

7.3 The Pythagorean Theorem

Essential Question How are the lengths of the sides of a right triangle related?

Pythagoras was a Greek mathematician and philosopher who discovered one of the most famous rules in mathematics. In mathematics, a rule is called a **theorem**. So, the rule that Pythagoras discovered is called the Pythagorean Theorem.

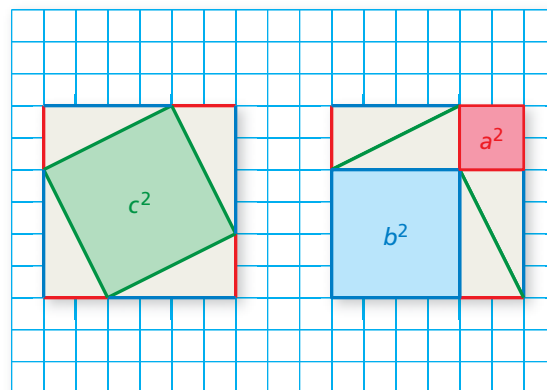
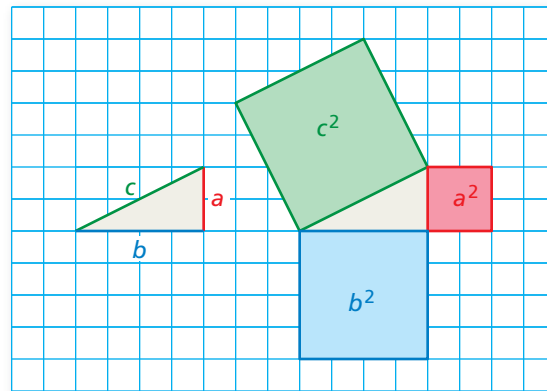


Pythagoras
(c. 570–c. 490 B.C.)

1 ACTIVITY: Discovering the Pythagorean Theorem

Work with a partner.

- On grid paper, draw any right triangle. Label the lengths of the two shorter sides a and b .
- Label the length of the longest side c .
- Draw squares along each of the three sides. Label the areas of the three squares a^2 , b^2 , and c^2 .
- Cut out the three squares. Make eight copies of the right triangle and cut them out. Arrange the figures to form two identical larger squares.
- MODELING** The Pythagorean Theorem describes the relationship among a^2 , b^2 , and c^2 . Use your result from part (d) to write an equation that describes this relationship.



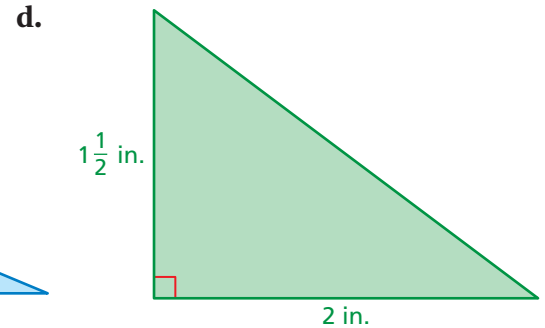
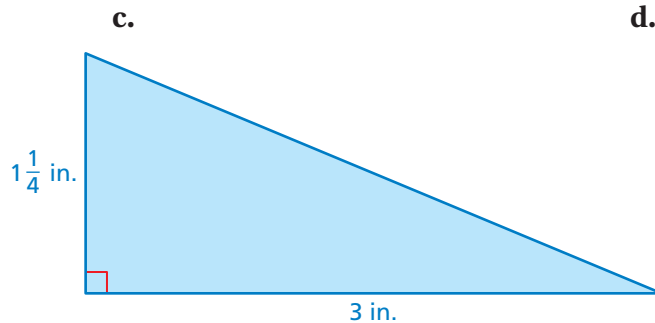
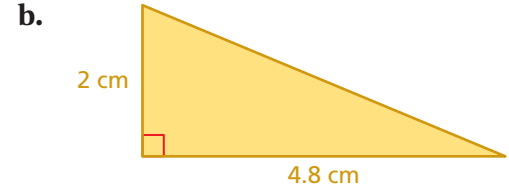
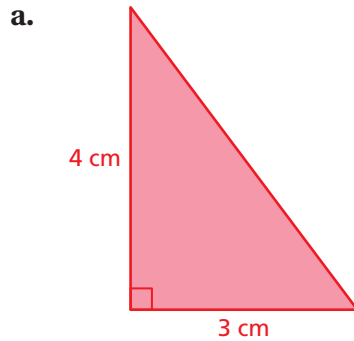
Pythagorean Theorem

In this lesson, you will

- provide geometric proof of the Pythagorean Theorem.
- use the Pythagorean Theorem to find missing side lengths of right triangles.
- solve real-life problems.

2 ACTIVITY: Using the Pythagorean Theorem in Two Dimensions

Work with a partner. Use a ruler to measure the longest side of each right triangle. Verify the result of Activity 1 for each right triangle.



3 ACTIVITY: Using the Pythagorean Theorem in Three Dimensions

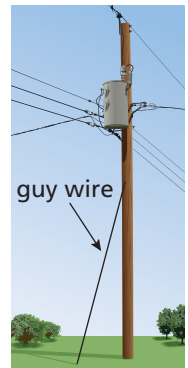
Math Practice

Use Definitions

How can you use what you know about the Pythagorean Theorem to describe the procedure for finding the length of the guy wire?

Work with a partner. A guy wire attached 24 feet above ground level on a telephone pole provides support for the pole.

- PROBLEM SOLVING** Describe a procedure that you could use to find the length of the guy wire without directly measuring the wire.
- Find the length of the wire when it meets the ground 10 feet from the base of the pole.



What Is Your Answer?

- IN YOUR OWN WORDS** How are the lengths of the sides of a right triangle related? Give an example using whole numbers.

Practice

Use what you learned about the Pythagorean Theorem to complete Exercises 3 and 4 on page 304.

Key Vocabulary

theorem, p. 300
 legs, p. 302
 hypotenuse, p. 302
 Pythagorean
 Theorem, p. 302

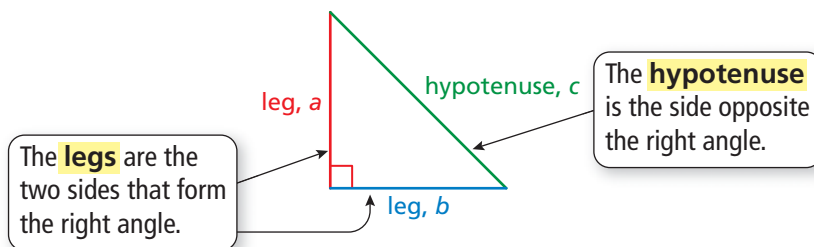
Study Tip

In a right triangle, the legs are the shorter sides and the hypotenuse is always the longest side.

Key Ideas

Sides of a Right Triangle

The sides of a right triangle have special names.

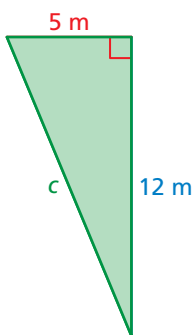


The Pythagorean Theorem

Words In any right triangle, the sum of the squares of the lengths of the legs is equal to the square of the length of the hypotenuse.

Algebra $a^2 + b^2 = c^2$

EXAMPLE 1 Finding the Length of a Hypotenuse



Find the length of the hypotenuse of the triangle.

$$a^2 + b^2 = c^2$$

Write the Pythagorean Theorem.

$$5^2 + 12^2 = c^2$$

Substitute 5 for a and 12 for b .

$$25 + 144 = c^2$$

Evaluate powers.

$$169 = c^2$$

Add.

$$\sqrt{169} = \sqrt{c^2}$$

Take positive square root of each side.

$$13 = c$$

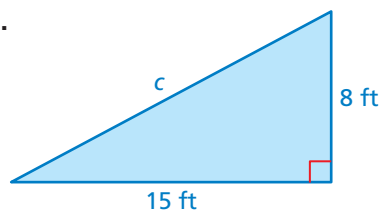
Simplify.

∴ The length of the hypotenuse is 13 meters.

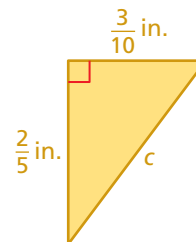
On Your Own

Find the length of the hypotenuse of the triangle.

1.



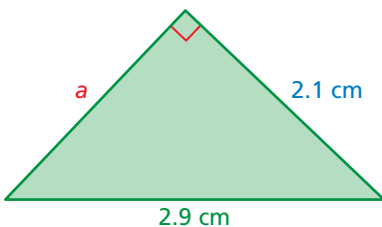
2.



Now You're Ready
Exercises 3 and 4

EXAMPLE 2 Finding the Length of a Leg

Find the missing length of the triangle.



$$a^2 + b^2 = c^2$$

$$a^2 + 2.1^2 = 2.9^2$$

$$a^2 + 4.41 = 8.41$$

$$a^2 = 4$$

$$a = 2$$

Write the Pythagorean Theorem.

Substitute 2.1 for b and 2.9 for c .

Evaluate powers.

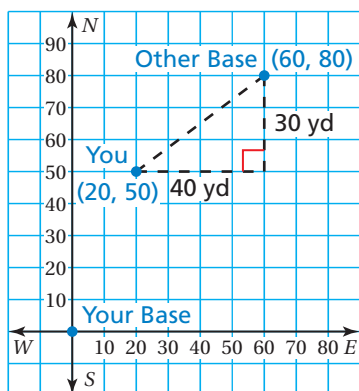
Subtract 4.41 from each side.

Take positive square root of each side.

∴ The missing length is 2 centimeters.

EXAMPLE 3 Real-Life Application

You are playing capture the flag. You are 50 yards north and 20 yards east of your team's base. The other team's base is 80 yards north and 60 yards east of your base. How far are you from the other team's base?



Step 1: Draw the situation in a coordinate plane. Let the origin represent your team's base. From the descriptions, you are at (20, 50) and the other team's base is at (60, 80).

Step 2: Draw a right triangle with a hypotenuse that represents the distance between you and the other team's base. The lengths of the legs are 30 yards and 40 yards.

Step 3: Use the Pythagorean Theorem to find the length of the hypotenuse.

$$a^2 + b^2 = c^2$$

$$30^2 + 40^2 = c^2$$

$$900 + 1600 = c^2$$

$$2500 = c^2$$

$$50 = c$$

Write the Pythagorean Theorem.

Substitute 30 for a and 40 for b .

Evaluate powers.

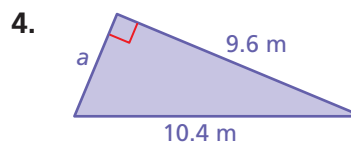
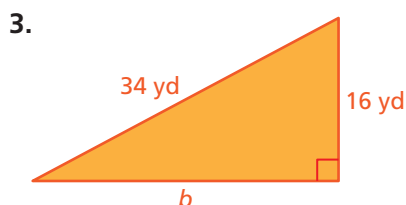
Add.

Take positive square root of each side.

∴ So, you are 50 yards from the other team's base.

On Your Own

Find the missing length of the triangle.



5. In Example 3, what is the distance between the bases?

Now You're Ready
Exercises 5–8

Vocabulary and Concept Check

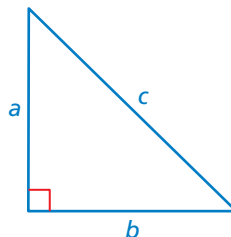
- VOCABULARY** In a right triangle, how can you tell which sides are the legs and which side is the hypotenuse?
- DIFFERENT WORDS, SAME QUESTION** Which is different? Find “both” answers.

Which side is the hypotenuse?

Which side is the longest?

Which side is a leg?

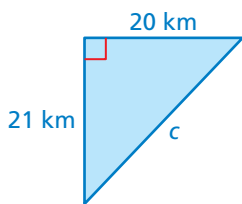
Which side is opposite the right angle?



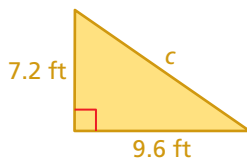
Practice and Problem Solving

Find the missing length of the triangle.

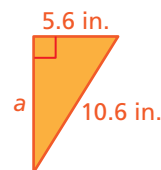
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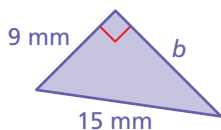
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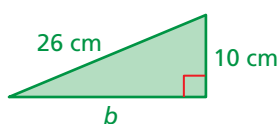
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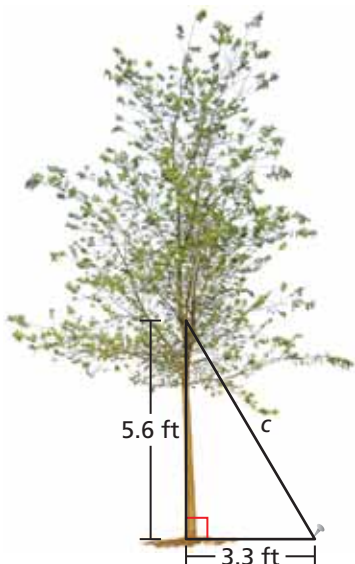
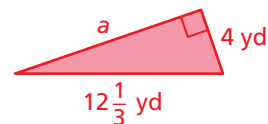
6.



7.



8.



- ERROR ANALYSIS** Describe and correct the error in finding the missing length of the triangle.

✗

$$a^2 + b^2 = c^2$$

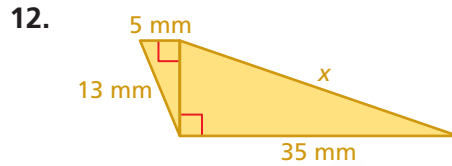
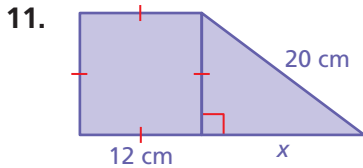
$$7^2 + 25^2 = c^2$$

$$674 = c^2$$

$$\sqrt{674} = c$$

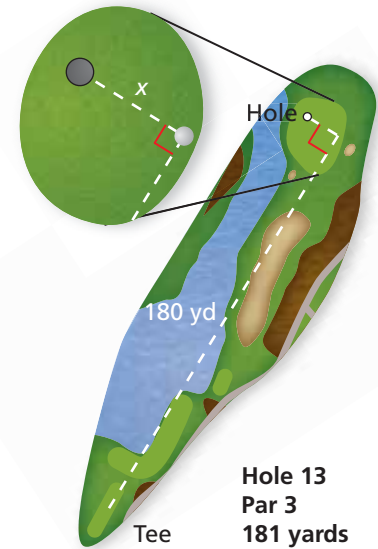
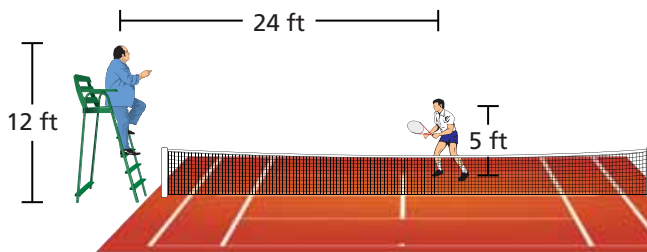
- TREE SUPPORT** How long is the wire that supports the tree?

Find the missing length of the figure.



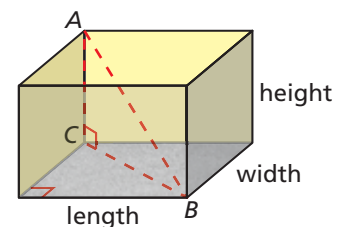
13. **GOLF** The figure shows the location of a golf ball after a tee shot. How many feet from the hole is the ball?

14. **TENNIS** A tennis player asks the referee a question. The sound of the player's voice travels only 30 feet. Can the referee hear the question? Explain.



15. **PROJECT** Measure the length, width, and height of a rectangular room. Use the Pythagorean Theorem to find length BC and length AB .

16. **ALGEBRA** The legs of a right triangle have lengths of 28 meters and 21 meters. The hypotenuse has a length of $5x$ meters. What is the value of x ?



17. **SNOWBALLS** You and a friend stand back-to-back. You run 20 feet forward, then 15 feet to your right. At the same time, your friend runs 16 feet forward, then 12 feet to her right. She stops and hits you with a snowball.

- Draw the situation in a coordinate plane.
- How far does your friend throw the snowball?

18. **Precision** A box has a length of 6 inches, a width of 8 inches, and a height of 24 inches. Can a cylindrical rod with a length of 63.5 centimeters fit in the box? Explain your reasoning.



Fair Game Review what you learned in previous grades & lessons

Find the square root(s). (Section 7.1)

19. $\pm\sqrt{36}$

20. $-\sqrt{121}$

21. $\sqrt{169}$

22. $-\sqrt{225}$

23. **MULTIPLE CHOICE** What is the solution of the system of linear equations $y = 4x + 1$ and $2x + y = 13$? (Section 5.2)

(A) $x = 1, y = 5$

(B) $x = 5, y = 3$

(C) $x = 2, y = 9$

(D) $x = 9, y = 2$