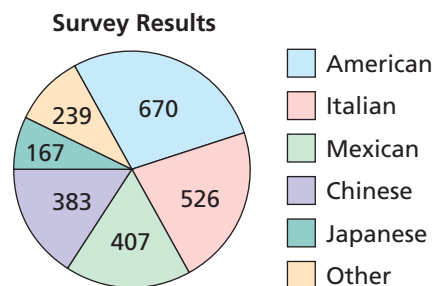
17. REASONING Refer to the spinner shown. The spinner is divided into sections with the same area.



- a. What is the theoretical probability that the spinner stops on a multiple of 3?
 - b. You spin the spinner 30 times. It stops on a multiple of 3 twenty times. What is the experimental probability of stopping on a multiple of 3?
 - c. Explain why the probability you found in part (b) is different than the probability you found in part (a).
- 18. OPEN-ENDED** Describe a real-life event that has a probability of 0. Then describe a real-life event that has a probability of 1.
- 19. DRAWING CONCLUSIONS** A survey of 2237 adults ages 18 and over asked which sport is their favorite. The results are shown in the figure. What is the probability that an adult chosen at random prefers auto racing? (See Example 6.)







20. DRAWING CONCLUSIONS A survey of 2392 adults ages 18 and over asked what type of food they would be most likely to choose at a restaurant. The results are shown in the figure. What is the probability that an adult chosen at random prefers Italian food?



21. **ANALYZING RELATIONSHIPS** Refer to the board in Exercise 13. Order the likelihoods that the dart lands in the given region from least likely to most likely.
- A. green B. not blue
C. red D. not yellow

22. **ANALYZING RELATIONSHIPS** Refer to the chart below. Order the following events from least likely to most likely.

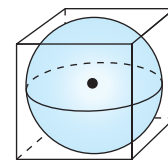
Four-Day Forecast			
Friday	Saturday	Sunday	Monday
			
Chance of Rain 5%	Chance of Rain 30%	Chance of Rain 80%	Chance of Rain 90%

- A. It rains on Sunday.
B. It does not rain on Saturday.
C. It rains on Monday.
D. It does not rain on Friday.
23. **USING TOOLS** Use the figure in Example 3 to answer each question.
- List the possible sums that result from rolling two six-sided dice.
 - Find the theoretical probability of rolling each sum.
 - The table below shows a simulation of rolling two six-sided dice three times. Use a random number generator to simulate rolling two six-sided dice 50 times. Compare the experimental probabilities of rolling each sum with the theoretical probabilities.

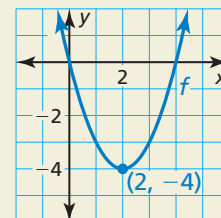
	A	B	C
1	First Die	Second Die	Sum
2	4	6	10
3	3	5	8
4	1	6	7
5			

24. **MAKING AN ARGUMENT** You flip a coin three times. It lands on heads twice and on tails once. Your friend concludes that the theoretical probability of the coin landing heads up is $P(\text{heads up}) = \frac{2}{3}$. Is your friend correct? Explain your reasoning.

25. **MATHEMATICAL CONNECTIONS** A sphere fits inside a cube so that it touches each side, as shown. What is the probability a point chosen at random inside the cube is also inside the sphere?



26. **HOW DO YOU SEE IT?** Consider the graph of f shown. What is the probability that the graph of $y = f(x) + c$ intersects the x -axis when c is a randomly chosen integer from 1 to 6? Explain.



27. **DRAWING CONCLUSIONS** A manufacturer tests 1200 computers and finds that 9 of them have defects. Find the probability that a computer chosen at random has a defect. Predict the number of computers with defects in a shipment of 15,000 computers. Explain your reasoning.

28. **THOUGHT PROVOKING** The tree diagram shows a sample space. Write a probability problem that can be represented by the sample space. Then write the answer(s) to the problem.

Box A	Box B	Outcomes	Sum	Product
1	1	(1, 1)	2	1
	2	(1, 2)	3	2
2	1	(2, 1)	3	2
	2	(2, 2)	4	4
3	1	(3, 1)	4	3
	2	(3, 2)	5	6

Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Find the product or quotient. (Section 7.3)

29. $\frac{3x}{y} \cdot \frac{2x^3}{y^2}$

30. $\frac{4x^2y}{3x^3} \cdot \frac{2xy}{8y^2}$

31. $\frac{x+3}{x^4-2} \cdot (x^2-7x+6)$

32. $\frac{2y}{5x} \div \frac{y}{6x}$

33. $\frac{3x}{12x-11} \div \frac{x+1}{5x}$

34. $\frac{3x^2+2x-13}{x^4} \div (x^2+9)$