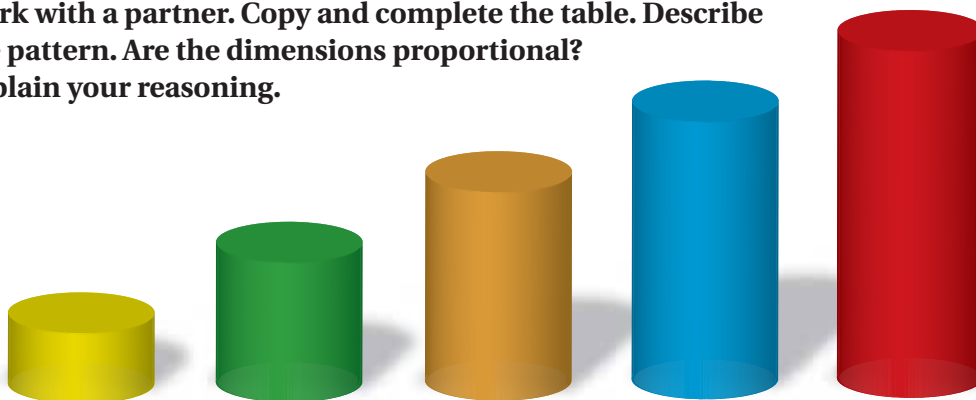


**Essential Question** When the dimensions of a solid increase by a factor of  $k$ , how does the surface area change? How does the volume change?

**1 ACTIVITY: Comparing Surface Areas and Volumes**

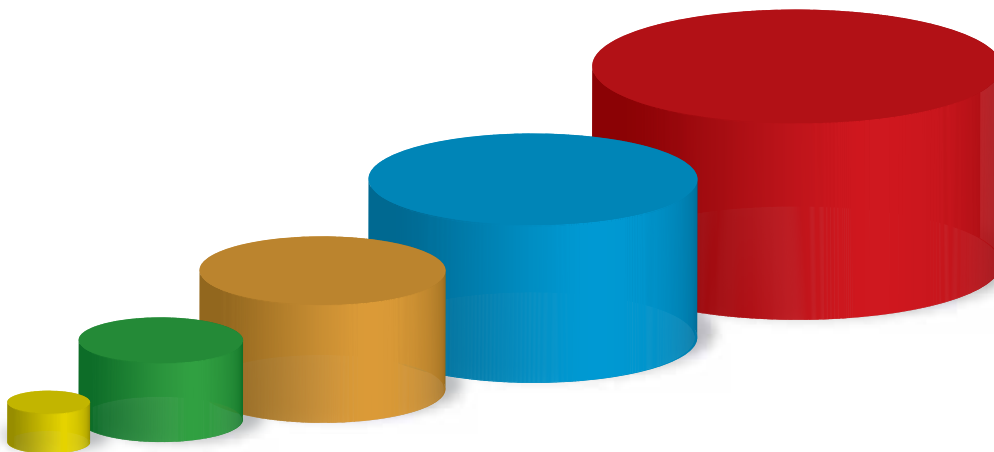
Work with a partner. Copy and complete the table. Describe the pattern. Are the dimensions proportional? Explain your reasoning.

a.



Radius	1	1	1	1	1
Height	1	2	3	4	5
Surface Area					
Volume					

b.



Radius	1	2	3	4	5
Height	1	2	3	4	5
Surface Area					
Volume					

### Geometry

In this lesson, you will

- identify similar solids.
- use properties of similar solids to find missing measures.
- understand the relationship between surface areas of similar solids.
- understand the relationship between volumes of similar solids.
- solve real-life problems.

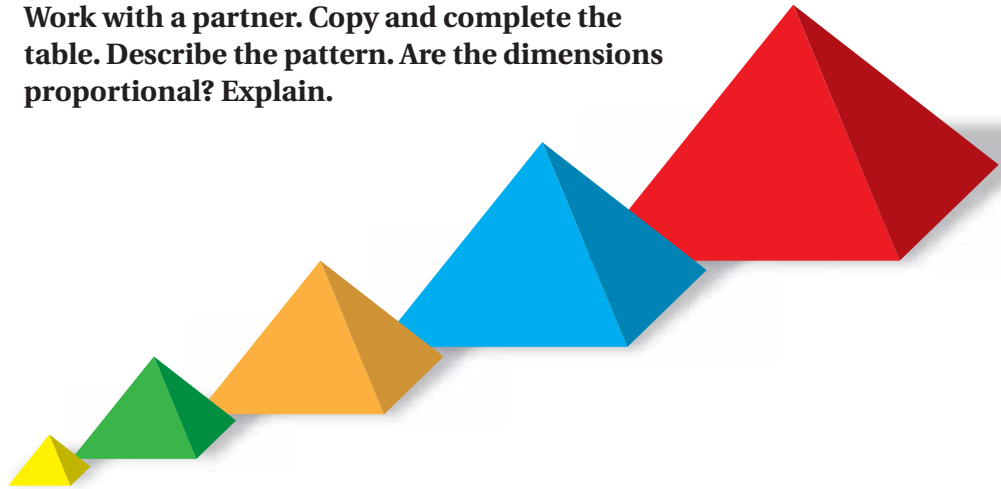
2

**ACTIVITY: Comparing Surface Areas and Volumes**

Work with a partner. Copy and complete the table. Describe the pattern. Are the dimensions proportional? Explain.

**Math Practice****Repeat Calculations**

Which calculations are repeated? How does this help you describe the pattern?



<b>Base Side</b>	6	12	18	24	30
<b>Height</b>	4	8	12	16	20
<b>Slant Height</b>	5	10	15	20	25
<b>Surface Area</b>					
<b>Volume</b>					

**What Is Your Answer?**

- IN YOUR OWN WORDS** When the dimensions of a solid increase by a factor of  $k$ , how does the surface area change?
- IN YOUR OWN WORDS** When the dimensions of a solid increase by a factor of  $k$ , how does the volume change?
- REPEATED REASONING** All the dimensions of a prism increase by a factor of 5.

a. How many times greater is the surface area? Explain.

5

10

25

125

b. How many times greater is the volume? Explain.

5


10

25

125

**Practice**

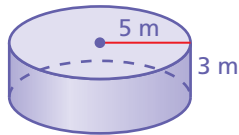
Use what you learned about surface areas and volumes of similar solids to complete Exercise 3 on page 359.

**Key Vocabulary**   
similar solids, p. 356

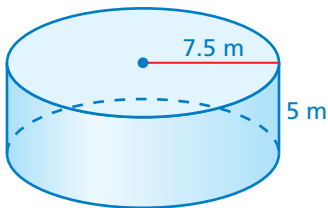
**Similar solids** are solids that have the same shape and proportional corresponding dimensions.

## EXAMPLE 1 Identifying Similar Solids

Cylinder B



Cylinder C



Which cylinder is similar to Cylinder A?

Check to see if corresponding dimensions are proportional.

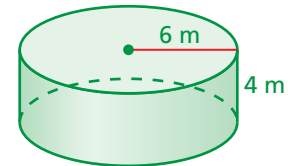
*Cylinder A and Cylinder B*

$$\frac{\text{Height of A}}{\text{Height of B}} = \frac{4}{3} \qquad \frac{\text{Radius of A}}{\text{Radius of B}} = \frac{6}{5}$$

*Cylinder A and Cylinder C*

$$\frac{\text{Height of A}}{\text{Height of C}} = \frac{4}{5} \qquad \frac{\text{Radius of A}}{\text{Radius of C}} = \frac{6}{7.5} = \frac{4}{5}$$

Cylinder A



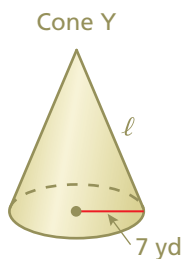
Not proportional

Proportional

So, Cylinder C is similar to Cylinder A.

## EXAMPLE 2 Finding Missing Measures in Similar Solids

The cones are similar. Find the missing slant height  $\ell$ .



$$\frac{\text{Radius of X}}{\text{Radius of Y}} = \frac{\text{Slant height of X}}{\text{Slant height of Y}}$$

$$\frac{5}{7} = \frac{13}{\ell} \qquad \text{Substitute.}$$

$$5\ell = 91 \qquad \text{Cross Products Property}$$

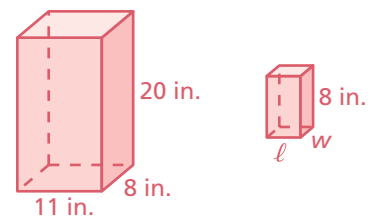
$$\ell = 18.2 \qquad \text{Divide each side by 5.}$$

The slant height is 18.2 yards.

### On Your Own

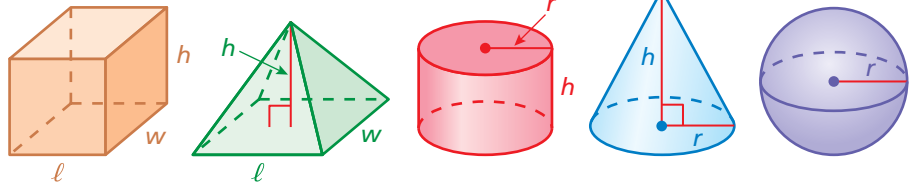
**Now You're Ready**  
Exercises 4–9

- Cylinder D has a radius of 7.5 meters and a height of 4.5 meters. Which cylinder in Example 1 is similar to Cylinder D?
- The prisms at the right are similar. Find the missing width and length.



## Key Ideas

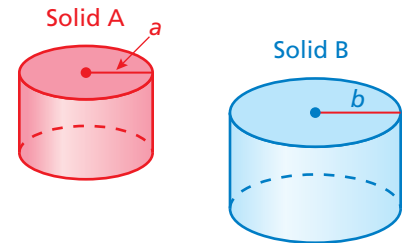
### Linear Measures



### Surface Areas of Similar Solids

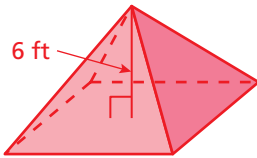
When two solids are similar, the ratio of their surface areas is equal to the square of the ratio of their corresponding linear measures.

$$\frac{\text{Surface Area of A}}{\text{Surface Area of B}} = \left(\frac{a}{b}\right)^2$$

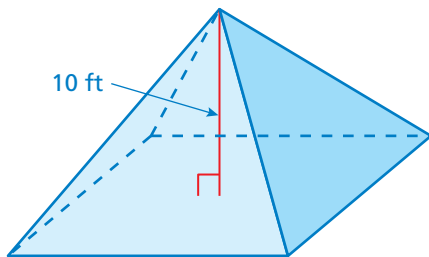


### EXAMPLE 3 Finding Surface Area

Pyramid A



Pyramid B



Surface Area =  $600 \text{ ft}^2$

The pyramids are similar. What is the surface area of Pyramid A?

$$\frac{\text{Surface Area of A}}{\text{Surface Area of B}} = \left(\frac{\text{Height of A}}{\text{Height of B}}\right)^2$$

$$\frac{S}{600} = \left(\frac{6}{10}\right)^2$$

Substitute.

$$\frac{S}{600} = \frac{36}{100}$$

Evaluate.

$$\frac{S}{600} \cdot 600 = \frac{36}{100} \cdot 600$$

Multiplication Property of Equality

$$S = 216$$

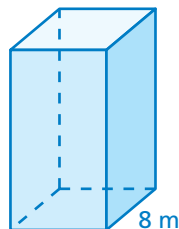
Simplify.

∴ The surface area of Pyramid A is 216 square feet.

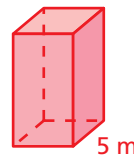
### On Your Own

The solids are similar. Find the surface area of the red solid. Round your answer to the nearest tenth.

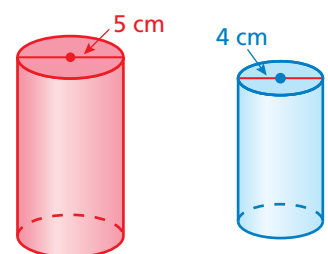
3.



Surface Area =  $608 \text{ m}^2$



4.



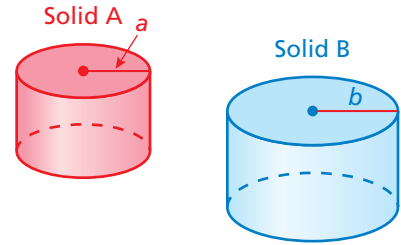
Surface Area =  $110 \text{ cm}^2$

## Key Idea

### Volumes of Similar Solids

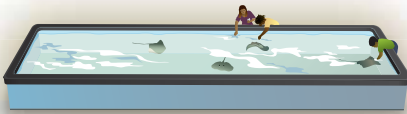
When two solids are similar, the ratio of their volumes is equal to the cube of the ratio of their corresponding linear measures.

$$\frac{\text{Volume of A}}{\text{Volume of B}} = \left(\frac{a}{b}\right)^3$$



## EXAMPLE 4 Finding Volume

Original Tank



Volume = 2000 ft<sup>3</sup>

The dimensions of the touch tank at an aquarium are doubled. What is the volume of the new touch tank?

- (A) 1500 ft<sup>3</sup>                      (B) 4000 ft<sup>3</sup>  
 (C) 8000 ft<sup>3</sup>                      (D) 16,000 ft<sup>3</sup>

The dimensions are doubled, so the ratio of the dimensions of the original tank to the dimensions of the new tank is 1 : 2.

$$\frac{\text{Original volume}}{\text{New volume}} = \left(\frac{\text{Original dimension}}{\text{New dimension}}\right)^3$$

$$\frac{2000}{V} = \left(\frac{1}{2}\right)^3 \quad \text{Substitute.}$$

$$\frac{2000}{V} = \frac{1}{8} \quad \text{Evaluate.}$$

$$16,000 = V \quad \text{Cross Products Property}$$

- ∴ The volume of the new tank is 16,000 cubic feet. So, the correct answer is (D).

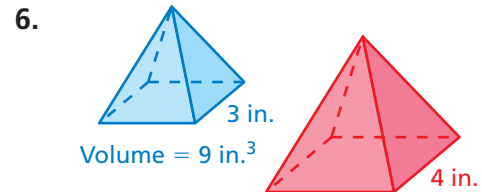
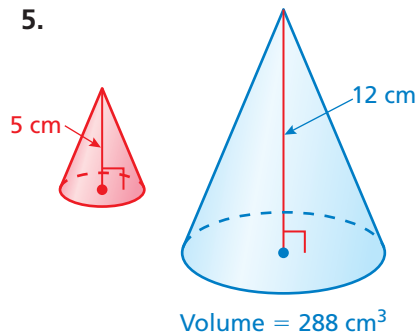
### Study Tip

When the dimensions of a solid are multiplied by  $k$ , the surface area is multiplied by  $k^2$  and the volume is multiplied by  $k^3$ .

## On Your Own

Now You're Ready  
Exercises 10–13

The solids are similar. Find the volume of the red solid. Round your answer to the nearest tenth.



## Vocabulary and Concept Check

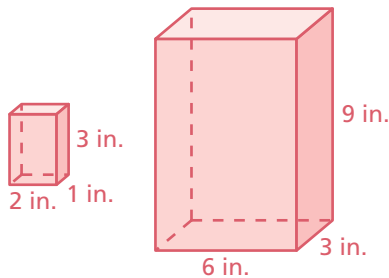
- VOCABULARY** What are similar solids?
- OPEN-ENDED** Draw two similar solids and label their corresponding linear measures.

## Practice and Problem Solving

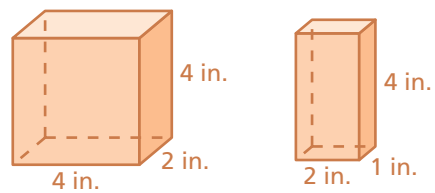
- NUMBER SENSE** All the dimensions of a cube increase by a factor of  $\frac{3}{2}$ .
  - How many times greater is the surface area? Explain.
  - How many times greater is the volume? Explain.

Determine whether the solids are similar.

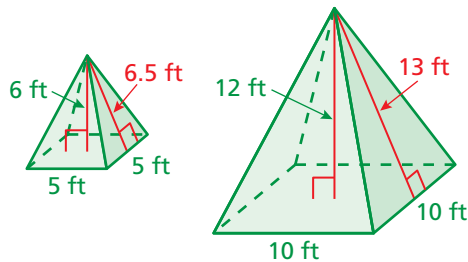
1 4.



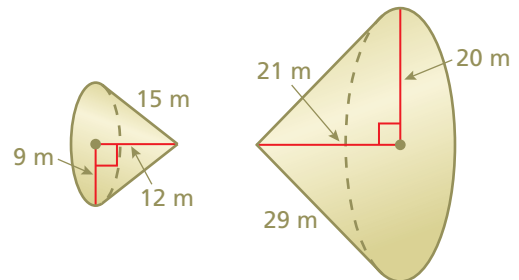
5.



6.

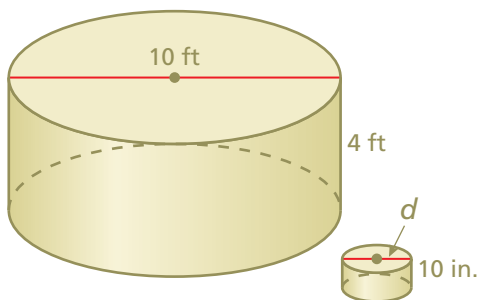


7.

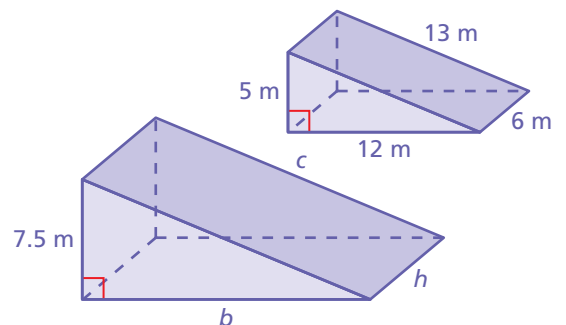


The solids are similar. Find the missing dimension(s).

2 8.

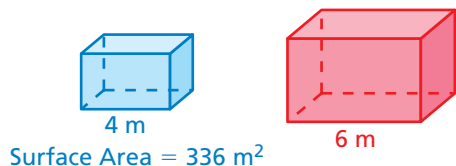


9.

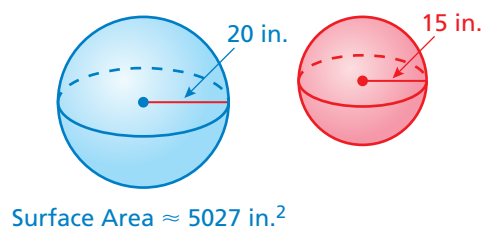


The solids are similar. Find the surface area  $S$  or volume  $V$  of the red solid. Round your answer to the nearest tenth.

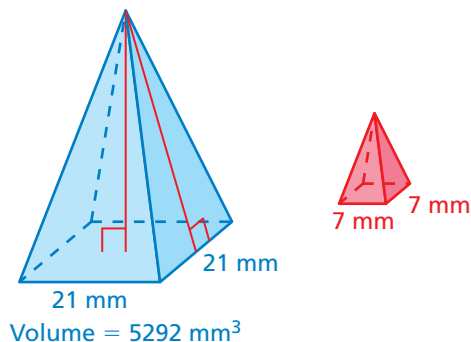
3 4 10.



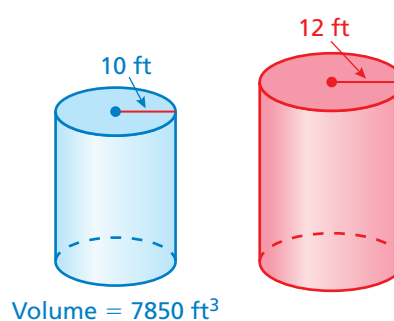
11.



12.



13.



14. **ERROR ANALYSIS** The ratio of the corresponding linear measures of two similar solids is 3 : 5. The volume of the smaller solid is 108 cubic inches. Describe and correct the error in finding the volume of the larger solid.

$$\times \quad \frac{108}{V} = \left(\frac{3}{5}\right)^2$$

$$\frac{108}{V} = \frac{9}{25}$$

$$300 = V$$

The volume of the larger solid is 300 cubic inches.

15. **MIXED FRUIT** The ratio of the corresponding linear measures of two similar cans of fruit is 4 to 7. The smaller can has a surface area of 220 square centimeters. Find the surface area of the larger can.

16. **CLASSIC MUSTANG** The volume of a 1968 Ford Mustang GT engine is 390 cubic inches. Which scale model of the Mustang has the greater engine volume, a 1 : 18 scale model or a 1 : 24 scale model? How much greater is it?





17. **MARBLE STATUE** You have a small marble statue of Wolfgang Mozart. It is 10 inches tall and weighs 16 pounds. The original statue is 7 feet tall.
- Estimate the weight of the original statue. Explain your reasoning.
  - If the original statue were 20 feet tall, how much would it weigh?



Wolfgang Mozart

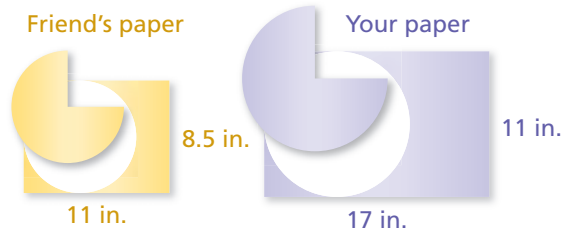
18. **REPEATED REASONING** The largest doll is 7 inches tall. Each of the other dolls is 1 inch shorter than the next larger doll. Make a table that compares the surface areas and the volumes of the seven dolls.



19. **Precision** You and a friend make paper cones to collect beach glass. You cut out the largest possible three-fourths circle from each piece of paper.



- Are the cones similar? Explain your reasoning.
- Your friend says that because your sheet of paper is twice as large, your cone will hold exactly twice the volume of beach glass. Is this true? Explain your reasoning.



## Fair Game Review what you learned in previous grades & lessons

Draw the figure and its reflection in the  $x$ -axis. Identify the coordinates of the image. (Section 2.3)

20.  $A(1, 1), B(3, 4), C(4, 2)$

21.  $J(-3, 0), K(-4, 3), L(-1, 4)$

22. **MULTIPLE CHOICE** Which system of linear equations has no solution? (Section 5.4)

(A)  $y = 4x + 1$   
 $y = -4x + 1$

(B)  $y = 2x - 7$   
 $y = 2x + 7$

(C)  $3x + y = 1$   
 $6x + 2y = 2$

(D)  $5x + y = 3$   
 $x + 5y = 15$