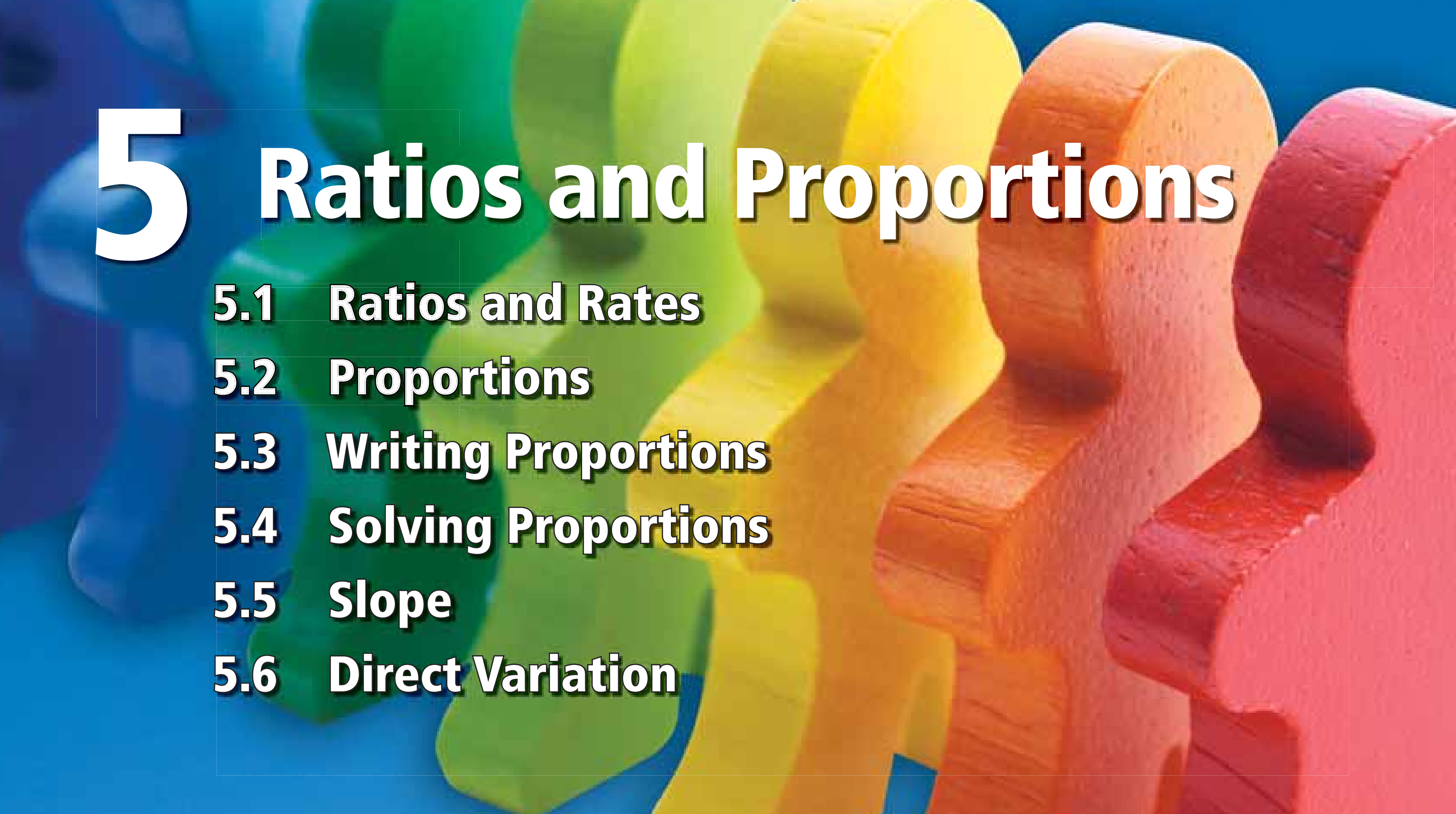
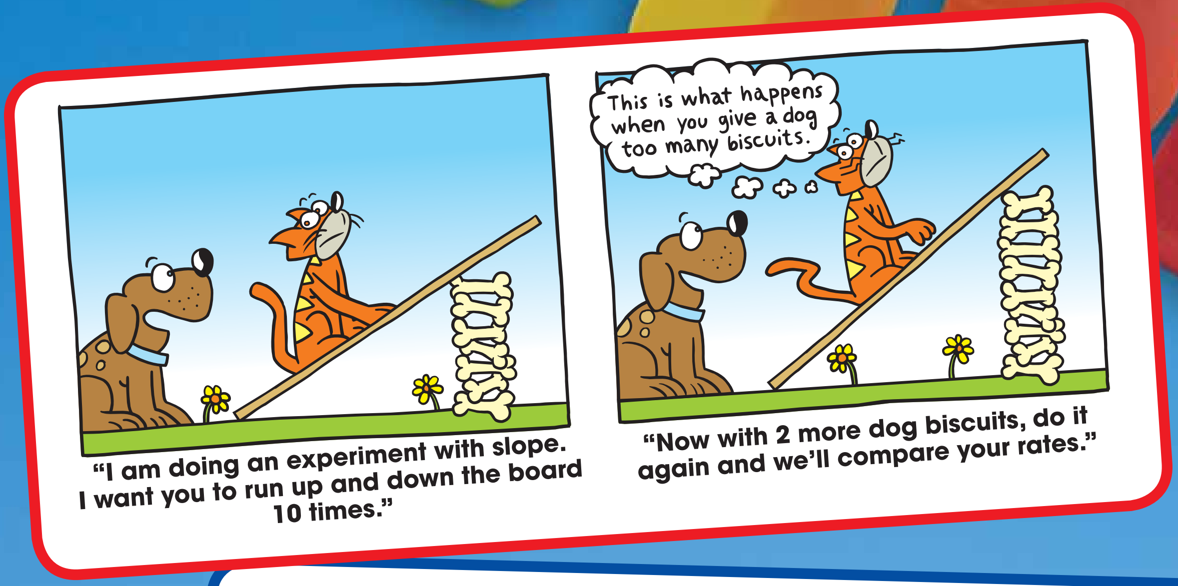


5 Ratios and Proportions



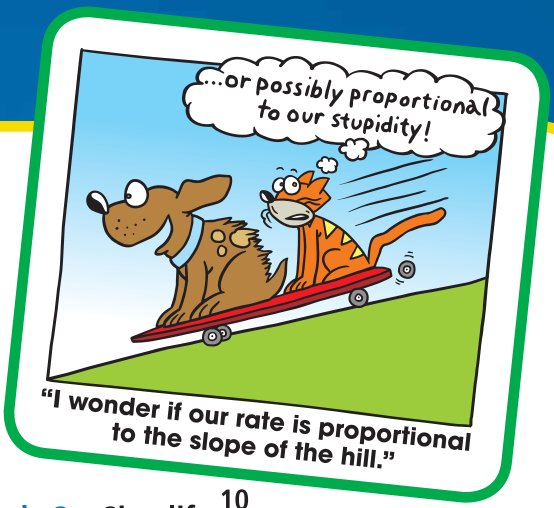
"I am doing an experiment with slope. I want you to run up and down the board 10 times."

"Now with 2 more dog biscuits, do it again and we'll compare your rates."

"Dear Sir: I counted the number of bacon, cheese, and chicken dog biscuits in the box I bought."

"There were 16 bacon, 12 cheese, and only 8 chicken. That's a ratio of 4:3:2. Please go back to the original ratio of 1:1:1."

What You Learned Before



"I wonder if our rate is proportional to the slope of the hill."

Simplifying Fractions

Example 1 **Simplify $\frac{4}{8}$.**

$$\frac{4 \div 4}{8 \div 4} = \frac{1}{2}$$

Example 2 **Simplify $\frac{10}{15}$.**

$$\frac{10 \div 5}{15 \div 5} = \frac{2}{3}$$

Identifying Equivalent Fractions

Example 3 **Is $\frac{1}{4}$ equivalent to $\frac{13}{52}$?**

$$\frac{13 \div 13}{52 \div 13} = \frac{1}{4}$$

∴ $\frac{1}{4}$ is equivalent to $\frac{13}{52}$.

Example 4 **Is $\frac{30}{54}$ equivalent to $\frac{5}{8}$?**

$$\frac{30 \div 6}{54 \div 6} = \frac{5}{9}$$

∴ $\frac{30}{54}$ is *not* equivalent to $\frac{5}{8}$.

Solving Equations

Example 5 **Solve $12x = 168$.**

$12x = 168$ Write the equation.

$\frac{12x}{12} = \frac{168}{12}$ Division Property of Equality

$x = 14$ Simplify.

Check

$12x = 168$

$12(14) \overset{?}{=} 168$

$168 = 168$ ✓

Try It Yourself

Simplify.

1. $\frac{12}{144}$ **2.** $\frac{15}{45}$ **3.** $\frac{75}{100}$ **4.** $\frac{16}{24}$

Are the fractions equivalent? Explain.

5. $\frac{15}{60} \overset{?}{=} \frac{3}{4}$ **6.** $\frac{2}{5} \overset{?}{=} \frac{24}{144}$ **7.** $\frac{15}{20} \overset{?}{=} \frac{3}{5}$ **8.** $\frac{2}{8} \overset{?}{=} \frac{16}{64}$

Solve the equation. Check your solution.

9. $\frac{y}{-5} = 3$ **10.** $0.6 = 0.2a$ **11.** $-2w = -9$ **12.** $\frac{1}{7}n = -4$

5.1 Ratios and Rates

Essential Question

How do rates help you describe real-life problems?

The Meaning of a Word ● Rate

When you rent snorkel gear at the beach, you should pay attention to the rental **rate**. The rental rate is in dollars per hour.

1 ACTIVITY: Finding Reasonable Rates

Work with a partner.

a. Match each description with a verbal rate.

b. Match each verbal rate with a numerical rate.

c. Give a reasonable numerical rate for each description. Then give an unreasonable rate.

Description	*Verbal Rate*	*Numerical Rate*
Your running rate in a 100-meter dash	Dollars per year	$\frac{\boxed{}\text{ in.}}{\text{yr}}$
The fertilization rate for an apple orchard	Inches per year	$\frac{\boxed{}\text{ lb}}{\text{acre}}$
The average pay rate for a professional athlete	Meters per second	$\frac{\$\boxed{}}{\text{yr}}$
The average rainfall rate in a rain forest	Pounds per acre	$\frac{\boxed{}\text{ m}}{\text{sec}}$

Ratios and Rates

In this lesson, you will

- find ratios, rates, and unit rates.
- find ratios and rates involving ratios of fractions.

2 ACTIVITY: Simplifying Expressions That Contain Fractions

Work with a partner. Describe a situation where the given expression may apply. Show how you can rewrite each expression as a division problem. Then simplify and interpret your result.

a. $\frac{\frac{1}{2}\text{ c}}{4\text{ fl oz}}$ **b.** $\frac{2\text{ in.}}{\frac{3}{4}\text{ sec}}$ **c.** $\frac{\frac{3}{8}\text{ c sugar}}{\frac{3}{5}\text{ c flour}}$ **d.** $\frac{\frac{5}{6}\text{ gal}}{\frac{2}{3}\text{ sec}}$

3 ACTIVITY: Using Ratio Tables to Find Equivalent Rates

Work with a partner. A communications satellite in orbit travels about 18 miles every 4 seconds.

a. Identify the rate in this problem.

b. Recall that you can use *ratio tables* to find and organize equivalent ratios and rates. Complete the ratio table below.

Time (seconds)	4	8	12	16	20
Distance (miles)					

c. How can you use a ratio table to find the speed of the satellite in miles per minute? miles per hour?

d. How far does the satellite travel in 1 second? Solve this problem (1) by using a ratio table and (2) by evaluating a quotient.

e. How far does the satellite travel in $\frac{1}{2}$ second? Explain your steps.

4 ACTIVITY: Unit Analysis

Math Practice

View as Components

What is the product of the numbers? What is the product of the units? Explain.

Work with a partner. Describe a situation where the product may apply. Then find each product and list the units.

a. $10 \text{ gal} \times \frac{22 \text{ mi}}{\text{gal}}$

b. $\frac{7}{2} \text{ lb} \times \frac{\$3}{\frac{1}{2} \text{ lb}}$

c. $\frac{1}{2} \text{ sec} \times \frac{30 \text{ ft}^2}{\text{sec}}$

What Is Your Answer?

5. **IN YOUR OWN WORDS** How do rates help you describe real-life problems? Give two examples.

6. To estimate the annual salary for a given hourly pay rate, multiply by 2 and insert “000” at the end.

Sample: \$10 per hour is about \$20,000 per year.

a. Explain why this works. Assume the person is working 40 hours a week.

b. Estimate the annual salary for an hourly pay rate of \$8 per hour.

c. You earn \$1 million per month. What is your annual salary?

d. Why is the cartoon funny?

“We had someone apply for the job. He says he would like \$1 million a month, but will settle for \$8 an hour.”

Practice Use what you discovered about ratios and rates to complete Exercises 7–10 on page 167.

5.1 Lesson

Check It Out
Lesson Tutorials
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Key Vocabulary

ratio, *p. 164*
rate, *p. 164*
unit rate, *p. 164*
complex fraction, *p. 165*

A **ratio** is a comparison of two quantities using division.

$$\frac{3}{4},\ 3 \text{ to } 4,\ 3:4$$

A **rate** is a ratio of two quantities with different units.

$$\frac{60 \text{ miles}}{2 \text{ hours}}$$

A rate with a denominator of 1 is called a **unit rate**.

$$\frac{30 \text{ miles}}{1 \text{ hour}}$$

EXAMPLE 1 Finding Ratios and Rates

There are 45 males and 60 females in a subway car. The subway car travels 2.5 miles in 5 minutes.

a. Find the ratio of males to females.

$$\frac{\text{males}}{\text{females}} = \frac{45}{60} = \frac{3}{4}$$

∴ The ratio of males to females is $\frac{3}{4}$.

b. Find the speed of the subway car.

$$2.5 \text{ miles in } 5 \text{ minutes} = \frac{2.5 \text{ mi}}{5 \text{ min}} = \frac{2.5 \text{ mi} \div 5}{5 \text{ min} \div 5} = \frac{0.5 \text{ mi}}{1 \text{ min}}$$

∴ The speed is 0.5 mile per minute.

EXAMPLE 2 Finding a Rate from a Ratio Table

The ratio table shows the costs for different amounts of artificial turf. Find the unit rate in dollars per square foot.

× 4 × 4 × 4

Amount (square feet)	25	100	400	1600
Cost (dollars)	100	400	1600	6400

× 4 × 4 × 4

Use a ratio from the table to find the unit rate.

$$\frac{\text{cost}}{\text{amount}} = \frac{\$100}{25 \text{ ft}^2} \qquad \text{Use the first ratio in the table.}$$

$$= \frac{\$4}{1 \text{ ft}^2} \qquad \text{Simplify.}$$

∴ So, the unit rate is \$4 per square foot.

Remember

The abbreviation ft^2 means *square feet*.

On Your Own

Now You're Ready
Exercises 11–24

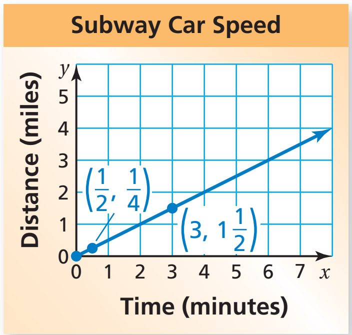
1. In Example 1, find the ratio of females to males.
2. In Example 1, find the ratio of females to total passengers.
3. The ratio table shows the distance that the *International Space Station* travels while orbiting Earth. Find the speed in miles per second.

Time (seconds)	3	6	9	12
Distance (miles)	14.4	28.8	43.2	57.6

A **complex fraction** has at least one fraction in the numerator, denominator, or both. You may need to simplify complex fractions when finding ratios and rates.

EXAMPLE 3 Finding a Rate from a Graph

The graph shows the speed of a subway car. Find the speed in miles per minute. Compare the speed to the speed of the subway car in Example 1.



Step 1: Choose and interpret a point on the line.

The point $\left(\frac{1}{2}, \frac{1}{4}\right)$ indicates that the subway car travels $\frac{1}{4}$ mile in $\frac{1}{2}$ minute.

Step 2: Find the speed.

$$\frac{\text{distance traveled}}{\text{elapsed time}} = \frac{\frac{1}{4} \leftarrow \boxed{\text{miles}}}{\frac{1}{2} \leftarrow \boxed{\text{minutes}}}$$

$$= \frac{1}{4} \div \frac{1}{2} \qquad \text{Rewrite the quotient.}$$

$$= \frac{1}{4} \cdot 2 = \frac{1}{2} \qquad \text{Simplify.}$$

The speed of the subway car is $\frac{1}{2}$ mile per minute.

Because $\frac{1}{2}$ mile per minute = 0.5 mile per minute, the speeds of the two subway cars are the same.

On Your Own

Now You're Ready
Exercise 28

4. You use the point $\left(3, 1\frac{1}{2}\right)$ to find the speed of the subway car. Does your answer change? Explain your reasoning.

EXAMPLE 4 Solving a Ratio Problem

STEM VIDEO

You mix $\frac{1}{2}$ cup of yellow paint for every $\frac{3}{4}$ cup of blue paint to make 15 cups of green paint. How much yellow paint and blue paint do you use?

Method 1: The ratio of yellow paint to blue paint is $\frac{1}{2}$ to $\frac{3}{4}$. Use a ratio table to find an equivalent ratio in which the total amount of yellow paint and blue paint is 15 cups.

	Yellow (cups)	Blue (cups)	Total (cups)	
	$\frac{1}{2}$	$\frac{3}{4}$	$\frac{1}{2} + \frac{3}{4} = \frac{5}{4}$	
× 4	2	3	5	× 4
× 3	6	9	15	× 3

So, you use 6 cups of yellow paint and 9 cups of blue paint.

Method 2: Use the fraction of the green paint that is made from yellow paint and the fraction of the green paint that is made from blue paint. You use $\frac{1}{2}$ cup of yellow paint for every $\frac{3}{4}$ cup of blue paint, so the fraction of the green paint that is made from yellow paint is

$$\begin{array}{l}\text{yellow} \rightarrow \\ \text{green} \rightarrow\end{array} \frac{\frac{1}{2}}{\frac{1}{2}+\frac{3}{4}} = \frac{\frac{1}{2}}{\frac{5}{4}} = \frac{1}{2} \cdot \frac{4}{5} = \frac{2}{5}.$$

Similarly, the fraction of the green paint that is made from blue paint is

$$\begin{array}{l}\text{blue} \rightarrow \\ \text{green} \rightarrow\end{array} \frac{\frac{3}{4}}{\frac{1}{2}+\frac{3}{4}} = \frac{\frac{3}{4}}{\frac{5}{4}} = \frac{3}{4} \cdot \frac{4}{5} = \frac{3}{5}.$$

So, you use $\frac{2}{5} \cdot 15 = 6$ cups of yellow paint and $\frac{3}{5} \cdot 15 = 9$ cups of blue paint.

Math Practice

Analyze Givens

What information is given in the problem? How does this help you know that the ratio table needs a "total" column? Explain.

On Your Own

Now You're Ready
Exercises 33 and 34

5. How much yellow paint and blue paint do you use to make 20 cups of green paint?

5.1 Exercises

Check It Out
Help with Homework
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Vocabulary and Concept Check

1. **VOCABULARY** How can you tell when a rate is a unit rate?
2. **WRITING** Why do you think rates are usually written as unit rates?
3. **OPEN-ENDED** Write a real-life rate that applies to you.

Estimate the unit rate.

4. \$74.75

5. \$1.19

6. \$2.35

Practice and Problem Solving

Find the product. List the units.

7. $8\text{ h} \times \frac{\$9}{\text{h}}$
8. $8\text{ lb} \times \frac{\$3.50}{\text{lb}}$
9. $\frac{29}{2}\text{ sec} \times \frac{60\text{ MB}}{\text{sec}}$
10. $\frac{3}{4}\text{ h} \times \frac{19\text{ mi}}{\frac{1}{4}\text{ h}}$

Write the ratio as a fraction in simplest form.

① 11. 25 to 45
12. 63 : 28
13. 35 girls : 15 boys
14. 51 correct : 9 incorrect
15. 16 dogs to 12 cats
16. $2\frac{1}{3}$ feet : $4\frac{1}{2}$ feet

Find the unit rate.

17. 180 miles in 3 hours
18. 256 miles per 8 gallons
19. \$9.60 for 4 pounds
20. \$4.80 for 6 cans
21. 297 words in 5.5 minutes
22. $21\frac{3}{4}$ meters in $2\frac{1}{2}$ hours

Use the ratio table to find the unit rate with the specified units.

② 23. servings per package

Packages	3	6	9	12
Servings	13.5	27	40.5	54

24. feet per year

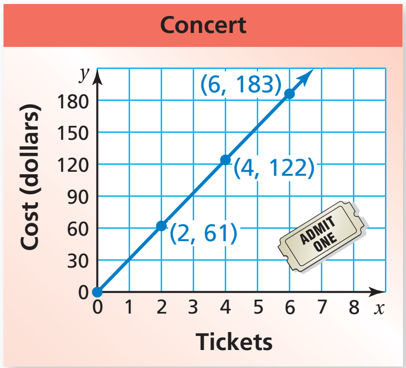
Years	2	6	10	14
Feet	7.2	21.6	36	50.4

25. **DOWNLOAD** At 1:00 P.M., you have 24 megabytes of a movie. At 1:15 P.M., you have 96 megabytes. What is the download rate in megabytes per minute?

26. **POPULATION** In 2007, the U.S. population was 302 million people. In 2012, it was 314 million. What was the rate of population change per year?

27. **PAINTING** A painter can paint 350 square feet in 1.25 hours. What is the painting rate in square feet per hour?

③ **28.** **TICKETS** The graph shows the cost of buying tickets to a concert.



a. What does the point (4, 122) represent?

b. What is the unit rate?

c. What is the cost of buying 10 tickets?

29. **CRITICAL THINKING** Are the two statements equivalent? Explain your reasoning.

- The ratio of boys to girls is 2 to 3.
- The ratio of girls to boys is 3 to 2.

30. **TENNIS** A sports store sells three different packs of tennis balls. Which pack is the best buy? Explain.

31. **FLOORING** It costs $68 for 16 square feet of flooring. How much does it cost for 12 square feet of flooring?

32. **OIL SPILL** An oil spill spreads 25 square meters every $\frac{1}{6}$ hour. How much area does the oil spill cover after 2 hours?

④ **33.** **JUICE** You mix $\frac{1}{4}$ cup of juice concentrate for every 2 cups of water to make 18 cups of juice. How much juice concentrate and water do you use?

34. **LANDSCAPING** A supplier sells $2\frac{1}{4}$ pounds of mulch for every $1\frac{1}{3}$ pounds of gravel. The supplier sells 172 pounds of mulch and gravel combined. How many pounds of each item does the supplier sell?

35. **HEART RATE** Your friend's heart beats 18 times in 15 seconds when at rest. While running, your friend's heart beats 25 times in 10 seconds.

a. Find the heart rate in beats per minute at rest and while running.

b. How many more times does your friend's heart beat in 3 minutes while running than while at rest?

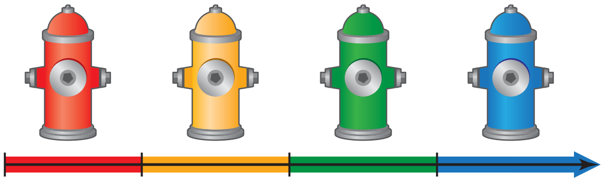
36. **PRECISION** The table shows nutritional information for three beverages.

a. Which has the most calories per fluid ounce?

b. Which has the least sodium per fluid ounce?

Beverage	Serving Size	Calories	Sodium
Whole milk	1 c	146	98 mg
Orange juice	1 pt	210	10 mg
Apple juice	24 fl oz	351	21 mg

37. **RESEARCH** Fire hydrants are painted one of four different colors to indicate the rate at which water comes from the hydrant.



a. Use the Internet to find the ranges of the rates for each color.

b. Research why a firefighter needs to know the rate at which water comes out of a hydrant.

38. **PAINT** You mix $\frac{2}{5}$ cup of red paint for every $\frac{1}{4}$ cup of blue paint to make $1\frac{5}{8}$ gallons of purple paint.

a. How much red paint and blue paint do you use?

b. You decide that you want to make a lighter purple paint. You make the new mixture by adding $\frac{1}{10}$ cup of white paint for every $\frac{2}{5}$ cup of red paint and $\frac{1}{4}$ cup of blue paint. How much red paint, blue paint, and white paint do you use to make $\frac{3}{8}$ gallon of lighter purple paint?

39. **Critical Thinking** You and a friend start hiking toward each other from opposite ends of a 17.5-mile hiking trail. You hike $\frac{2}{3}$ mile every $\frac{1}{4}$ hour. Your friend hikes $2\frac{1}{3}$ miles per hour.

a. Who hikes faster? How much faster?

b. After how many hours do you meet?

c. When you meet, who hiked farther? How much farther?

Fair Game Review What you learned in previous grades & lessons

Copy and complete the statement using <, >, or =. *(Section 2.1)*

40. $\frac{9}{2} \square \frac{8}{3}$

41. $-\frac{8}{15} \square \frac{10}{18}$

42. $\frac{-6}{24} \square \frac{-2}{8}$

43. **MULTIPLE CHOICE** Which fraction is greater than $-\frac{2}{3}$ and less than $-\frac{1}{2}$? *(Section 2.1)*

Ⓐ $-\frac{3}{4}$ Ⓑ $-\frac{7}{12}$ Ⓒ $-\frac{5}{12}$ Ⓓ $-\frac{3}{8}$

5.2 Proportions

Essential Question How can proportions help you decide when things are "fair"?

The Meaning of a Word ● Proportional

When you work toward a goal, your success is usually **proportional** to the amount of work you put in.

An equation stating that two ratios are equal is a **proportion**.

1 ACTIVITY: Determining Proportions

Work with a partner. Tell whether the two ratios are equivalent. If they are not equivalent, change the next day to make the ratios equivalent. Explain your reasoning.

a. On the first day, you pay $5 for 2 boxes of popcorn. The next day, you pay $7.50 for 3 boxes.

First Day Next Day

$$\frac{\$5.00}{2 \text{ boxes}} \overset{?}{=} \frac{\$7.50}{3 \text{ boxes}}$$

b. On the first day, it takes you $3\frac{1}{2}$ hours to drive 175 miles. The next day, it takes you 5 hours to drive 200 miles.

First Day Next Day

$$\frac{3\frac{1}{2} \text{ h}}{175 \text{ mi}} \overset{?}{=} \frac{5 \text{ h}}{200 \text{ mi}}$$

c. On the first day, you walk 4 miles and burn 300 calories. The next day, you walk $3\frac{1}{3}$ miles and burn 250 calories.

First Day Next Day

$$\frac{4 \text{ mi}}{300 \text{ cal}} \overset{?}{=} \frac{3\frac{1}{3} \text{ mi}}{250 \text{ cal}}$$

d. On the first day, you paint 150 square feet in $2\frac{1}{2}$ hours. The next day, you paint 200 square feet in 4 hours.

First Day Next Day

$$\frac{150 \text{ ft}^2}{2\frac{1}{2} \text{ h}} \overset{?}{=} \frac{200 \text{ ft}^2}{4 \text{ h}}$$

Proportions

In this lesson, you will

- use equivalent ratios to determine whether two ratios form a proportion.
- use the Cross Products Property to determine whether two ratios form a proportion.

2 ACTIVITY: Checking a Proportion

Work with a partner.

a. It is said that "one year in a dog's life is equivalent to seven years in a human's life." Explain why Newton thinks he has a score of 105 points. Did he solve the proportion correctly?

$$\frac{1 \text{ year}}{7 \text{ years}} \overset{?}{=} \frac{15 \text{ points}}{105 \text{ points}}$$

b. If Newton thinks his score is 98 points, how many points does he actually have? Explain your reasoning.

"I got 15 on my online test. That's 105 in dog points! Isn't that an A+?"

3 ACTIVITY: Determining Fairness

Math Practice

Justify Conclusions

What information can you use to justify your conclusion?

Work with a partner. Write a ratio for each sentence. Compare the ratios. If they are equal, then the answer is "It is fair." If they are not equal, then the answer is "It is not fair." Explain your reasoning.

a. You pay $184 for 2 tickets to a concert. & I pay $266 for 3 tickets to the same concert. → **Is this fair?**

b. You get 75 points for answering 15 questions correctly. & I get 70 points for answering 14 questions correctly. → **Is this fair?**

c. You trade 24 football cards for 15 baseball cards. & I trade 20 football cards for 32 baseball cards. → **Is this fair?**

What Is Your Answer?

4. Find a recipe for something you like to eat. Then show how two of the ingredient amounts are proportional when you double or triple the recipe.

5. **IN YOUR OWN WORDS** How can proportions help you decide when things are "fair"? Give an example.

Practice Use what you discovered about proportions to complete Exercises 15–20 on page 174.

5.2 Lesson

Check It Out
Lesson Tutorials
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Key Vocabulary
proportion, *p. 172*
proportional, *p. 172*
cross products, *p. 173*

Key Idea

Proportions

Words A **proportion** is an equation stating that two ratios are equivalent. Two quantities that form a proportion are **proportional**.

Numbers $\frac{2}{3} = \frac{4}{6}$ The proportion is read "2 is to 3 as 4 is to 6."

EXAMPLE 1 Determining Whether Ratios Form a Proportion

Tell whether $\frac{6}{4}$ and $\frac{8}{12}$ form a proportion.

Compare the ratios in simplest form.

$$\frac{6}{4} = \frac{6 \div 2}{4 \div 2} = \frac{3}{2}$$

$$\frac{8}{12} = \frac{8 \div 4}{12 \div 4} = \frac{2}{3}$$

The ratios are *not* equivalent.

So, $\frac{6}{4}$ and $\frac{8}{12}$ do *not* form a proportion.

EXAMPLE 2 Determining Whether Two Quantities Are Proportional

Tell whether x and y are proportional.

x	y
$\frac{1}{2}$	3
1	6
$\frac{3}{2}$	9
2	12

Compare each ratio x to y in simplest form.

$$\frac{\frac{1}{2}}{3} = \frac{1}{6} \qquad \frac{1}{6} \qquad \frac{\frac{3}{2}}{9} = \frac{1}{6} \qquad \frac{2}{12} = \frac{1}{6}$$

The ratios are equivalent.

So, x and y are proportional.

Reading

Two quantities that are proportional are in a *proportional relationship*.

On Your Own

Now You're Ready
Exercises 5–14

Tell whether the ratios form a proportion.

1. $\frac{1}{2}, \frac{5}{10}$
2. $\frac{4}{6}, \frac{18}{24}$
3. $\frac{10}{3}, \frac{5}{6}$
4. $\frac{25}{20}, \frac{15}{12}$
5. Tell whether x and y are proportional.

Birdhouses Built, *x*	1	2	4	6
Nails Used, *y*	12	24	48	72

Key Ideas

Cross Products

In the proportion $\frac{a}{b} = \frac{c}{d}$, the products $a \cdot d$ and $b \cdot c$ are called **cross products**.

Cross Products Property

Words The cross products of a proportion are equal.

Numbers

$$\frac{2}{3} = \frac{4}{6}$$

$2 \cdot 6 = 3 \cdot 4$

Algebra

$$\frac{a}{b} = \frac{c}{d}$$

$ad = bc$,
where $b \neq 0$ and $d \neq 0$

Study Tip

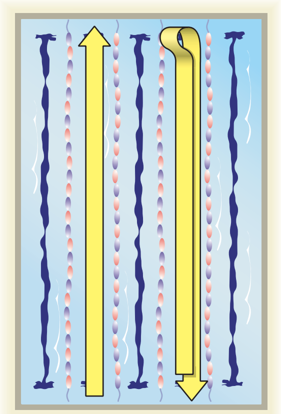
You can use the Multiplication Property of Equality to show that the cross products are equal.

$$\frac{a}{b} = \frac{c}{d}$$

$$\cancel{b}d \cdot \frac{a}{\cancel{b}} = b\cancel{d} \cdot \frac{c}{\cancel{d}}$$

$$ad = bc$$

EXAMPLE 3 Identifying Proportional Relationships



You swim your first 4 laps in 2.4 minutes. You complete 16 laps in 12 minutes. Is the number of laps proportional to your time?

Method 1: Compare unit rates.

÷ 4 ÷ 16

$$\frac{2.4 \text{ min}}{4 \text{ laps}} = \frac{0.6 \text{ min}}{1 \text{ lap}} \qquad \frac{12 \text{ min}}{16 \text{ laps}} = \frac{0.75 \text{ min}}{1 \text{ lap}}$$

÷ 4 ÷ 16

The unit rates are *not* equivalent.

So, the number of laps is *not* proportional to the time.

Method 2: Use the Cross Products Property.

$$\frac{2.4 \text{ min}}{4 \text{ laps}} \stackrel{?}{=} \frac{12 \text{ min}}{16 \text{ laps}}$$ Test to see if the rates are equivalent.

$$2.4 \cdot 16 \stackrel{?}{=} 4 \cdot 12$$ Find the cross products.

$$38.4 \neq 48$$ The cross products are *not* equal.

So, the number of laps is *not* proportional to the time.

On Your Own

Now You're Ready
Exercises 15–20

6. You read the first 20 pages of a book in 25 minutes. You read 36 pages in 45 minutes. Is the number of pages read proportional to your time?

5.2 Exercises

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Vocabulary and Concept Check

1. **VOCABULARY** What does it mean for two ratios to form a proportion?

2. **VOCABULARY** What are two ways you can tell that two ratios form a proportion?

3. **OPEN-ENDED** Write two ratios that are equivalent to $\frac{3}{5}$.

4. **WHICH ONE DOESN'T BELONG?** Which ratio does *not* belong with the other three? Explain your reasoning.

$\frac{4}{10}$ $\frac{2}{5}$ $\frac{3}{5}$ $\frac{6}{15}$

Practice and Problem Solving

Tell whether the ratios form a proportion.

① **5.** $\frac{1}{3}, \frac{7}{21}$ **6.** $\frac{1}{5}, \frac{6}{30}$ **7.** $\frac{3}{4}, \frac{24}{18}$ **8.** $\frac{2}{5}, \frac{40}{16}$

9. $\frac{48}{9}, \frac{16}{3}$ **10.** $\frac{18}{27}, \frac{33}{44}$ **11.** $\frac{7}{2}, \frac{16}{6}$ **12.** $\frac{12}{10}, \frac{14}{12}$

Tell whether *x* and *y* are proportional.

② **13.**

x	1	2	3	4
y	7	8	9	10

14.

x	2	4	6	8
y	5	10	15	20

Tell whether the two rates form a proportion.

③ **15.** 7 inches in 9 hours; 42 inches in 54 hours

16. 12 players from 21 teams; 15 players from 24 teams

17. 440 calories in 4 servings; 300 calories in 3 servings

18. 120 units made in 5 days; 88 units made in 4 days

19. 66 wins in 82 games; 99 wins in 123 games

20. 68 hits in 172 at bats; 43 hits in 123 at bats

21. **FITNESS** You can do 90 sit-ups in 2 minutes. Your friend can do 135 sit-ups in 3 minutes. Do these rates form a proportion? Explain.

22. **HEART RATES** Find the heart rates of you and your friend. Do these rates form a proportion? Explain.

	Heartbeats	Seconds
You	22	20
Friend	18	15

Tell whether the ratios form a proportion.

23. $\frac{2.5}{4}, \frac{7}{11.2}$

24. 2 to 4, 11 to $\frac{11}{2}$

25. $2 : \frac{4}{5}, \frac{3}{4} : \frac{3}{10}$

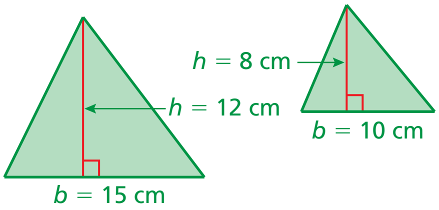
26. **PAY RATE** You earn \$56 walking your neighbor's dog for 8 hours. Your friend earns \$36 painting your neighbor's fence for 4 hours.

a. What is your pay rate?

b. What is your friend's pay rate?

c. Are the pay rates equivalent? Explain.

27. **GEOMETRY** Are the heights and bases of the two triangles proportional? Explain.



28. **BASEBALL** A pitcher coming back from an injury limits the number of pitches thrown in bull pen sessions as shown.

Session Number, x	Pitches, y	Curveballs, z
1	10	4
2	20	8
3	30	12
4	40	16

a. Which quantities are proportional?

b. How many pitches that are not curveballs do you think the pitcher will throw in Session 5?

29. **NAIL POLISH** A specific shade of red nail polish requires 7 parts red to 2 parts yellow. A mixture contains 35 quarts of red and 8 quarts of yellow. How can you fix the mixture to make the correct shade of red?

30. **COIN COLLECTION** The ratio of quarters to dimes in a coin collection is 5 : 3. You add the same number of new quarters as dimes to the collection.

a. Is the ratio of quarters to dimes still 5 : 3?

b. If so, illustrate your answer with an example. If not, show why with a "counterexample."

31. **AGE** You are 13 years old, and your cousin is 19 years old. As you grow older, is your age proportional to your cousin's age? Explain your reasoning.

32. Critical Thinking Ratio *A* is equivalent to Ratio *B*. Ratio *B* is equivalent to Ratio *C*. Is Ratio *A* equivalent to Ratio *C*? Explain.

Fair Game Review What you learned in previous grades & lessons

Add or subtract. *(Section 1.2 and Section 1.3)*

33. $-28 + 15$

34. $-6 + (-11)$

35. $-10 - 8$

36. $-17 - (-14)$

37. **MULTIPLE CHOICE** Which fraction is not equivalent to $\frac{2}{6}$? *(Skills Review Handbook)*

Ⓐ $\frac{1}{3}$ Ⓑ $\frac{12}{36}$ Ⓒ $\frac{4}{12}$ Ⓓ $\frac{6}{9}$

Extension 5.2 Graphing Proportional Relationships

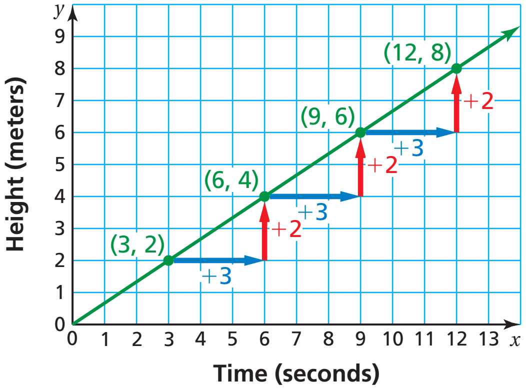
Check It Out
Lesson Tutorials
BigIdeasMath.com

Recall that you can graph the values from a ratio table.

Time, x (seconds)	Height, y (meters)
3	2
6	4
9	6
12	8

+ 3 + 3 + 3

+ 2 + 2 + 2



The structure in the ratio table shows why the graph has a constant *rate of change*. You can use the constant rate of change to show that the graph passes through the origin. The graph of every proportional relationship is a line through the origin.

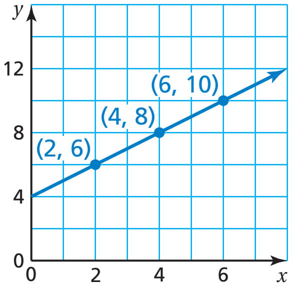
EXAMPLE 1 Determining Whether Two Quantities Are Proportional

Use a graph to tell whether x and y are in a proportional relationship.

a.

x	2	4	6
y	6	8	10

Plot (2, 6), (4, 8), and (6, 10). Draw a line through the points.



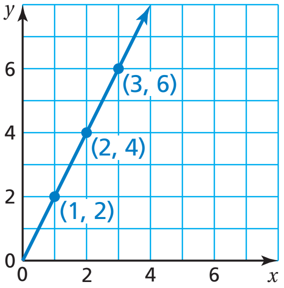
The graph is a line that does not pass through the origin.

∴ So, x and y are not in a proportional relationship.

b.

x	1	2	3
y	2	4	6

Plot (1, 2), (2, 4), and (3, 6). Draw a line through the points.



The graph is a line that passes through the origin.

∴ So, x and y are in a proportional relationship.

Proportions

In this extension, you will

- use graphs to determine whether two ratios form a proportion.
- interpret graphs of proportional relationships.

Practice

Use a graph to tell whether x and y are in a proportional relationship.

1.

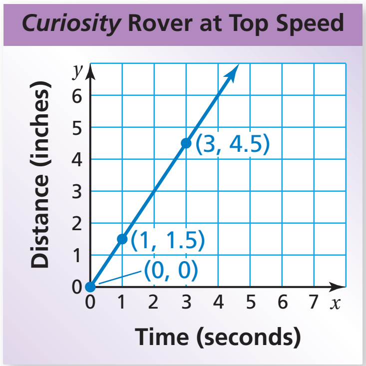
x	1	2	3	4
y	3	4	5	6

2.

x	1	3	5	7
y	0.5	1.5	2.5	3.5

EXAMPLE 2 Interpreting the Graph of a Proportional Relationship

The graph shows that the distance traveled by the Mars rover *Curiosity* is proportional to the time traveled. Interpret each plotted point in the graph.



(0, 0): The rover travels 0 inches in 0 seconds.

(1, 1.5): The rover travels 1.5 inches in 1 second. So, the unit rate is 1.5 inches per second.

(3, 4.5): The rover travels 4.5 inches in 3 seconds. Because the relationship is proportional, you can also use this point to find the unit rate.

$$\frac{4.5 \text{ in.}}{3 \text{ sec}} = \frac{1.5 \text{ in.}}{1 \text{ sec}}, \text{ or } 1.5 \text{ inches per second}$$

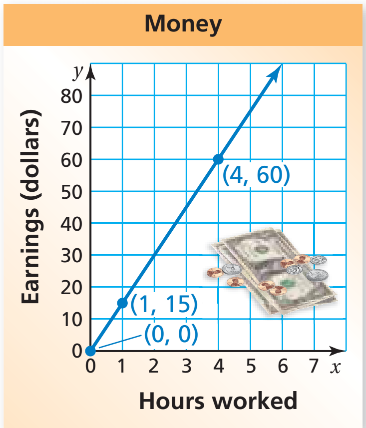
Study Tip

In the graph of a proportional relationship, you can find the unit rate from the point (1, *y*).

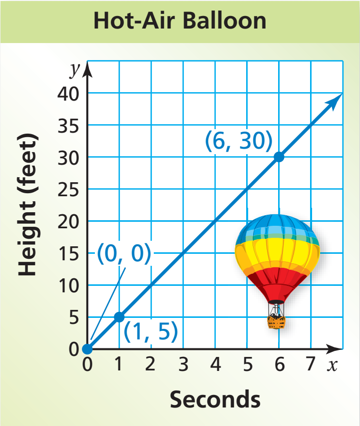
Practice

Interpret each plotted point in the graph of the proportional relationship.

3.



4.



Tell whether *x* and *y* are in a proportional relationship. If so, find the unit rate.

5.

***x* (hours)**	1	4	7	10
***y* (feet)**	5	20	35	50

6. Let *y* be the temperature *x* hours after midnight. The temperature is 60°F at midnight and decreases 2°F every $\frac{1}{2}$ hour.

7. **REASONING** The graph of a proportional relationship passes through (12, 16) and (1, *y*). Find *y*.

8. **MOVIE RENTAL** You pay $1 to rent a movie plus an additional $0.50 per day until you return the movie. Your friend pays $1.25 per day to rent a movie.

a. Make tables showing the costs to rent a movie up to 5 days.

b. Which person pays an amount proportional to the number of days rented?

5.3 Writing Proportions

Essential Question How can you write a proportion that solves a problem in real life?

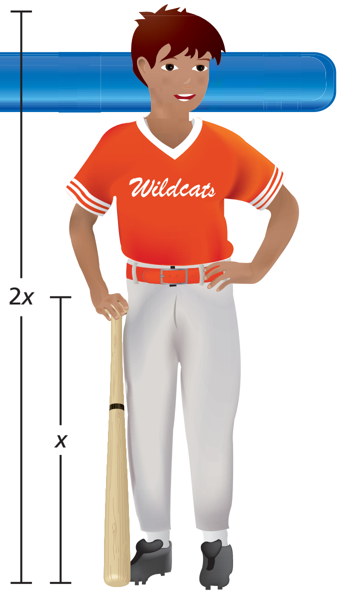
1 ACTIVITY: Writing Proportions

Work with a partner. A rough rule for finding the correct bat length is "the bat length should be half of the batter's height." So, a 62-inch-tall batter uses a bat that is 31 inches long. Write a proportion to find the bat length for each given batter height.

a. 58 inches

b. 60 inches

c. 64 inches



2 ACTIVITY: Bat Lengths

Work with a partner. Here is a more accurate table for determining the bat length for a batter. Find all the batter heights and corresponding weights for which the rough rule in Activity 1 is exact.

Weight of Batter (pounds)	Height of Batter (inches)							
	45–48	**49–52**	**53–56**	**57–60**	**61–64**	**65–68**	**69–72**	**Over 72**
Under 61	28	29	29					
61–70	28	29	30	30				
71–80	28	29	30	30	31			
81–90	29	29	30	30	31	32		
91–100	29	30	30	31	31	32		
101–110	29	30	30	31	31	32		
111–120	29	30	30	31	31	32		
121–130	29	30	30	31	32	33	33	
131–140	30	30	31	31	32	33	33	
141–150	30	30	31	31	32	33	33	
151–160	30	31	31	32	32	33	33	33
161–170		31	31	32	32	33	33	34
171–180				32	33	33	34	34
Over 180					33	33	34	34

Proportions

In this lesson, you will

- write proportions.
- solve proportions using mental math.

3 ACTIVITY: Writing Proportions

Math Practice

Evaluate Results

How do you know if your results are reasonable? Explain.

Work with a partner. The batting average of a baseball player is the number of "hits" divided by the number of "at bats."

$$\text{batting average} = \frac{\text{hits } (H)}{\text{at bats } (A)}$$

A player whose batting average is 0.250 is said to be "batting 250."

Actual hits

Actual at bats

$$\frac{20\text{ hits}}{80\text{ at bats}} = 0.250 = \frac{250\text{ hits}}{1000\text{ at bats}}$$

Batting 250 out of 1000

Batting average

Write a proportion to find how many hits *H* a player needs to achieve the given batting average. Then solve the proportion.

a. 50 times at bat; batting average is 0.200.

b. 84 times at bat; batting average is 0.250.

c. 80 times at bat; batting average is 0.350.

d. 1 time at bat; batting average is 1.000.

What Is Your Answer?

4. **IN YOUR OWN WORDS** How can you write a proportion that solves a problem in real life?

5. Two players have the same batting average.

	At Bats	Hits	Batting Average
Player 1	132	45	
Player 2	132	45	

Player 1 gets four hits in the next five at bats. Player 2 gets three hits in the next three at bats.

a. Who has the higher batting average?

b. Does this seem fair? Explain your reasoning.

Practice Use what you discovered about proportions to complete Exercises 4–7 on page 182.

5.3 Lesson

Check It Out
Lesson Tutorials
BigIdeasMath.com

One way to write a proportion is to use a table.

	Last Month	This Month
Purchase	2 ringtones	3 ringtones
Total Cost	6 dollars	x dollars

Use the columns or the rows to write a proportion.

Use columns:

$$\frac{2 \text{ ringtones}}{6 \text{ dollars}} = \frac{3 \text{ ringtones}}{x \text{ dollars}}$$

Numerators have the same units.

Denominators have the same units.

Use rows:

$$\frac{2 \text{ ringtones}}{3 \text{ ringtones}} = \frac{6 \text{ dollars}}{x \text{ dollars}}$$

The units are the same on each side of the proportion.

EXAMPLE 1 Writing a Proportion

Black Bean Soup

1.5 cups black beans
0.5 cup salsa
2 cups water
1 tomato
2 teaspoons seasoning

A chef increases the amounts of ingredients in a recipe to make a proportional recipe. The new recipe has 6 cups of black beans. Write a proportion that gives the number x of tomatoes in the new recipe.

Organize the information in a table.

	Original Recipe	New Recipe
Black Beans	1.5 cups	6 cups
Tomatoes	1 tomato	x tomatoes

∴ One proportion is $\dfrac{1.5 \text{ cups beans}}{1 \text{ tomato}} = \dfrac{6 \text{ cups beans}}{x \text{ tomatoes}}$.

On Your Own

Now You're Ready
Exercises 8–11

1. Write a different proportion that gives the number x of tomatoes in the new recipe.
2. Write a proportion that gives the amount y of water in the new recipe.

EXAMPLE 2 Solving Proportions Using Mental Math

Solve $\frac{3}{2} = \frac{x}{8}$.

Step 1: Think: The product of 2 and what number is 8?

$$\frac{3}{2} = \frac{x}{8}$$

2 × ? = 8

Step 2: Because the product of 2 and 4 is 8, multiply the numerator by 4 to find x.

3 × 4 = 12

$$\frac{3}{2} = \frac{x}{8}$$

2 × 4 = 8

The solution is $x = 12$.

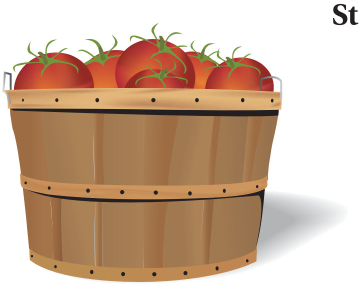
EXAMPLE 3 Solving Proportions Using Mental Math

In Example 1, how many tomatoes are in the new recipe?

Solve the proportion $\frac{1.5}{1} = \frac{6}{x}$.

cups black beans

tomatoes



Step 1: Think: The product of 1.5 and what number is 6?

1.5 × ? = 6

$$\frac{1.5}{1} = \frac{6}{x}$$

Step 2: Because the product of 1.5 and 4 is 6, multiply the denominator by 4 to find x.

1.5 × 4 = 6

$$\frac{1.5}{1} = \frac{6}{x}$$

1 × 4 = 4

So, there are 4 tomatoes in the new recipe.

On Your Own

Now You're Ready
Exercises 16–21

Solve the proportion.

3. $\frac{5}{8} = \frac{20}{d}$

4. $\frac{7}{z} = \frac{14}{10}$

5. $\frac{21}{24} = \frac{x}{8}$

6. A school has 950 students. The ratio of female students to all students is $\frac{48}{95}$. Write and solve a proportion to find the number f of students who are female.

5.3 Exercises

Vocabulary and Concept Check

1. **WRITING** Describe two ways you can use a table to write a proportion.

2. **WRITING** What is your first step when solving $\frac{x}{15} = \frac{3}{5}$? Explain.

3. **OPEN-ENDED** Write a proportion using an unknown value x and the ratio 5 : 6. Then solve it.

Practice and Problem Solving

Write a proportion to find how many points a student needs to score on the test to get the given score.

4. test worth 50 points; test score of 40%

5. test worth 50 points; test score of 78%

6. test worth 80 points; test score of 80%

7. test worth 150 points; test score of 96%

Use the table to write a proportion.

① 8.

	Game 1	Game 2
Points	12	18
Shots	14	w

9.

	May	June
Winners	n	34
Entries	85	170

10.

	Today	Yesterday
Miles	15	m
Hours	2.5	4

11.

	Race 1	Race 2
Meters	100	200
Seconds	x	22.4

12. **ERROR ANALYSIS** Describe and correct the error in writing the proportion.

✗

	Monday	Tuesday
Dollars	2.08	d
Ounces	8	16

$$\frac{2.08}{16} = \frac{d}{8}$$

13. **T-SHIRTS** You can buy 3 T-shirts for \$24. Write a proportion that gives the cost c of buying 7 T-shirts.

14. **COMPUTERS** A school requires 2 computers for every 5 students. Write a proportion that gives the number c of computers needed for 145 students.

15. **SWIM TEAM** The school team has 80 swimmers. The ratio of seventh-grade swimmers to all swimmers is 5 : 16. Write a proportion that gives the number s of seventh-grade swimmers.

Solve the proportion.

② ③ **16.** $\frac{1}{4} = \frac{z}{20}$

17. $\frac{3}{4} = \frac{12}{y}$

18. $\frac{35}{k} = \frac{7}{3}$

19. $\frac{15}{8} = \frac{45}{c}$

20. $\frac{b}{36} = \frac{5}{9}$

21. $\frac{1.4}{2.5} = \frac{g}{25}$

22. **ORCHESTRA** In an orchestra, the ratio of trombones to violas is 1 to 3.

a. There are 9 violas. Write a proportion that gives the number t of trombones in the orchestra.

b. How many trombones are in the orchestra?

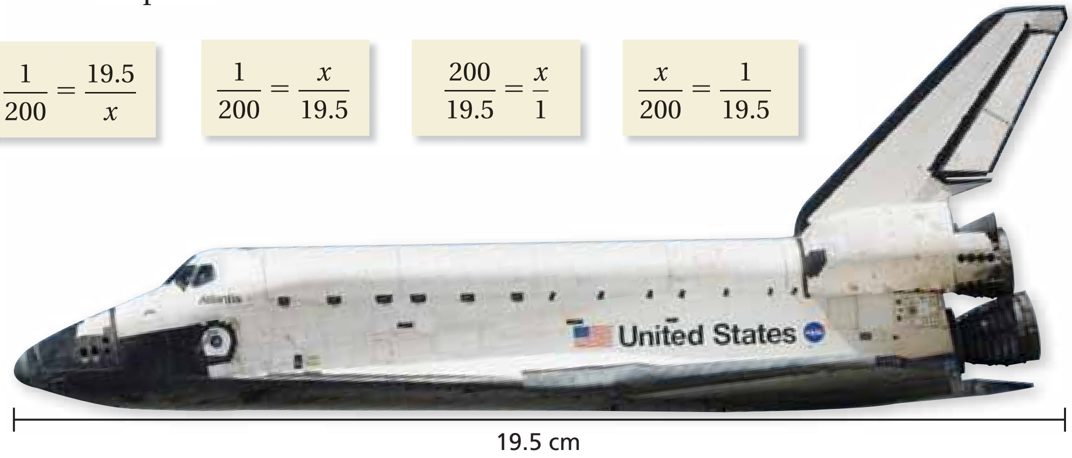
23. **ATLANTIS** Your science teacher has a 1 : 200 scale model of the space shuttle *Atlantis*. Which of the proportions can you use to find the actual length x of *Atlantis*? Explain.

$\frac{1}{200} = \frac{19.5}{x}$

$\frac{1}{200} = \frac{x}{19.5}$

$\frac{200}{19.5} = \frac{x}{1}$

$\frac{x}{200} = \frac{1}{19.5}$



24. **YOU BE THE TEACHER** Your friend says "$48x = 6 \cdot 12$." Is your friend right? Explain.

Solve $\frac{6}{x} = \frac{12}{48}$.

25. **Reasoning** There are 180 white lockers in the school. There are 3 white lockers for every 5 blue lockers. How many lockers are in the school?

Fair Game Review What you learned in previous grades & lessons

Solve the equation. *(Section 3.4)*

26. $\frac{x}{6} = 25$

27. $8x = 72$

28. $150 = 2x$

29. $35 = \frac{x}{4}$

30. **MULTIPLE CHOICE** What is the value of $-\frac{9}{4} + \left| -\frac{8}{5} \right| - 2\frac{1}{2}$? *(Section 2.3)*

Ⓐ $-6\frac{7}{20}$

Ⓑ $-5\frac{7}{20}$

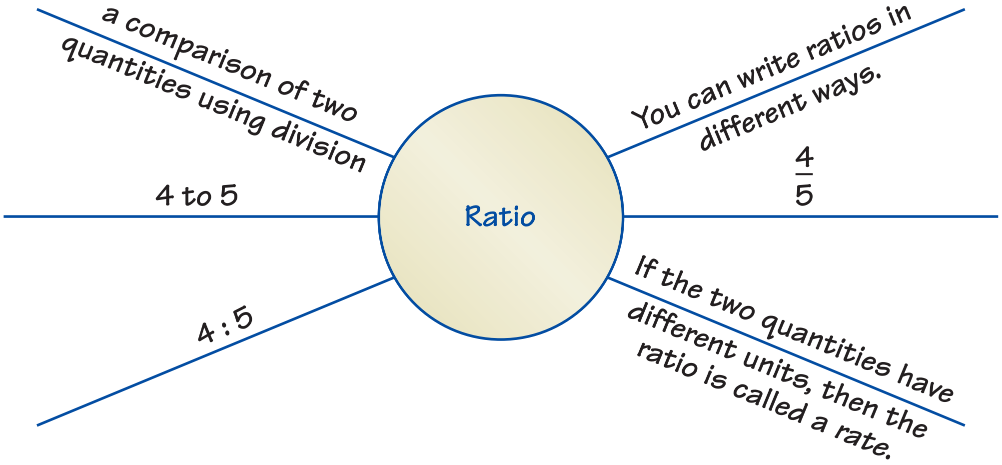
Ⓒ $-3\frac{3}{20}$

Ⓓ $-2\frac{3}{20}$

5 Study Help

Check It Out
Graphic Organizer
BigIdeasMath.com

You can use an **information wheel** to organize information about a concept. Here is an example of an information wheel for ratio.



On Your Own

Make information wheels to help you study these topics.

1. rate
2. unit rate
3. proportion
4. cross products
5. graphing proportional relationships

After you complete this chapter, make information wheels for the following topics.

6. solving proportions
7. slope
8. direct variation

"My **information wheel** summarizes how cats act when they get baths."

5.1–5.3 Quiz

Check It Out
Progress Check
BigIdeasMath.com

Write the ratio as a fraction in simplest form. *(Section 5.1)*

1. 18 red buttons : 12 blue buttons

2. $\frac{5}{4}$ inches to $\frac{2}{3}$ inch

Use the ratio table to find the unit rate with the specified units. *(Section 5.1)*

3. cost per song

Songs	0	2	4	6
Cost	\$0	\$1.98	\$3.96	\$5.94

4. gallons per hour

Hours	3	6	9	12
Gallons	10.5	21	31.5	42

Tell whether the ratios form a proportion. *(Section 5.2)*

5. $\frac{1}{8}, \frac{4}{32}$

6. $\frac{2}{3}, \frac{10}{30}$

7. $\frac{7}{4}, \frac{28}{16}$

Tell whether the two rates form a proportion. *(Section 5.2)*

8. 75 miles in 3 hours; 140 miles in 4 hours

9. 12 gallons in 4 minutes; 21 gallons in 7 minutes

10. 150 steps in 50 feet; 72 steps in 24 feet

11. 3 rotations in 675 days; 2 rotations in 730 days

Use the table to write a proportion. *(Section 5.3)*

12.

	Monday	**Tuesday**
Dollars	42	56
Hours	6	*h*

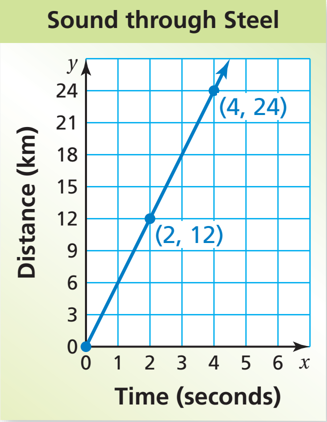
13.

	Series 1	**Series 2**
Games	*g*	6
Wins	4	3

14. **MUSIC DOWNLOAD** The amount of time needed to download music is shown in the table. Find the unit rate in megabytes per second. *(Section 5.1)*

Seconds	6	12	18	24
Megabytes	2	4	6	8

15. **SOUND** The graph shows the distance that sound travels through steel. Interpret each plotted point in the graph of the proportional relationship. *(Section 5.2)*



16. **GAMING** You advance 3 levels in 15 minutes. Your friend advances 5 levels in 20 minutes. Do these rates form a proportion? Explain. *(Section 5.2)*

17. **CLASS TIME** You spend 150 minutes in 3 classes. Write and solve a proportion to find how many minutes you spend in 5 classes. *(Section 5.3)*

5.4 Solving Proportions

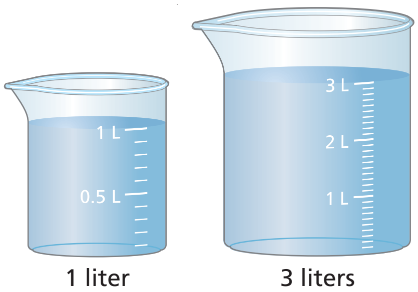
Essential Question How can you use ratio tables and cross products to solve proportions?

1 ACTIVITY: Solving a Proportion in Science

Work with a partner. You can use ratio tables to determine the amount of a compound (like salt) that is dissolved in a solution. Determine the unknown quantity. Explain your procedure.

a. Salt Water

Salt Water	1 L	3 L
Salt	250 g	x g



$$\frac{1\text{ L}}{\square} = \frac{\square}{\square}$$ Write proportion.

$$1 \cdot \square = \square \cdot \square$$ Set cross products equal.

$$\square = \square$$ Simplify.

There are ☐ grams of salt in the 3-liter solution.

b. White Glue Solution

Water	$\frac{1}{2}$ cup	1 cup
White Glue	$\frac{1}{2}$ cup	x cups

c. Borax Solution

Borax	1 tsp	2 tsp
Water	1 cup	x cups

d. Slime (See recipe.)

Borax Solution	$\frac{1}{2}$ cup	1 cup
White Glue Solution	y cups	x cups

Recipe for SLIME

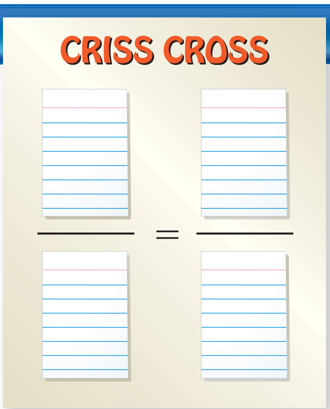
1. Add $\frac{1}{2}$ cup of water and $\frac{1}{2}$ cup white glue. Mix thoroughly. This is your white glue solution.
2. Add a couple drops of food coloring to the white glue solution. Mix thoroughly.
3. Add 1 teaspoon of borax to 1 cup of water. Mix thoroughly. This is your borax solution (about 1 cup).
4. Pour the borax solution and the glue solution into a separate bowl.
5. Place the slime that forms into a plastic bag. Squeeze the mixture repeatedly to mix it up.

Proportions

In this lesson, you will

- solve proportions using multiplication or the Cross Products Property.
- use a point on a graph to write and solve proportions.

2 ACTIVITY: The Game of Criss Cross



Math Practice

Use Operations

How can you use the name of the game to determine which operation to use?

Preparation:

- Cut index cards to make 48 playing cards.
- Write each number on a card.

  1, 1, 1, 2, 2, 2, 3, 3, 3, 4, 4, 4, 5, 5, 5, 6, 6, 6, 7, 7, 7, 8, 8, 8, 9, 9, 9, 10, 10, 10, 12, 12, 12, 13, 13, 13, 14, 14, 14, 15, 15, 15, 16, 16, 16, 18, 20, 25
- Make a copy of the game board.

To Play:

- Play with a partner.
- Deal eight cards to each player.
- Begin by drawing a card from the remaining cards. Use four of your cards to try to form a proportion.
- Lay the four cards on the game board. If you form a proportion, then say "Criss Cross." You earn 4 points. Place the four cards in a discard pile. Now it is your partner's turn.
- If you cannot form a proportion, then it is your partner's turn.
- When the original pile of cards is empty, shuffle the cards in the discard pile. Start again.
- The first player to reach 20 points wins.

What Is Your Answer?

3. **IN YOUR OWN WORDS** How can you use ratio tables and cross products to solve proportions? Give an example.

4. **PUZZLE** Use each number once to form three proportions.

1	2	10	4	12	20
15	5	16	6	8	3

Practice Use what you discovered about solving proportions to complete Exercises 10–13 on page 190.

5.4 Lesson

Check It Out
Lesson Tutorials
BigIdeasMath.com

Key Idea

Solving Proportions

Method 1 Use mental math. *(Section 5.3)*

Method 2 Use the Multiplication Property of Equality. *(Section 5.4)*

Method 3 Use the Cross Products Property. *(Section 5.4)*

EXAMPLE 1 Solving Proportions Using Multiplication

Solve $\frac{5}{7} = \frac{x}{21}$.

$\frac{5}{7} = \frac{x}{21}$ Write the proportion.

$21 \cdot \frac{5}{7} = 21 \cdot \frac{x}{21}$ Multiplication Property of Equality

$15 = x$ Simplify.

∴ The solution is 15.

On Your Own

Now You're Ready
Exercises 4–9

Use multiplication to solve the proportion.

1. $\frac{w}{6} = \frac{6}{9}$ **2.** $\frac{12}{10} = \frac{a}{15}$ **3.** $\frac{y}{6} = \frac{2}{4}$

EXAMPLE 2 Solving Proportions Using the Cross Products Property

Solve each proportion.

a. $\frac{x}{8} = \frac{7}{10}$ **b.** $\frac{9}{y} = \frac{3}{17}$

$x \cdot 10 = 8 \cdot 7$ Cross Products Property $9 \cdot 17 = y \cdot 3$

$10x = 56$ Multiply. $153 = 3y$

$x = 5.6$ Divide. $51 = y$

∴ The solution is 5.6. ∴ The solution is 51.

On Your Own

Now You're Ready
Exercises 10–21

Use the Cross Products Property to solve the proportion.

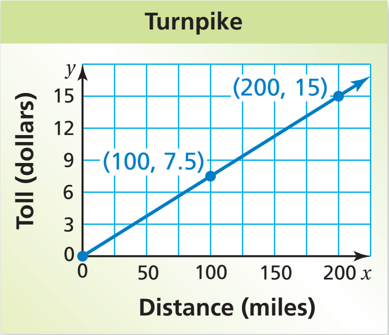
4. $\frac{2}{7} = \frac{x}{28}$

5. $\frac{12}{5} = \frac{6}{y}$

6. $\frac{40}{z+1} = \frac{15}{6}$

EXAMPLE 3 Real-Life Application

The graph shows the toll *y* due on a turnpike for driving *x* miles. Your toll is $7.50. How many *kilometers* did you drive?



The point (100, 7.5) on the graph shows that the toll is $7.50 for driving 100 miles. Convert 100 miles to kilometers.

Method 1: Convert using a ratio.

1 mi ≈ 1.61 km

$$100\,\cancel{\text{mi}} \times \frac{1.61\text{ km}}{1\,\cancel{\text{mi}}} = 161\text{ km}$$

So, you drove about 161 kilometers.

Method 2: Convert using a proportion.

Let x be the number of kilometers equivalent to 100 miles.

$$\begin{array}{r} \text{kilometers} \rightarrow \\ \text{miles} \rightarrow \end{array} \frac{1.61}{1} = \frac{x}{100} \begin{array}{l} \leftarrow \text{kilometers} \\ \leftarrow \text{miles} \end{array}$$ Write a proportion. Use 1.61 km ≈ 1 mi.

$$1.61 \cdot 100 = 1 \cdot x$$ Cross Products Property

$$161 = x$$ Simplify.

So, you drove about 161 kilometers.

On Your Own

Now You're Ready
Exercises 28–30

Write and solve a proportion to complete the statement. Round to the nearest hundredth, if necessary.

7. 7.5 in. ≈ ☐ cm

8. 100 g ≈ ☐ oz

9. 2 L ≈ ☐ qt

10. 4 m ≈ ☐ ft

5.4 Exercises

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Vocabulary and Concept Check

1. **WRITING** What are three ways you can solve a proportion?

2. **OPEN-ENDED** Which way would you choose to solve $\frac{3}{x} = \frac{6}{14}$? Explain your reasoning.

3. **NUMBER SENSE** Does $\frac{x}{4} = \frac{15}{3}$ have the same solution as $\frac{x}{15} = \frac{4}{3}$? Use the Cross Products Property to explain your answer.

Practice and Problem Solving

Use multiplication to solve the proportion.

① **4.** $\frac{9}{5} = \frac{z}{20}$
5. $\frac{h}{15} = \frac{16}{3}$
6. $\frac{w}{4} = \frac{42}{24}$

7. $\frac{35}{28} = \frac{n}{12}$
8. $\frac{7}{16} = \frac{x}{4}$
9. $\frac{y}{9} = \frac{44}{54}$

Use the Cross Products Property to solve the proportion.

② **10.** $\frac{a}{6} = \frac{15}{2}$
11. $\frac{10}{7} = \frac{8}{k}$
12. $\frac{3}{4} = \frac{v}{14}$
13. $\frac{5}{n} = \frac{16}{32}$

14. $\frac{36}{42} = \frac{24}{r}$
15. $\frac{9}{10} = \frac{d}{6.4}$
16. $\frac{x}{8} = \frac{3}{12}$
17. $\frac{8}{m} = \frac{6}{15}$

18. $\frac{4}{24} = \frac{c}{36}$
19. $\frac{20}{16} = \frac{d}{12}$
20. $\frac{30}{20} = \frac{w}{14}$
21. $\frac{2.4}{1.8} = \frac{7.2}{k}$

22. **ERROR ANALYSIS** Describe and correct the error in solving the proportion $\frac{m}{8} = \frac{15}{24}$.

✗

$$\frac{m}{8} = \frac{15}{24}$$
$$8 \cdot m = 24 \cdot 15$$
$$m = 45$$

23. **PENS** Forty-eight pens are packaged in 4 boxes. How many pens are packaged in 9 boxes?

24. **PIZZA PARTY** How much does it cost to buy 10 medium pizzas?

Solve the proportion.

25. $\frac{2x}{5} = \frac{9}{15}$
26. $\frac{5}{2} = \frac{d-2}{4}$
27. $\frac{4}{k+3} = \frac{8}{14}$

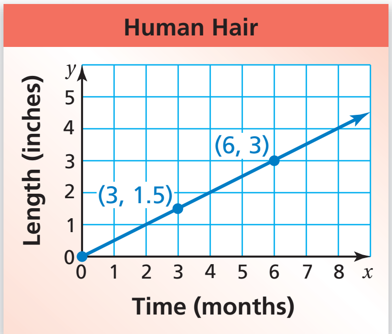
Write and solve a proportion to complete the statement. Round to the nearest hundredth if necessary.

③ **28.** 6 km ≈ ☐ mi **29.** 2.5 L ≈ ☐ gal **30.** 90 lb ≈ ☐ kg

31. TRUE OR FALSE? Tell whether the statement is *true* or *false*. Explain.

$$\text{If } \frac{a}{b} = \frac{2}{3}, \text{ then } \frac{3}{2} = \frac{b}{a}.$$

32. CLASS TRIP It costs \$95 for 20 students to visit an aquarium. How much does it cost for 162 students?



33. GRAVITY A person who weighs 120 pounds on Earth weighs 20 pounds on the Moon. How much does a 93-pound person weigh on the Moon?

34. HAIR The length of human hair is proportional to the number of months it has grown.

a. What is the hair length in *centimeters* after 6 months?

b. How long does it take hair to grow 8 inches?

c. Use a different method than the one in part (b) to find how long it takes hair to grow 20 inches.

35. SWING SET It takes 6 hours for 2 people to build a swing set. Can you use the proportion $\frac{2}{6} = \frac{5}{h}$ to determine the number of hours h it will take 5 people to build the swing set? Explain.

36. REASONING There are 144 people in an audience. The ratio of adults to children is 5 to 3. How many are adults?

37. PROBLEM SOLVING Three pounds of lawn seed covers 1800 square feet. How many bags are needed to cover 8400 square feet?

38. **Critical Thinking** Consider the proportions $\frac{m}{n} = \frac{1}{2}$ and $\frac{n}{k} = \frac{2}{5}$. What is the ratio $\frac{m}{k}$? Explain your reasoning.

Fair Game Review What you learned in previous grades & lessons

Plot the ordered pair in a coordinate plane. *(Skills Review Handbook)*

39. $A(-5, -2)$ **40.** $B(-3, 0)$ **41.** $C(-1, 2)$ **42.** $D(1, 4)$

43. MULTIPLE CHOICE Which expression is equivalent to $(3w - 8) - 4(2w + 3)$? *(Section 3.2)*

Ⓐ $11w + 4$ Ⓑ $-5w - 5$ Ⓒ $-5w + 4$ Ⓓ $-5w - 20$

5.5 Slope

Essential Question How can you compare two rates graphically?

1 ACTIVITY: Comparing Unit Rates

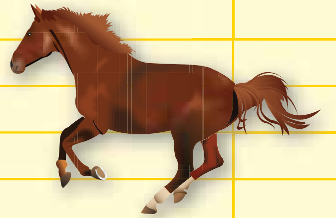
Work with a partner. The table shows the maximum speeds of several animals.

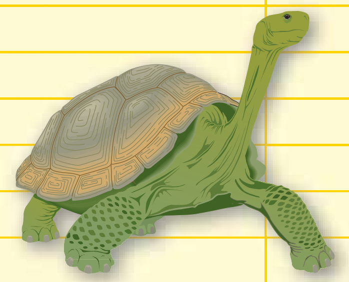
a. Find the missing speeds. Round your answers to the nearest tenth.

b. Which animal is fastest? Which animal is slowest?

c. Explain how you convert between the two units of speed.

Animal	Speed (miles per hour)	Speed (feet per second)
Antelope	61.0	
Black mamba snake		29.3
Cheetah		102.6
Chicken		13.2
Coyote	43.0	
Domestic pig		16.0
Elephant		36.6
Elk		66.0
Giant tortoise	0.2	
Giraffe	32.0	
Gray fox		61.6
Greyhound	39.4	
Grizzly bear		44.0
Human		41.0
Hyena	40.0	
Jackal	35.0	
Lion		73.3
Peregrine falcon	200.0	
Quarter horse	47.5	
Spider		1.76
Squirrel	12.0	
Thomson's gazelle	50.0	
Three-toed sloth		0.2
Tuna	47.0	





Slope

In this lesson, you will

- find the slopes of lines.
- interpret the slopes of lines as rates.

2 ACTIVITY: Comparing Two Rates Graphically

Math Practice

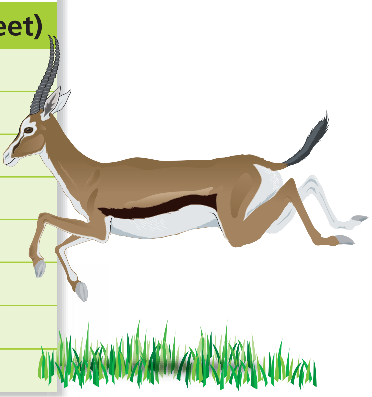
Apply Mathematics

How can you use the graph to determine which animal has the greater speed?

Work with a partner. A cheetah and a Thomson's gazelle run at maximum speed.

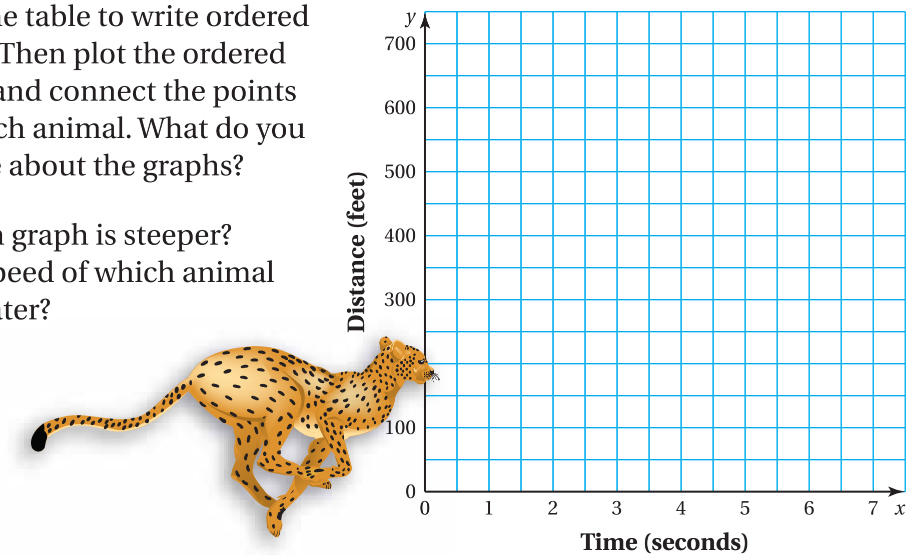
a. Use the table in Activity 1 to calculate the missing distances.

	Cheetah	Gazelle
Time (seconds)	**Distance (feet)**	**Distance (feet)**
0		
1		
2		
3		
4		
5		
6		
7		



b. Use the table to write ordered pairs. Then plot the ordered pairs and connect the points for each animal. What do you notice about the graphs?

c. Which graph is steeper? The speed of which animal is greater?



What Is Your Answer?

3. **IN YOUR OWN WORDS** How can you compare two rates graphically? Explain your reasoning. Give some examples with your answer.

4. **REPEATED REASONING** Choose 10 animals from Activity 1.

- **a.** Make a table for each animal similar to the table in Activity 2.
- **b.** Sketch a graph of the distances for each animal.
- **c.** Compare the steepness of the 10 graphs. What can you conclude?

5.5 Lesson

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Key Vocabulary
slope, *p. 194*

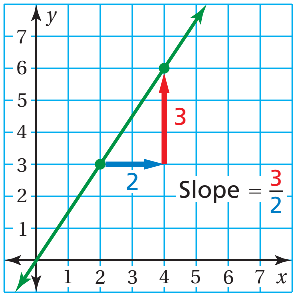
Key Idea

Slope

Slope is the rate of change between any two points on a line. It is a measure of the *steepness* of a line.

To find the slope of a line, find the ratio of the change in y (vertical change) to the change in x (horizontal change).

$$\text{slope} = \frac{\text{change in } y}{\text{change in } x}$$



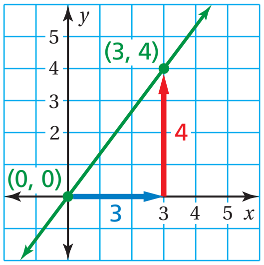
Study Tip

The slope of a line is the same between any two points on the line because lines have a *constant* rate of change.

EXAMPLE 1 Finding Slopes

Find the slope of each line.

a.

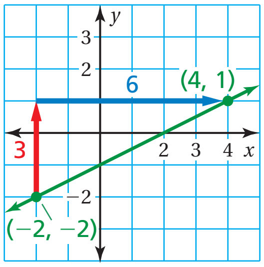


$$\text{slope} = \frac{\text{change in } y}{\text{change in } x}$$

$$= \frac{4}{3}$$

The slope of the line is $\frac{4}{3}$.

b.



$$\text{slope} = \frac{\text{change in } y}{\text{change in } x}$$

$$= \frac{3}{6} = \frac{1}{2}$$

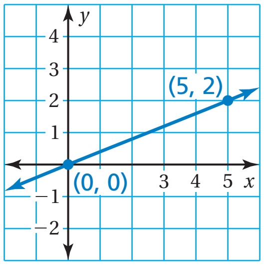
The slope of the line is $\frac{1}{2}$.

On Your Own

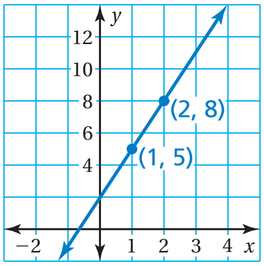
Now You're Ready
Exercises 4–9

Find the slope of the line.

1.



2.



EXAMPLE 2 Interpreting a Slope

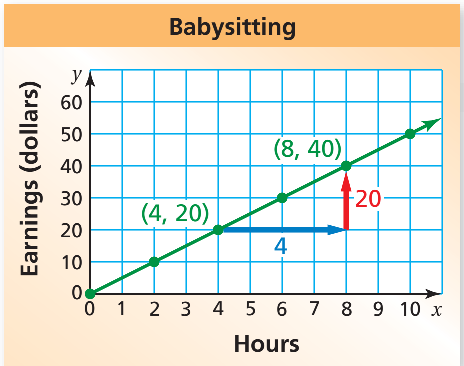
The table shows your earnings for babysitting.

a. Graph the data.

b. Find and interpret the slope of the line through the points.

Hours, x	0	2	4	6	8	10
Earnings, y (dollars)	0	10	20	30	40	50

a. Graph the data. Draw a line through the points.



b. Choose any two points to find the slope of the line.

$$\text{slope} = \frac{\text{change in } y}{\text{change in } x}$$

$$= \frac{20}{4} \begin{array}{l} \leftarrow \boxed{\text{dollars}} \\ \leftarrow \boxed{\text{hours}} \end{array}$$

$$= 5$$

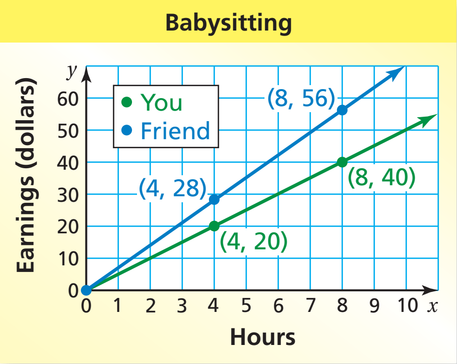
The slope of the line represents the unit rate. The slope is 5. So, you earn $5 per hour babysitting.

On Your Own

Now You're Ready
Exercises 10 and 11

3. In Example 2, use two other points to find the slope. Does the slope change?

4. The graph shows the amounts you and your friend earn babysitting.



a. Compare the steepness of the lines. What does this mean in the context of the problem?

b. Find and interpret the slope of the blue line.

5.5 Exercises

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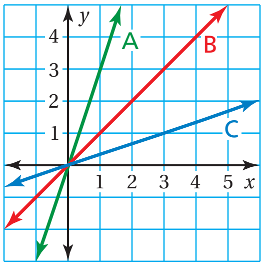
Vocabulary and Concept Check

1. **VOCABULARY** Is there a connection between rate and slope? Explain.

2. **REASONING** Which line has the greatest slope?

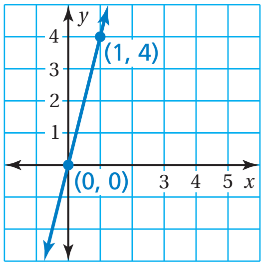


3. **REASONING** Is it more difficult to run up a ramp with a slope of $\frac{1}{5}$ or a ramp with a slope of 5? Explain.

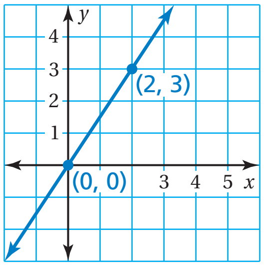
Practice and Problem Solving

Find the slope of the line.

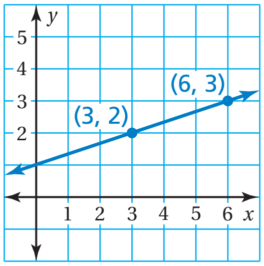
① 4.



5.



6.



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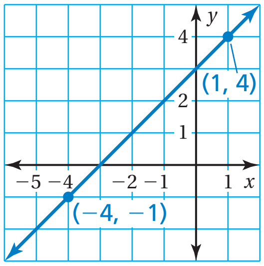

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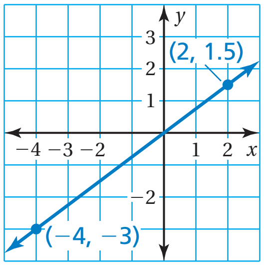

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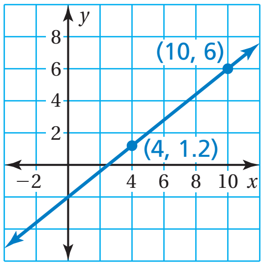


Graph the data. Then find and interpret the slope of the line through the points.

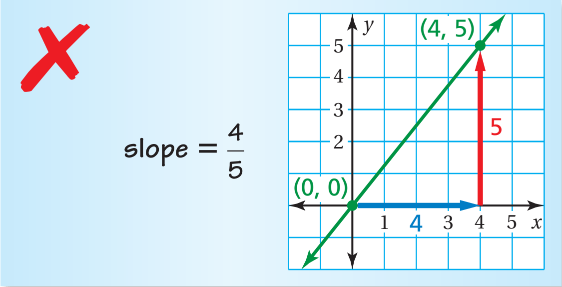
② 10.

Minutes, x	3	5	7	9
Words, y	135	225	315	405

11.

Gallons, x	5	10	15	20
Miles, y	162.5	325	487.5	650

12. **ERROR ANALYSIS** Describe and correct the error in finding the slope of the line passing through (0, 0) and (4, 5).



$\text{slope} = \frac{4}{5}$

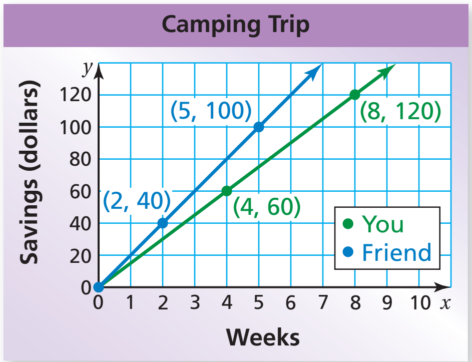
Graph the line that passes through the two points. Then find the slope of the line.

13. $(0, 0), \left(\frac{1}{3}, \frac{7}{3}\right)$

14. $\left(-\frac{3}{2}, -\frac{3}{2}\right), \left(\frac{3}{2}, \frac{3}{2}\right)$

15. $\left(1, \frac{5}{2}\right), \left(-\frac{1}{2}, -\frac{1}{4}\right)$

16. **CAMPING** The graph shows the amount of money you and a friend are saving for a camping trip.



a. Compare the steepness of the lines. What does this mean in the context of the problem?

b. Find the slope of each line.

c. How much more money does your friend save each week than you?

d. The camping trip costs \$165. How long will it take you to save enough money?

17. **MAPS** An atlas contains a map of Ohio. The table shows data from the key on the map.

Distance on Map (mm), *x*	10	20	30	40
Actual Distance (mi), *y*	25	50	75	100

a. Graph the data.

b. Find the slope of the line. What does this mean in the context of the problem?

c. The map distance between Toledo and Columbus is 48 millimeters. What is the actual distance?

d. Cincinnati is about 225 miles from Cleveland. What is the distance between these cities on the map?

18. **CRITICAL THINKING** What is the slope of a line that passes through the points (2, 0) and (5, 0)? Explain.

19. Number Sense A line has a slope of 2. It passes through the points (1, 2) and (3, y). What is the value of y?

Fair Game Review What you learned in previous grades & lessons

Multiply. *(Section 2.4)*

20. $-\frac{3}{5} \times \frac{8}{6}$

21. $1\frac{1}{2} \times \left(-\frac{6}{15}\right)$

22. $-2\frac{1}{4} \times \left(-1\frac{1}{3}\right)$

23. **MULTIPLE CHOICE** You have 18 stamps from Mexico in your stamp collection. These stamps represent $\frac{3}{8}$ of your collection. The rest of the stamps are from the United States. How many stamps are from the United States? *(Section 3.4)*

Ⓐ 12 Ⓑ 24 Ⓒ 30 Ⓓ 48

5.6 Direct Variation

Essential Question How can you use a graph to show the relationship between two quantities that vary directly? How can you use an equation?

1 ACTIVITY: Math in Literature

Gulliver's Travels was written by Jonathan Swift and published in 1726. Gulliver was shipwrecked on the island Lilliput, where the people were only 6 inches tall. When the Lilliputians decided to make a shirt for Gulliver, a Lilliputian tailor stated that he could determine Gulliver's measurements by simply measuring the distance around Gulliver's thumb. He said "Twice around the thumb equals once around the wrist. Twice around the wrist is once around the neck. Twice around the neck is once around the waist."

Direct Variation

In this lesson, you will

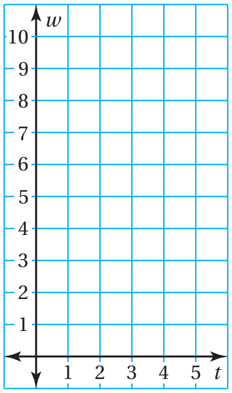
- identify direct variation from graphs or equations.
- use direct variation models to solve problems.

Work with a partner. Use the tailor's statement to complete the table.

Thumb, t	Wrist, w	Neck, n	Waist, x
0 in.			
1 in.			
	4 in.		
		12 in.	
			32 in.
	10 in.		

2 ACTIVITY: Drawing a Graph

Work with a partner. Use the information from Activity 1.



a. In your own words, describe the relationship between t and w.

b. Use the table to write the ordered pairs (t, w). Then plot the ordered pairs.

c. What do you notice about the graph of the ordered pairs?

d. Choose two points and find the slope of the line between them.

e. The quantities t and w are said to *vary directly*. An equation that describes the relationship is

$$w = \square\, t.$$

3 ACTIVITY: Drawing a Graph and Writing an Equation

Work with a partner. Use the information from Activity 1 to draw a graph of the relationship. Write an equation that describes the relationship between the two quantities.

a. Thumb t and neck n $(n = \square\, t)$

b. Wrist w and waist x $(x = \square\, w)$

c. Wrist w and thumb t $(t = \square\, w)$

d. Waist x and wrist w $(w = \square\, x)$

Math Practice

Label Axes

How do you know which labels to use for the axes? Explain.

What Is Your Answer?

4. **IN YOUR OWN WORDS** How can you use a graph to show the relationship between two quantities that vary directly? How can you use an equation?

5. **STRUCTURE** How are all the graphs in Activity 3 alike?

6. Give a real-life example of two variables that vary directly.

7. Work with a partner. Use string to find the distance around your thumb, wrist, and neck. Do your measurements agree with the tailor's statement in *Gulliver's Travels*? Explain your reasoning.

Practice Use what you learned about quantities that vary directly to complete Exercises 4 and 5 on page 202.

5.6 Lesson

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Key Vocabulary

direct variation, *p. 200*

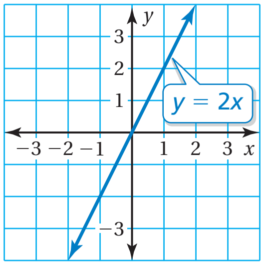
constant of proportionality, *p. 200*

Key Idea

Direct Variation

Words Two quantities x and y show **direct variation** when $y = kx$, where k is a number and $k \neq 0$. The number k is called the **constant of proportionality**.

Graph The graph of $y = kx$ is a line with a slope of k that passes through the origin. So, two quantities that show direct variation are in a proportional relationship.



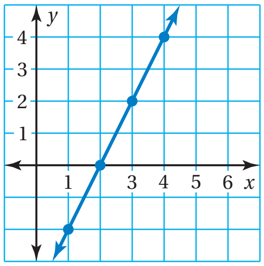
EXAMPLE 1 Identifying Direct Variation

Tell whether x and y show direct variation. Explain your reasoning.

a.

x	1	2	3	4
y	−2	0	2	4

Plot the points. Draw a line through the points.

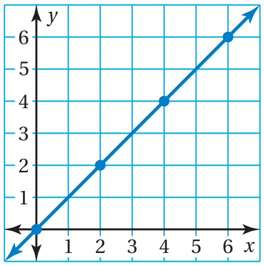


The line does *not* pass through the origin. So, x and y do *not* show direct variation.

b.

x	0	2	4	6
y	0	2	4	6

Plot the points. Draw a line through the points.



The line passes through the origin. So, x and y show direct variation.

Study Tip

Other ways to say that x and y show direct variation are "y varies directly with x" and "x and y are directly proportional."

EXAMPLE 2 Identifying Direct Variation

Tell whether x and y show direct variation. Explain your reasoning.

a. $y + 1 = 2x$

$y = 2x - 1$ Solve for y.

The equation *cannot* be written as $y = kx$. So, x and y do *not* show direct variation.

b. $\frac{1}{2}y = x$

$y = 2x$ Solve for y.

The equation can be written as $y = kx$. So, x and y show direct variation.

On Your Own

Now You're Ready
Exercises 6–17

Tell whether x and y show direct variation. Explain your reasoning.

1.

x	y
0	-2
1	1
2	4
3	7

2.

x	y
1	4
2	8
3	12
4	16

3.

x	y
-2	4
-1	2
0	0
1	2

4. $xy = 3$

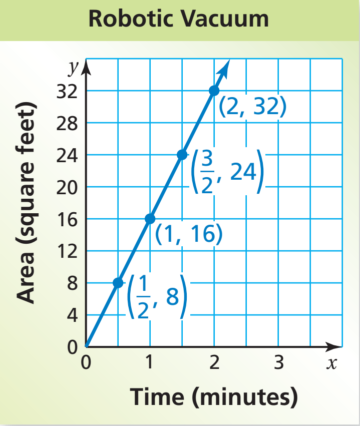
5. $x = \frac{1}{3}y$

6. $y + 1 = x$

EXAMPLE 3 Real-Life Application

x	y
$\frac{1}{2}$	8
1	16
$\frac{3}{2}$	24
2	32

The table shows the area y (in square feet) that a robotic vacuum cleans in x minutes.



a. Graph the data. Tell whether x and y are directly proportional.

Graph the data. Draw a line through the points.

⁝ The graph is a line through the origin. So, x and y are directly proportional.

b. Write an equation that represents the line.

Choose any two points to find the slope of the line.

$$\text{slope} = \frac{\text{change in } y}{\text{change in } x} = \frac{16}{1} = 16$$

⁝ The slope of the line is the constant of proportionality, k. So, an equation of the line is $y = 16x$.

c. Use the equation to find the area cleaned in 10 minutes.

$y = 16x$ Write the equation.

$= 16(10)$ Substitute 10 for x.

$= 160$ Multiply.

⁝ So, the vacuum cleans 160 square feet in 10 minutes.

On Your Own

Now You're Ready
Exercise 19

7. **WHAT IF?** The battery weakens and the robot begins cleaning less and less area each minute. Do x and y show direct variation? Explain.

5.6 Exercises

Vocabulary and Concept Check

1. **VOCABULARY** What does it mean for x and y to vary directly?

2. **WRITING** What point is on the graph of every direct variation equation?

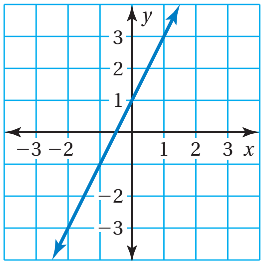
3. **DIFFERENT WORDS, SAME QUESTION** Which is different? Find "both" answers.

Do x and y show direct variation?

Are x and y in a proportional relationship?

Is the graph of the relationship a line?

Does y vary directly with x?



Practice and Problem Solving

Graph the ordered pairs in a coordinate plane. Do you think that graph shows that the quantities vary directly? Explain your reasoning.

4. (−1, −1), (0, 0), (1, 1), (2, 2)

5. (−4, −2), (−2, 0), (0, 2), (2, 4)

Tell whether x and y show direct variation. Explain your reasoning. If so, find k.

① 6.

x	1	2	3	4
y	2	4	6	8

7.

x	−2	−1	0	1
y	0	2	4	6

8.

x	−1	0	1	2
y	−2	−1	0	1

9.

x	3	6	9	12
y	2	4	6	8

② 10. $y - x = 4$

11. $x = \frac{2}{5}y$

12. $y + 3 = x + 6$

13. $y - 5 = 2x$

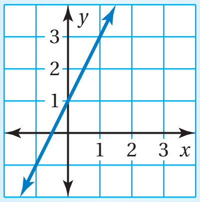
14. $x - y = 0$

15. $\frac{x}{y} = 2$

16. $8 = xy$

17. $x^2 = y$

18. **ERROR ANALYSIS** Describe and correct the error in telling whether x and y show direct variation.



The graph is a line, so it shows direct variation.

③ 19. **RECYCLING** The table shows the profit y for recycling x pounds of aluminum. Graph the data. Tell whether x and y show direct variation. If so, write an equation that represents the line.

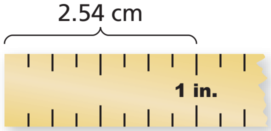
Aluminum (lb), x	10	20	30	40
Profit, y	\$4.50	\$9.00	\$13.50	\$18.00

The variables x and y vary directly. Use the values to find the constant of proportionality. Then write an equation that relates x and y.

20. $y = 72; x = 3$

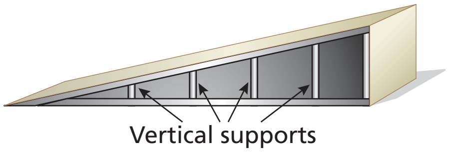
21. $y = 20; x = 12$

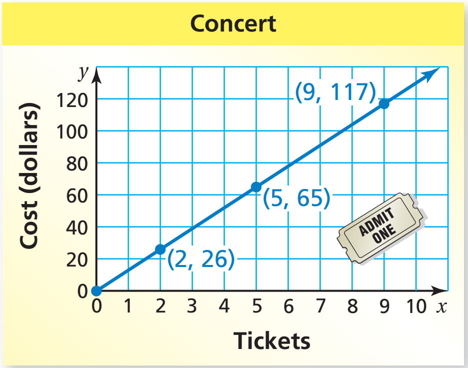
22. $y = 45; x = 40$



23. **MEASUREMENT** Write a direct variation equation that relates x inches to y centimeters.

24. **MODELING** Design a waterskiing ramp. Show how you can use direct variation to plan the heights of the vertical supports.





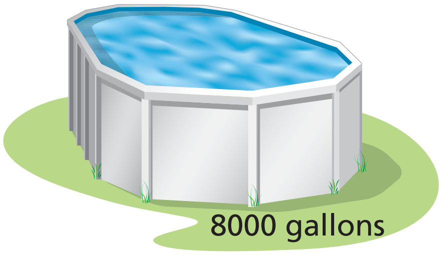
25. **REASONING** Use $y = kx$ to show why the graph of a proportional relationship always passes through the origin.

26. **TICKETS** The graph shows the cost of buying concert tickets. Tell whether x and y show direct variation. If so, find and interpret the constant of proportionality. Then write an equation and find the cost of 14 tickets.

27. **CELL PHONE PLANS** Tell whether x and y show direct variation. If so, write an equation of direct variation.

Minutes, x	500	700	900	1200
Cost, y	\$40	\$50	\$60	\$75

28. **CHLORINE** The amount of chlorine in a swimming pool varies directly with the volume of water. The pool has 2.5 milligrams of chlorine per liter of water. How much chlorine is in the pool?



29. **Critical Thinking** Is the graph of every direct variation equation a line? Does the graph of every line represent a direct variation equation? Explain your reasoning.

Fair Game Review What you learned in previous grades & lessons

Write the fraction as a decimal. *(Section 2.1)*

30. $\frac{13}{20}$

31. $\frac{9}{16}$

32. $\frac{21}{40}$

33. $\frac{24}{25}$

34. **MULTIPLE CHOICE** Which rate is *not* equivalent to 180 feet per 8 seconds? *(Section 5.1)*

Ⓐ $\frac{225\text{ ft}}{10\text{ sec}}$

Ⓑ $\frac{45\text{ ft}}{2\text{ sec}}$

Ⓒ $\frac{135\text{ ft}}{6\text{ sec}}$

Ⓓ $\frac{180\text{ ft}}{1\text{ sec}}$

5.4–5.6 Quiz

Check It Out
Progress Check
BigIdeasMath.com

Solve the proportion. *(Section 5.4)*

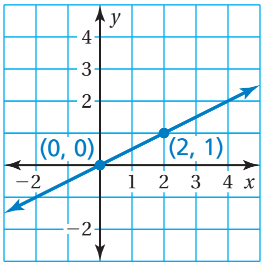
1. $\frac{7}{n} = \frac{42}{48}$

2. $\frac{x}{2} = \frac{40}{16}$

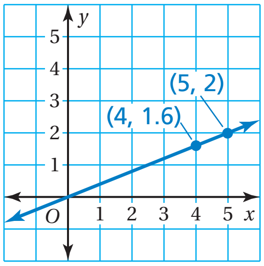
3. $\frac{3}{11} = \frac{27}{z}$

Find the slope of the line. *(Section 5.5)*

4.



5.



Graph the data. Then find and interpret the slope of the line through the points. *(Section 5.5)*

6.

Hours, x	2	4	6	8
Miles, y	10	20	30	40

7.

Packages, x	6	10	14	18
Servings, y	9	15	21	27

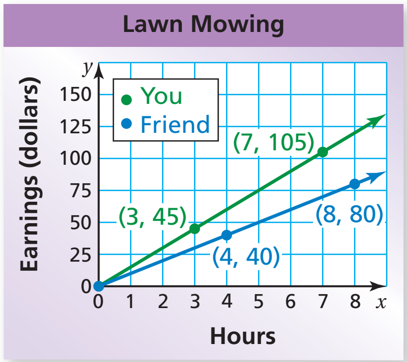
Tell whether x and y show direct variation. Explain your reasoning. *(Section 5.6)*

8. $y - 9 = 6 + x$

9. $x = \frac{5}{8}y$

10. **CONCERT** A benefit concert with three performers lasts 8 hours. At this rate, how many hours is a concert with four performers? *(Section 5.4)*

11. **LAWN MOWING** The graph shows how much you and your friend each earn mowing lawns. *(Section 5.5)*



a. Compare the steepness of the lines. What does this mean in the context of the problem?

b. Find and interpret the slope of each line.

c. How much more money do you earn per hour than your friend?

12. **PIE SALE** The table shows the profits of a pie sale. Tell whether x and y show direct variation. If so, write the equation of direct variation. *(Section 5.6)*

Pies Sold, x	10	12	14	16
Profit, y	\$79.50	\$95.40	\$111.30	\$127.20

5 Chapter Review

Check It Out
Vocabulary Help
BigIdeasMath.com

Review Key Vocabulary

ratio, *p. 164*
rate, *p. 164*
unit rate, *p. 164*
complex fraction, *p. 165*
proportion, *p. 172*
proportional, *p. 172*
cross products, *p. 173*
slope, *p. 194*
direct variation, *p. 200*
constant of proportionality, *p. 200*

Review Examples and Exercises

5.1 Ratios and Rates *(pp. 162–169)*

There are 15 orangutans and 25 gorillas in a nature preserve. One of the orangutans swings 75 feet in 15 seconds on a rope.

a. Find the ratio of orangutans to gorillas.

b. How fast is the orangutan swinging?

a. $\frac{\text{orangutans}}{\text{gorillas}} = \frac{15}{25} = \frac{3}{5}$

∴ The ratio of orangutans to gorillas is $\frac{3}{5}$.

b. $$\text{75 feet in 15 seconds} = \frac{75 \text{ ft}}{15 \text{ sec}} = \frac{75 \text{ ft} \div 15}{15 \text{ sec} \div 15} = \frac{5 \text{ ft}}{1 \text{ sec}}$$

∴ The orangutan is swinging 5 feet per second.

Exercises

Find the unit rate.

1. 289 miles on 10 gallons

2. $6\frac{2}{5}$ revolutions in $2\frac{2}{3}$ seconds

3. calories per serving

Servings	2	4	6	8
Calories	240	480	720	960

5.2 Proportions *(pp. 170–177)*

Tell whether the ratios $\frac{9}{12}$ and $\frac{6}{8}$ form a proportion.

$$\frac{9}{12} = \frac{9 \div 3}{12 \div 3} = \frac{3}{4}$$

$$\frac{6}{8} = \frac{6 \div 2}{8 \div 2} = \frac{3}{4}$$

The ratios are equivalent.

∴ So, $\frac{9}{12}$ and $\frac{6}{8}$ form a proportion.

Exercises

Tell whether the ratios form a proportion.

4. $\frac{4}{9}, \frac{2}{3}$

5. $\frac{12}{22}, \frac{18}{33}$

6. $\frac{8}{50}, \frac{4}{10}$

7. $\frac{32}{40}, \frac{12}{15}$

8. Use a graph to determine whether x and y are in a proportional relationship.

x	1	3	6	8
y	4	12	24	32

5.3 Writing Proportions *(pp. 178–183)*

Write a proportion that gives the number r of returns on Saturday.

	Friday	Saturday
Sales	40	85
Returns	32	r

One proportion is $\frac{40 \text{ sales}}{32 \text{ returns}} = \frac{85 \text{ sales}}{r \text{ returns}}$.

Exercises

Use the table to write a proportion.

9.

	Game 1	Game 2
Penalties	6	8
Minutes	16	m

10.

	Concert 1	Concert 2
Songs	15	18
Hours	2.5	h

5.4 Solving Proportions *(pp. 186–191)*

Solve $\frac{15}{2} = \frac{30}{y}$.

$15 \cdot y = 2 \cdot 30$ Cross Products Property

$15y = 60$ Multiply.

$y = 4$ Divide.

The solution is 4.

Exercises

Solve the proportion.

11. $\frac{x}{4} = \frac{2}{5}$

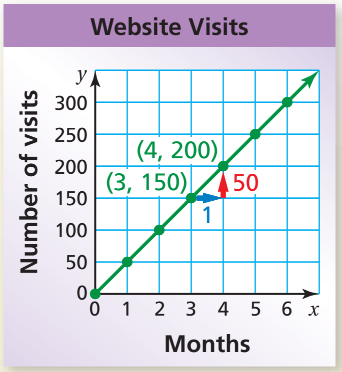
12. $\frac{5}{12} = \frac{y}{15}$

13. $\frac{8}{20} = \frac{6}{w}$

14. $\frac{s+1}{4} = \frac{4}{8}$

5.5 Slope (pp. 192–197)

The graph shows the number of visits your website received over the past 6 months. Find and interpret the slope.



Choose any two points to find the slope of the line.

$$\text{slope} = \frac{\text{change in } y}{\text{change in } x}$$

$$= \frac{50}{1}$$

visits

months

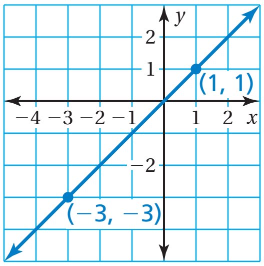
$$= 50$$

The slope of the line represents the unit rate. The slope is 50. So, the number of visits increased by 50 each month.

Exercises

Find the slope of the line.

15.



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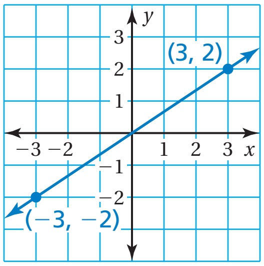
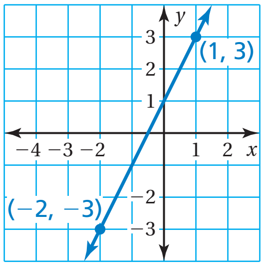


17.



5.6 Direct Variation (pp. 198–203)

Tell whether x and y show direct variation. Explain your reasoning.

a. $x + y - 1 = 3$

$y = 4 - x$ Solve for y.

The equation *cannot* be written as $y = kx$. So, x and y do *not* show direct variation.

b. $x = 8y$

$\frac{1}{8}x = y$ Solve for y.

The equation can be written as $y = kx$. So, x and y show direct variation.

Exercises

Tell whether x and y show direct variation. Explain your reasoning.

18. $x + y = 6$ **19.** $y - x = 0$ **20.** $\frac{x}{y} = 20$ **21.** $x = y + 2$

5 Chapter Test

Check It Out
Test Practice
BigIdeasMath.com

Find the unit rate.

1. 84 miles in 12 days

2. $2\frac{2}{5}$ kilometers in $3\frac{3}{4}$ minutes

Tell whether the ratios form a proportion.

3. $\frac{1}{9}, \frac{6}{54}$

4. $\frac{9}{12}, \frac{8}{72}$

Use a graph to tell whether *x* and *y* are in a proportional relationship.

5.

x	2	4	6	8
y	10	20	30	40

6.

x	1	3	5	7
y	3	7	11	15

Use the table to write a proportion.

7.

	Monday	Tuesday
Gallons	6	8
Miles	180	*m*

8.

	Thursday	Friday
Classes	6	*c*
Hours	8	4

Solve the proportion.

9. $\frac{x}{8} = \frac{9}{4}$

10. $\frac{17}{3} = \frac{y}{6}$

Graph the line that passes through the two points. Then find the slope of the line.

11. $(15, 9), (-5, -3)$

12. $(2, 9), (4, 18)$

Tell whether *x* and *y* show direct variation. Explain your reasoning.

13. $xy - 11 = 5$

14. $x = \frac{3}{y}$

15. $\frac{y}{x} = 8$

16. **MOVIE TICKETS** Five movie tickets cost $36.25. What is the cost of 8 movie tickets?

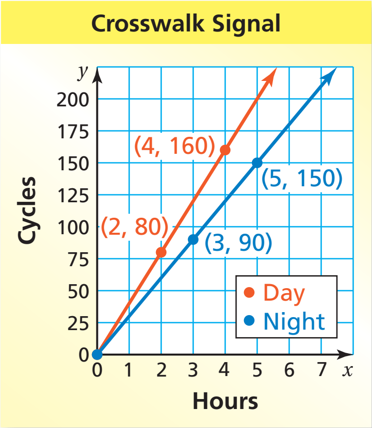
17. **CROSSWALK** The graph shows the number of cycles of a crosswalk signal during the day and during the night.

a. Compare the steepness of the lines. What does this mean in the context of the problem?

b. Find and interpret the slope of each line.

Don't Walk

Walk

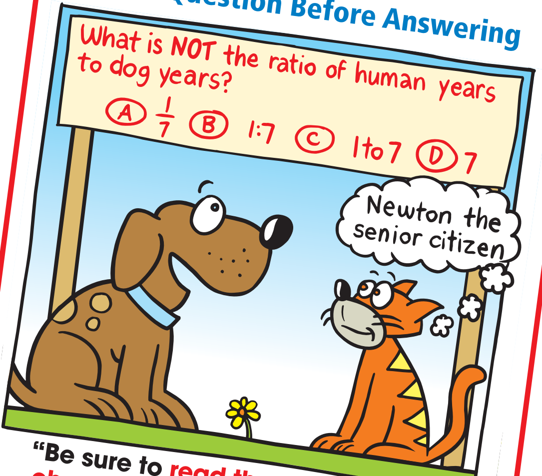


18. **GLAZE** A specific shade of green glaze requires 5 parts blue to 3 parts yellow. A glaze mixture contains 25 quarts of blue and 9 quarts of yellow. How can you fix the mixture to make the specific shade of green glaze?

5 Cumulative Assessment

Test-Taking Strategy

Read Question Before Answering



"Be sure to read the question before choosing your answer. You may find a word that changes the meaning."

1. The school store sells 4 pencils for $0.80. What is the unit cost of a pencil?

A. $0.20

B. $0.80

C. $3.20

D. $5.00

2. Which expressions do *not* have a value of 3?

I. $2 + (-1)$

II. $2 - (-1)$

III. $-3 \times (-1)$

IV. $-3 \div (-1)$

F. I only

G. III and IV

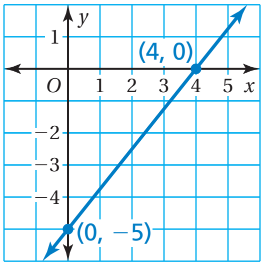
H. II only

I. I, III, and IV

3. What is the value of the expression below?

$$-4 \times (-6) - (-5)$$

4. What is the slope of the line shown?



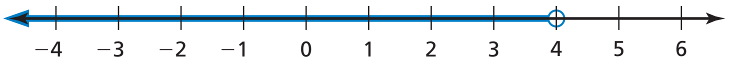
A. $\frac{4}{5}$

B. $\frac{5}{4}$

C. 4

D. 5

5. The graph below represents which inequality?



F. $-3 - 6x < -27$

G. $2x + 6 \geq 14$

H. $5 - 3x > -7$

I. $2x + 3 \leq 11$

6. The quantities *x* and *y* are proportional. What is the missing value in the table?

x	*y*
$\frac{2}{3}$	6
$\frac{4}{3}$	12
$\frac{8}{3}$	24
5	

A. 30

B. 36

C. 45

D. 48

7. You are selling tomatoes. You have already earned \$16 today. How many additional pounds of tomatoes do you need to sell to earn a total of \$60?

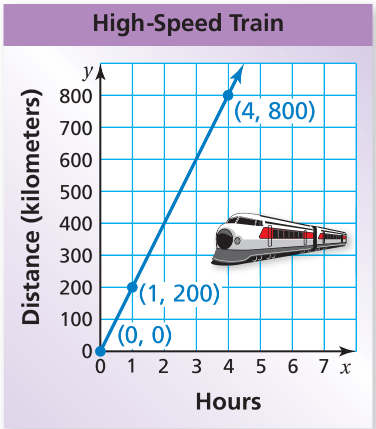
F. 4

G. 11

H. 15

I. 19

8. The distance traveled by the a high-speed train is proportional to the number of hours traveled. Which of the following is *not* a valid interpretation of the graph below?



A. The train travels 0 kilometers in 0 hours.

B. The unit rate is 200 kilometers per hour.

C. After 4 hours, the train is traveling 800 kilometers per hour.

D. The train travels 800 kilometers in 4 hours.

9. Regina was evaluating the expression below. What should Regina do to correct the error she made?

$$\begin{aligned} -\frac{3}{2} \div \left(-\frac{8}{7}\right) &= -\frac{2}{3} \times \left(-\frac{7}{8}\right) \\ &= \frac{2 \times 7}{3 \times 8} \\ &= \frac{14}{24} \\ &= \frac{7}{12} \end{aligned}$$

F. Rewrite $-\frac{3}{2} \div \left(-\frac{8}{7}\right)$ as $-\frac{2}{3} \times \left(-\frac{8}{7}\right)$.

G. Rewrite $-\frac{3}{2} \div \left(-\frac{8}{7}\right)$ as $-\frac{3}{2} \times \left(-\frac{7}{8}\right)$.

H. Rewrite $-\frac{3}{2} \div \left(-\frac{8}{7}\right)$ as $-\frac{3}{7} \times \left(-\frac{8}{2}\right)$.

I. Rewrite $-\frac{2}{3} \times \left(-\frac{7}{8}\right)$ as $-\frac{2 \times 7}{3 \times 8}$.

10. What is the least value of t for which the inequality is true?

$$3 - 6t \leq -15$$

11. You can mow 800 square feet of lawn in 15 minutes. At this rate, how many minutes will you take to mow a lawn that measures 6000 square feet?

Part A Write a proportion to represent the problem. Use m to represent the number of minutes. Explain your reasoning.

Part B Solve the proportion you wrote in Part A. Then use it to answer the problem. Show your work.

12. What value of p makes the equation below true?

$$6 - 2p = -48$$

A. -27

B. -21

C. 21

D. 27