7.4 Writing Equations in Two Variables

Essential Question How can you write an equation in two variables?

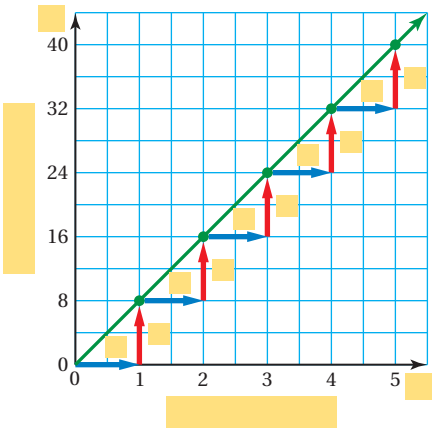
1 ACTIVITY: Writing an Equation in Two Variables

Work with a partner. You earn $8 per hour working part-time at a store.

a. Complete the table.

Hours Worked	Money Earned (dollars)
1	
2	
3	
4	
5	

b. Use the values from the table to complete the graph. Then answer each question below.



- What does the horizontal axis represent? What variable did you use to identify it?
- What does the vertical axis represent? What variable did you use to identify it?
- How are the ordered pairs in the graph related to the values in the table?
- How are the horizontal and vertical distances shown on the graph related to the values in the table?

c. How can you write an equation that shows how the two variables are related?

d. What does the green line in the graph represent?

Common Core

Writing Equations

In this lesson, you will

- identify independent and dependent variables.
- write equations in two variables.
- use tables and graphs to analyze the relationship between two variables.

Learning Standard 6.EE.9

2 ACTIVITY: Describing Variables

Work with a partner. Use the equation you wrote in Activity 1.

a. How is this equation different from the equations earlier in this chapter?

b. One of the variables in this equation *depends* on the other variable. Determine which variable is which by answering the following questions:

- Does the amount of money you earn *depend* on the number of hours you work?
- Does the number of hours you work *depend* on the amount of money you earn?

What do you think is the significance of having two types of variables? How do you think you can use these types of variables in real life?

3 ACTIVITY: Describing a Formula in Two Variables

Math Practice 7

Look for Patterns

What pattern do you notice in the table for the perimeter of the square?

Work with a partner. Recall that the perimeter of a square is 4 times its side length.

a. Write the formula for the perimeter of a square. Tell what each variable represents.

b. Describe how the perimeter of a square changes as its side length increases by 1 unit. Use a table and a graph to support your answer.

c. In your formula, which variable depends on which?

What Is Your Answer?

4. IN YOUR OWN WORDS How can you write an equation in two variables?

5. The equation $y = 7.75x$ shows how the number of movie tickets is related to the total amount of money spent. Describe what each part of the equation represents.

6. CHOOSE TOOLS In Activity 1, you want to know the amount of money you earn after working 30.5 hours during a week. Would you use the table, the graph, or the equation to find your earnings? What are your earnings? Explain your reasoning.

7. Give an example of another real-life situation that you can model by an equation in two variables.

Practice Use what you learned about equations in two variables to complete Exercises 4 and 5 on page 319.

7.4 Lesson

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An **equation in two variables** represents two quantities that change in relationship to one another. A **solution of an equation in two variables** is an ordered pair that makes the equation true.

EXAMPLE 1 Identifying Solutions of Equations in Two Variables

Key Vocabulary
- equation in two variables, *p. 316*
- solution of an equation in two variables, *p. 316*
- independent variable, *p. 316*
- dependent variable, *p. 316*

Tell whether the ordered pair is a solution of the equation.

a. $y = 2x$; (3, 6)

$6 \stackrel{?}{=} 2(3)$ Substitute.

$6 = 6$ ✓ Compare.

∴ So, (3, 6) is a solution.

b. $y = 4x - 3$; (4, 12)

$12 \stackrel{?}{=} 4(4) - 3$

$12 \neq 13$ ✗

∴ So, (4, 12) is *not* a solution.

You can use equations in two variables to represent situations involving two related quantities. The variable representing the quantity that can change freely is the **independent variable**. The other variable is called the **dependent variable** because its value *depends* on the independent variable.

EXAMPLE 2 Using an Equation in Two Variables

The equation $y = 128 - 8x$ gives the amount y (in fluid ounces) of milk remaining in a gallon jug after you pour x cups.

a. Identify the independent and dependent variables.

∴ Because the amount y remaining depends on the number x of cups you pour, y is the dependent variable and x is the independent variable.

b. How much milk remains in the jug after you pour 10 cups?

Use the equation to find the value of y when $x = 10$.

$y = 128 - 8x$ Write the equation.

$= 128 - 8(10)$ Substitute 10 for x.

$= 48$ Simplify.

∴ There are 48 fluid ounces remaining.

On Your Own

Now You're Ready
Exercises 6–11 and 13–17

Tell whether the ordered pair is a solution of the equation.

1. $y = 7x$; (2, 21)

2. $y = 5x + 1$; (3, 16)

3. The equation $y = 10x + 25$ gives the amount y (in dollars) in your savings account after x weeks.

a. Identify the independent and dependent variables.

b. How much is in your savings account after 8 weeks?

Key Idea

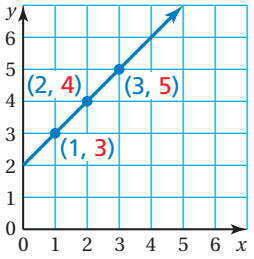
Study Tip

When you draw a line through the points, you graph *all* the solutions of the equation.

Tables, Graphs, and Equations

You can use tables and graphs to represent equations in two variables. The table and graph below represent the equation $y = x + 2$.

Independent Variable, x	Dependent Variable, y	Ordered Pair, (x, y)
1	3	(1, 3)
2	4	(2, 4)
3	5	(3, 5)



EXAMPLE 3 Writing and Graphing an Equation in Two Variables

An athlete burns 200 calories weight lifting. The athlete then works out on an elliptical trainer and burns 10 calories for every minute. Write and graph an equation in two variables that represents the total number of calories burned during the workout.

Reading

Make sure you read and understand the context of the problem. Because you cannot have a negative number of minutes, use only whole number values of m.

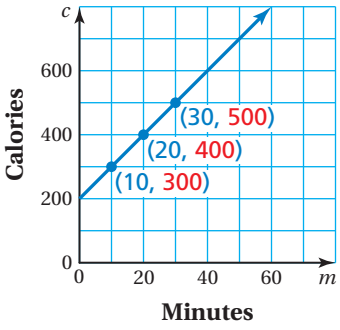
Words The total number of calories burned equals calories burned weight lifting plus calories burned per minute times the number of minutes.

Variables Let c be the total number of calories burned, and let m be the number of minutes on the elliptical trainer.

Equation $c = 200 + 10 \cdot m$

To graph the equation, first make a table. Then plot the ordered pairs and draw a line through the points.

Minutes, m	$c = 200 + 10m$	Calories, c	Ordered Pair, (m, c)
10	$c = 200 + 10(10)$	300	(10, 300)
20	$c = 200 + 10(20)$	400	(20, 400)
30	$c = 200 + 10(30)$	500	(30, 500)



On Your Own

Now You're Ready

Exercises 22 and 23

4. It costs \$25 to rent a kayak plus \$8 for each hour. Write and graph an equation in two variables that represents the total cost of renting the kayak.

You can model many rate problems by using the *distance formula* $d = rt$, where d is the distance traveled, r is the speed, and t is the time. When you are given a speed, you can use the formula to write an equation in two variables that represents the situation.

Remember

Speed is an example of a rate.

Key Idea

Distance Formula

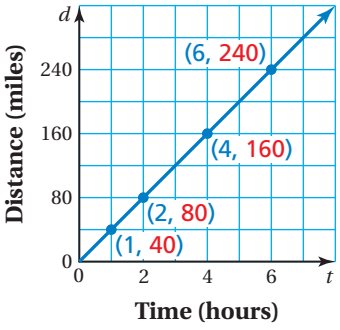
Words To find the distance traveled d, multiply the speed r by the time t.

Algebra $d = rt$

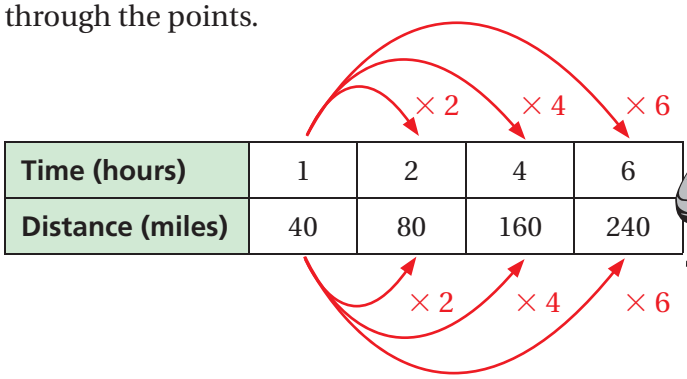
EXAMPLE 4 Real-Life Application

A train averages 40 miles per hour between two cities. Use a graph to show the relationship between the time and the distance traveled.

Method 1: Use a ratio table.



You can use a ratio table and multiplication to find equivalent rates. Then plot the ordered pairs (time, distance) from the table and draw a line through the points.



Method 2: Use an equation in two variables.

Use the distance formula to write the equation $d = 40t$. Use the equation to make a table. Then plot the ordered pairs and draw a line through the points, as shown in the graph above.

Time (hours), t	$d = 40t$	Distance (miles), d	Ordered Pair, (t, d)
1	$d = 40(1)$	40	(1, 40)
2	$d = 40(2)$	80	(2, 80)
4	$d = 40(4)$	160	(4, 160)
6	$d = 40(6)$	240	(6, 240)

On Your Own

Now You're Ready
Exercise 25

5. **WHAT IF?** The train averages 50 miles per hour. Use a graph to show the relationship between the time and the distance traveled.

7.4 Exercises

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Vocabulary and Concept Check

1. **VOCABULARY** How are independent variables and dependent variables different?
2. **PRECISION** Explain how to graph an equation in two variables.
3. **WHICH ONE DOESN'T BELONG?** Which one does *not* belong with the other three? Explain your reasoning.

$y = 12x + 25$ | $c = 10t - 5$ | $a = 7b + 11$ | $n = 4n - 6$

Practice and Problem Solving

Write a formula for the given measure. Tell what each variable represents. Identify which variable depends on which in the formula.

4. the perimeter of a rectangle with a length of 5 inches
5. the area of a trapezoid with base lengths of 7 feet and 11 feet

Tell whether the ordered pair is a solution of the equation.

1 6. $y = 4x$; (0, 4)
7. $y = 3x$; (2, 6)
8. $y = 5x - 10$; (3, 5)
9. $y = x + 7$; (1, 6)
10. $y = 7x + 2$; (2, 0)
11. $y = 2x - 3$; (4, 5)

12. **ERROR ANALYSIS** Describe and correct the error in finding a solution of the equation in two variables.

✗
$y = 3x + 2$; (5, 1)
$5 \stackrel{?}{=} 3(1) + 2$
$5 = 5$
So, (5, 1) is a solution.

Identify the independent and dependent variables.

2 13. The equation $A = 25w$ gives the area A (in square feet) of a rectangular dance floor with a width of w feet.

14. The equation $c = 0.09s$ gives the amount c (in dollars) of commission a salesperson receives for making a sale of s dollars.

15. The equation $t = 12p + 12$ gives the total cost t (in dollars) of a meal with a tip of p percent (in decimal form).

16. The equation $h = 60 - 4m$ gives the height h (in inches) of the water in a tank m minutes after it starts to drain.

17. **DRUM SET** The equation $b = 540 - 30m$ gives the balance b (in dollars) that you owe on a drum set after m monthly payments. What is the balance after 9 monthly payments?

OPEN-ENDED **Complete the table by describing possible independent or dependent variables.**

	Independent Variable	Dependent Variable
18.	The number of hours you study for a test	
19.	The speed you are pedaling a bike	
20.		Your monthly cell phone bill
21.		The amount of money you earn

3 **22.** **PIZZA** A cheese pizza costs $5. Additional toppings cost $1.50 each. Write and graph an equation in two variables that represents the total cost of a pizza.

23. **GYM MEMBERSHIP** It costs $35 to join a gym. The monthly fee is $25. Write and graph an equation in two variables that represents the total cost of a gym membership.

24. **TEXTING** The maximum size of a text message is 160 characters. A space counts as one character.

a. Write an equation in two variables that represents the remaining (unused) characters in a text message as you type.

b. Identify the independent and dependent variables.

c. How many characters remain in the message shown?

4 **25.** **CHOOSE TOOLS** A car averages 60 miles per hour on a road trip. Use a graph to show the relationship between the time and the distance traveled. What method did you use to create your graph?

Write and graph an equation in two variables that shows the relationship between the time and the distance traveled.

26.

Moves 2 meters every 3 hours.

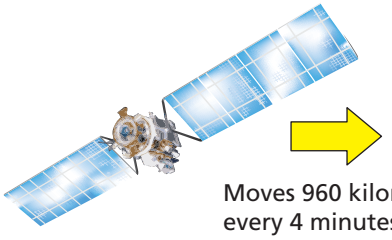
27.

Rises 5 stories every 6 seconds.

28.

Moves 660 feet every 10 seconds.

29.



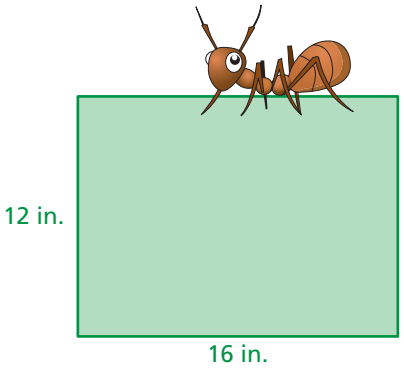
Moves 960 kilometers every 4 minutes.

Fill in the blank so that the ordered pair is a solution of the equation.

30. $y = 8x + 3$; (1, ____)

31. $y = 12x + 2$; (____, 14)

32. $y = 22 - 9x$; (____, 4)



33. CRITICAL THINKING Can the dependent variable cause a change in the independent variable? Explain.

34. OPEN-ENDED Write an equation in two variables that has (3, 4) as a solution.

35. WALKING You walk 5 city blocks in 12 minutes. How many city blocks can you walk in 2 hours?

36. ANT How fast should the ant walk to go around the rectangle in 4 minutes?

37. LIGHTNING To estimate how far you are from lightning (in miles), count the number of seconds between a lightning flash and the thunder that follows. Then divide the number of seconds by 5. Use a graph to show the relationship between the time and the distance. Describe the method you used to create your graph.

38. PROBLEM SOLVING You and a friend start biking in opposite directions from the same point. You travel 108 feet every 8 seconds. Your friend travels 63 feet every 6 seconds.

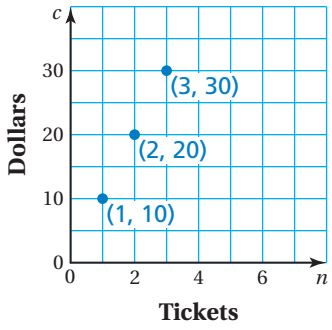
a. How far apart are you and your friend after 15 minutes?

b. After 20 minutes, you take a 5-minute rest, but your friend does not. How far apart are you and your friend after 40 minutes? Explain your reasoning.

39. Reasoning The graph represents the cost c (in dollars) of buying n tickets to a baseball game.

a. Should the points be connected with a line to show all the solutions? Explain your reasoning.

b. Write an equation in two variables that represents the graph.

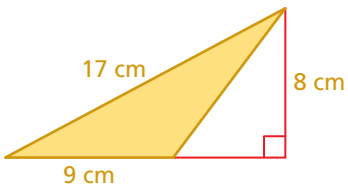


Fair Game Review What you learned in previous grades & lessons

Write the fraction as a percent. *(Section 5.5)*

40. $\frac{3}{10}$ **41.** $\frac{4}{5}$ **42.** $\frac{9}{20}$ **43.** $\frac{17}{25}$

44. MULTIPLE CHOICE What is the area of the triangle? *(Section 4.2)*



Ⓐ 36 cm^2 Ⓑ 68 cm^2

Ⓒ 72 cm^2 Ⓓ 76.5 cm^2