

# 11.3 Simplifying Rational Expressions

**Essential Question** How can you simplify a rational expression? What are the excluded values of a rational expression?

## 1 ACTIVITY: Simplifying a Rational Expression

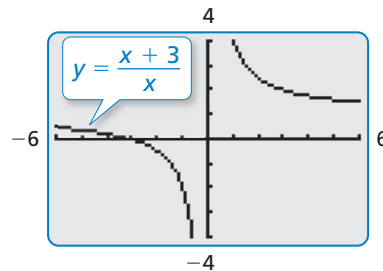
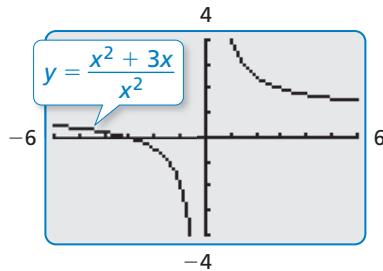
Work with a partner.

**Sample:** You can see that the rational expressions

$$\frac{x^2 + 3x}{x^2} \quad \text{and} \quad \frac{x + 3}{x}$$

are equivalent by graphing the related functions

$$y = \frac{x^2 + 3x}{x^2} \quad \text{and} \quad y = \frac{x + 3}{x}$$



COMMON  
CORE

### Rational Expressions

In this lesson, you will

- simplify rational expressions.

Learning Standard  
A.SSE.2

Both functions have the same graph.

**Match each rational expression with its equivalent rational expression.**

**Use a graphing calculator to check your answers.**

a.  $\frac{x^2 + x}{x^2}$       b.  $\frac{x^2}{x^2 + x}$       c.  $\frac{x + 1}{x^2 - 1}$       d.  $\frac{x + 1}{x^2 + 2x + 1}$       e.  $\frac{x^2 + 2x + 1}{x + 1}$

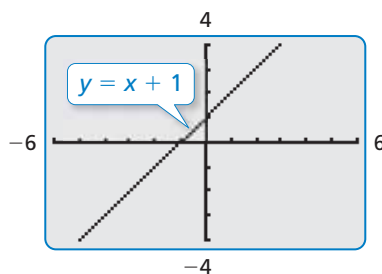
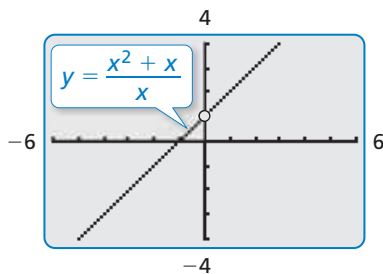
A.  $\frac{1}{x + 1}$       B.  $x + 1$       C.  $\frac{x + 1}{x}$       D.  $\frac{1}{x - 1}$       E.  $\frac{x}{x + 1}$

## 2 ACTIVITY: Finding Excluded Values

Work with a partner. Are the graphs of

$$y = \frac{x^2 + x}{x} \quad \text{and} \quad y = x + 1$$

*exactly* the same? Explain your reasoning.



## 3 ACTIVITY: Simplifying and Finding Excluded Values

### Math Practice 1

#### Explain the Meaning

What does it mean for a simplified expression to have an excluded value?

Work with a partner. Simplify each rational expression, if possible. Then compare the excluded value(s) of the original expression with the excluded value(s) of the simplified expression.

a.  $\frac{x^2 + 2x}{x^2}$

b.  $\frac{x^2}{x^2 + 2x}$

c.  $\frac{x^2}{x}$

d.  $\frac{x^2 + 4x + 4}{x + 2}$

e.  $\frac{x - 2}{x^2 - 4}$

f.  $\frac{1}{x^2 + 1}$

## What Is Your Answer?

4. **IN YOUR OWN WORDS** How can you simplify a rational expression? What are the excluded values of a rational expression? Include the following rational expressions in your answer.

a.  $\frac{x(x + 1)}{x}$

b.  $\frac{x^2 + 3x + 2}{x + 2}$

c.  $\frac{x + 3}{x^2 - 9}$

### Practice

Use what you learned about simplifying rational expressions to complete Exercises 3–5 on page 564.

# 11.3 Lesson

## Key Vocabulary

rational expression,  
p. 562  
simplest form of a  
rational expression,  
p. 562

## Study Tip

You can see why you can *divide out* common factors by rewriting the expression.

$$\frac{ac}{bc} = \frac{a}{b} \cdot \frac{c}{c} = \frac{a}{b} \cdot 1 = \frac{a}{b}$$

A **rational expression** is an expression that can be written as a fraction whose numerator and denominator are polynomials. Values that make the denominator of the expression zero are *excluded values*.

## Key Idea

### Simplifying Rational Expressions

**Words** A rational expression is in **simplest form** when the numerator and denominator have no common factors except 1. To simplify a rational expression, factor the numerator and denominator and *divide out* any common factors.

**Algebra** Let  $a$ ,  $b$ , and  $c$  be polynomials, where  $b, c \neq 0$ .

$$\frac{ac}{bc} = \frac{a \cdot \cancel{c}}{b \cdot \cancel{c}} = \frac{a}{b}$$

### Example

$$\frac{2(x+1)}{5(x+1)} = \frac{2}{5}; x \neq -1$$

## EXAMPLE 1 Simplifying Rational Expressions

Simplify each rational expression, if possible. State the excluded value(s).

a.  $\frac{12}{2x^2} = \frac{\cancel{2} \cdot 2 \cdot 3}{\cancel{2} \cdot x \cdot x}$  Divide out the common factor.  
 $= \frac{6}{x^2}$  Simplify.

∴ The excluded value is  $x = 0$ .

b.  $\frac{n}{n+8}$

∴ The expression is in simplest form. The excluded value is  $n = -8$ .

c.  $\frac{3y^2}{6y(y-7)} = \frac{\cancel{3} \cdot \cancel{y} \cdot y}{2 \cdot \cancel{3} \cdot \cancel{y} \cdot (y-7)}$  Divide out the common factors.  
 $= \frac{y}{2(y-7)}$  Simplify.

∴ The excluded values are  $y = 0$  and  $y = 7$ .

## On Your Own

Simplify the rational expression, if possible. State the excluded value(s).

1.  $\frac{5y^3}{2y^2}$

2.  $\frac{8x(x+1)}{12x^2}$

3.  $\frac{m+1}{m(m+3)}$

Now You're Ready  
Exercises 3–8

## EXAMPLE 2 Simplifying Rational Expressions

Simplify each rational expression, if possible. State the excluded value(s).

a. 
$$\frac{1 - z^2}{z - 1} = \frac{(1 - z)(1 + z)}{z - 1}$$
 Difference of Two Squares Pattern

$$= \frac{-(z - 1)(1 + z)}{z - 1}$$
 Rewrite  $1 - z$  as  $-(z - 1)$ .

$$= \frac{-\cancel{(z - 1)}(1 + z)}{\cancel{z - 1}}$$
 Divide out the common factor.

$$= -z - 1$$
 Simplify.

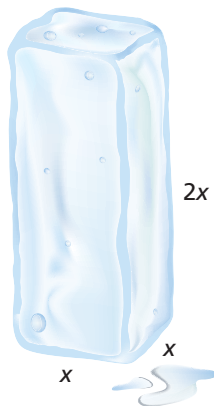
∴ The excluded value is  $z = 1$ .

b. 
$$\frac{c^2 + c - 12}{c^2 - c - 20} = \frac{\cancel{(c + 4)}(c - 3)}{\cancel{(c + 4)}(c - 5)}$$
 Factor. Divide out the common factor.

$$= \frac{c - 3}{c - 5}$$
 Simplify.

∴ The excluded values are  $c = -4$  and  $c = 5$ .

## EXAMPLE 3 Real-Life Application



In general, as the surface area to volume ratio of a substance increases, it reacts faster with other substances. Write and simplify this ratio for a block of ice that has the shape shown.

$$\frac{\text{Surface area}}{\text{Volume}} = \frac{2(x^2) + 4(2x^2)}{x(x)(2x)}$$
 Write an expression.

$$= \frac{10x^2}{2x^3}$$
 Simplify. Divide out the common factors.

$$= \frac{5}{x}$$
 Simplify.

### On Your Own

**Now You're Ready**  
Exercises 10–15

Simplify the rational expression, if possible. State the excluded value(s).

4.  $\frac{2b + 8}{7b + 28}$

5.  $\frac{2a - 6}{4a^2 - 12a}$

6.  $\frac{z^2 - 6z - 16}{8 - z}$

7. What is the surface area to volume ratio of a cube-shaped substance with edge length  $x$ ?

## Vocabulary and Concept Check

- VOCABULARY** Is  $\frac{\sqrt{x}-1}{x+3}$  a rational expression? Explain.
- REASONING** Why is it necessary to state excluded values of a rational expression?

## Practice and Problem Solving

Simplify the rational expression, if possible. State the excluded value(s).

3.  $\frac{6}{18x}$
4.  $\frac{15y^3}{5y^2}$
5.  $\frac{n-1}{n+1}$
6.  $\frac{9w^3}{12w^4}$
7.  $\frac{4t^2}{2t(t+1)}$
8.  $\frac{16x^2y}{24xy^3}$

- ERROR ANALYSIS** Describe and correct the error in stating the excluded value(s).

$$\frac{x^3}{x^2(x-3)} = \frac{x}{x-3}$$

The excluded value is  $x = 3$ .

Simplify the rational expression. State the excluded value(s).

10.  $\frac{3b+9}{8b+24}$
11.  $\frac{5-2z}{2z-5}$
12.  $\frac{6a^2+12a}{9a^3+18a^2}$
13.  $\frac{4-y^2}{y^2-3y-10}$
14.  $\frac{n^2+5n+6}{n^2+8n+15}$
15.  $\frac{3x^3-12x}{6x^3-24x^2+24x}$

- WRITING** Is  $\frac{(x+2)(x-5)}{(x-2)(5-x)}$  in simplest form? Explain.

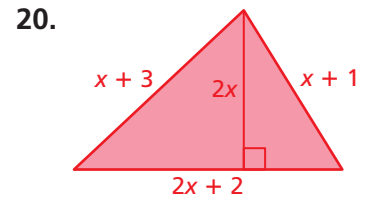
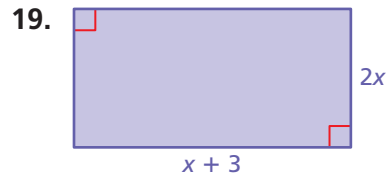
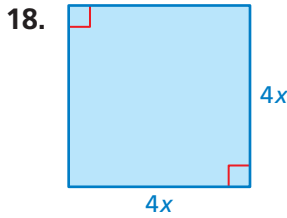
- RECYCLING** You hang recycling posters on bulletin boards at your school. Simplify the dimensions of the poster.



$$\frac{(x+3)^3}{(x+3)^2}$$

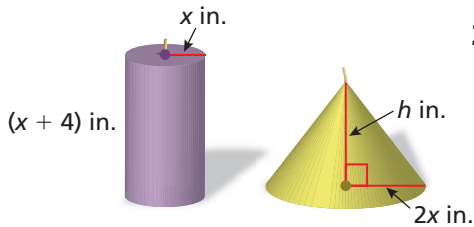
$$\frac{x^2-3x}{2x-6}$$

Write and simplify a rational expression for the ratio of the perimeter of the figure to its area.



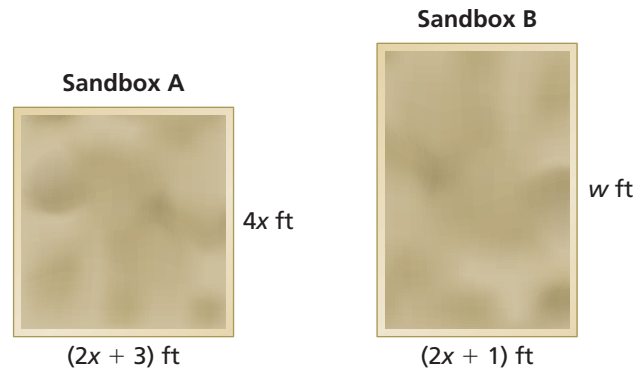
21. **OPEN-ENDED** Write a rational expression whose excluded values are  $-3$  and  $-5$ .

22. **WRITING** Is  $\frac{x^2 - 4}{x + 2}$  equivalent to  $x - 2$ ? Justify your answer.



23. **PROBLEM SOLVING** The candles shown have the same volume. Write and simplify an expression for the height of the cone-shaped candle.

24. **SANDBOX** The area of Sandbox B is 4 square feet greater than the area of Sandbox A. Write and simplify an expression for the width  $w$  of Sandbox B.



25. **Critical Thinking** Find two polynomials whose simplified ratio is  $\frac{4x + 1}{2x - 1}$  and whose sum is  $6x^2 + 12x$ . Explain your reasoning.



## Fair Game Review what you learned in previous grades & lessons

Graph the function. Is the domain discrete or continuous? (Section 5.2)

26.

Input Boxes, $x$	Output Number of Shoes, $y$
1	2
2	4
3	6

27.

Input Months, $x$	Output Height of Plant, $y$ (inches)
1	1.3
2	2.1
3	2.9

28. **MULTIPLE CHOICE** Consider  $f(x) = 2x - 4$ . What is the value of  $x$  so that  $f(x) = 8$ ? (Section 5.4)

(A) 2

(B) 4

(C) 6

(D) 7