6.3 Linear Functions

Essential Question How can you use a function to describe a linear pattern?

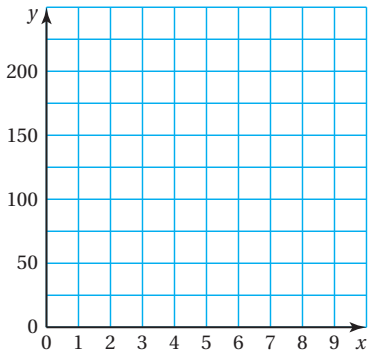
1 ACTIVITY: Finding Linear Patterns

Work with a partner.

- **Plot the points from the table in a coordinate plane.**
- **Write a linear equation for the function represented by the graph.**

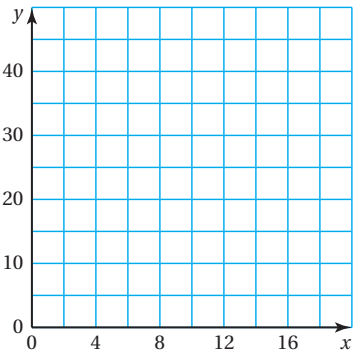
a.

x	0	2	4	6	8
y	150	125	100	75	50



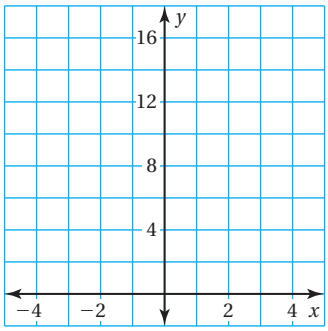
b.

x	4	6	8	10	12
y	15	20	25	30	35



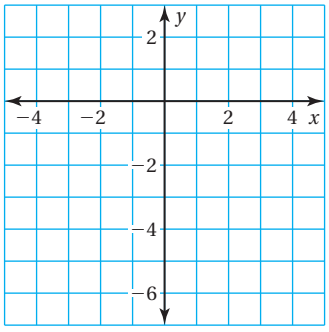
c.

x	−4	−2	0	2	4
y	4	6	8	10	12



d.

x	−4	−2	0	2	4
y	1	0	−1	−2	−3



Functions

In this lesson, you will

- understand that the equation $y = mx + b$ defines a linear function.
- write linear functions using graphs or tables.
- compare linear functions.

2 ACTIVITY: Finding Linear Patterns

Math Practice

Label Axes

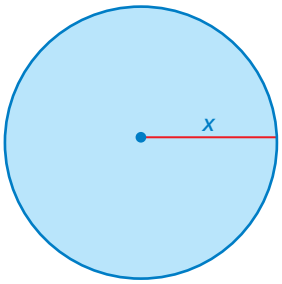
How do you know what to label the axes? How does this help you accurately graph the data?

Work with a partner. The table shows a familiar linear pattern from geometry.

- **Write a function that relates y to x.**
- **What do the variables x and y represent?**
- **Graph the function.**

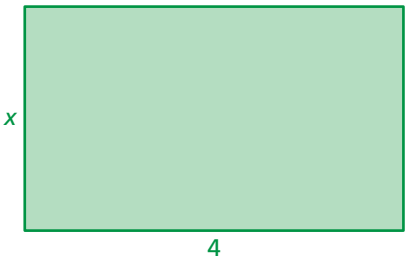
a.

x	1	2	3	4	5
y	2π	4π	6π	8π	10π



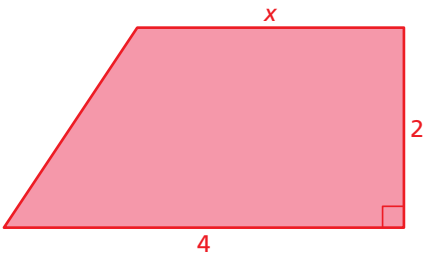
b.

x	1	2	3	4	5
y	10	12	14	16	18



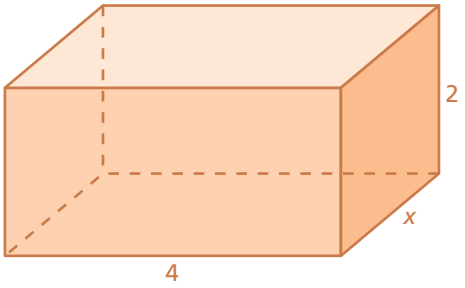
c.

x	1	2	3	4	5
y	5	6	7	8	9



d.

x	1	2	3	4	5
y	28	40	52	64	76



What Is Your Answer?

3. **IN YOUR OWN WORDS** How can you use a function to describe a linear pattern?

4. Describe the strategy you used to find the functions in Activities 1 and 2.

Practice Use what you learned about linear patterns to complete Exercises 3 and 4 on page 261.

6.3 Lesson

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Key Vocabulary
linear function, *p. 258*

A **linear function** is a function whose graph is a nonvertical line. A linear function can be written in the form $y = mx + b$, where m is the slope and b is the y-intercept.

EXAMPLE 1 Writing a Linear Function Using a Graph

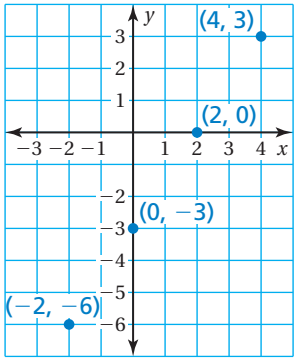
Use the graph to write a linear function that relates y to x.

The points lie on a line. Find the slope by using the points (2, 0) and (4, 3).

$$m = \frac{\text{change in } y}{\text{change in } x} = \frac{3 - 0}{4 - 2} = \frac{3}{2}$$

Because the line crosses the y-axis at (0, −3), the y-intercept is −3.

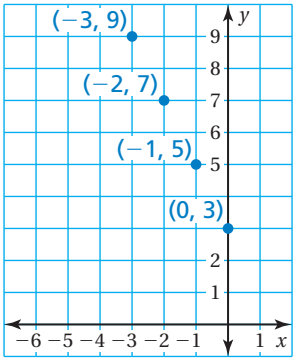
So, the linear function is $y = \frac{3}{2}x - 3$.



EXAMPLE 2 Writing a Linear Function Using a Table

Use the table to write a linear function that relates y to x.

x	−3	−2	−1	0
y	9	7	5	3



Plot the points in the table.

The points lie on a line. Find the slope by using the points (−2, 7) and (−3, 9).

$$m = \frac{\text{change in } y}{\text{change in } x} = \frac{9 - 7}{-3 - (-2)} = \frac{2}{-1} = -2$$

Because the line crosses the y-axis at (0, 3), the y-intercept is 3.

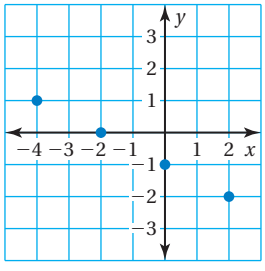
So, the linear function is $y = -2x + 3$.

On Your Own

Now You're Ready
Exercises 5–10

Use the graph or table to write a linear function that relates y to x.

1.



2.

x	−2	−1	0	1
y	2	2	2	2

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EXAMPLE 3 Real-Life Application

Minutes, x	Height (thousands of feet), y
0	65
10	60
20	55
30	50

You are controlling an unmanned aerial vehicle (UAV) for surveillance. The table shows the height y (in thousands of feet) of the UAV x minutes after you start its descent from cruising altitude.

a. Write a linear function that relates y to x. Interpret the slope and the y-intercept.

You can write a linear function that relates the dependent variable y to the independent variable x because the table shows a constant rate of change. Find the slope by using the points (0, 65) and (10, 60).

$$m = \frac{\text{change in } y}{\text{change in } x} = \frac{60 - 65}{10 - 0} = \frac{-5}{10} = -0.5$$

Because the line crosses the y-axis at (0, 65), the y-intercept is 65.

So, the linear function is $y = -0.5x + 65$. The slope indicates that the height decreases 500 feet per minute. The y-intercept indicates that the descent begins at a cruising altitude of 65,000 feet.

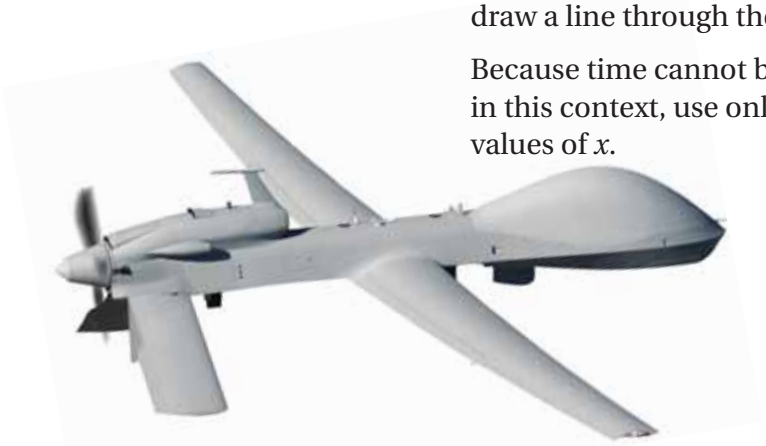
Common Error

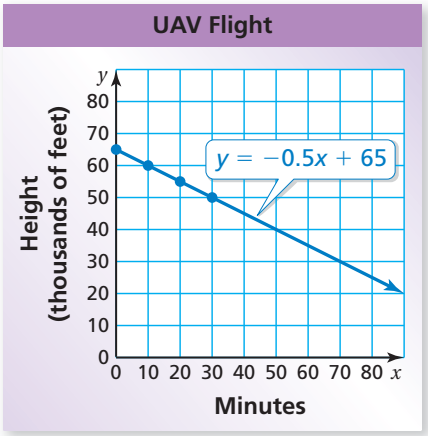
Make sure you consider the units when interpreting the slope and the y-intercept.

b. Graph the linear function.

Plot the points in the table and draw a line through the points.

Because time cannot be negative in this context, use only positive values of x.





c. Find the height of the UAV when you stop the descent after 1 hour.

Because 1 hour = 60 minutes, find the value of y when $x = 60$.

$y = -0.5x + 65$ Write the equation.

$= -0.5(60) + 65$ Substitute 60 for x.

$= 35$ Simplify.

So, the descent of the UAV stops at a height of 35,000 feet.

On Your Own

3. WHAT IF? You double the rate of descent. Repeat parts (a)–(c).

Now You're Ready
Exercises 11–13

EXAMPLE 4 Comparing Linear Functions

The earnings y (in dollars) of a nighttime employee working x hours are represented by the linear function $y = 7.5x + 30$. The table shows the earnings of a daytime employee.

	+1	+1	+1	
Time (hours), x	1	2	3	4
Earnings (dollars), y	12.50	25.00	37.50	50.00
	+12.50	+12.50	+12.50	

a. Which employee has a higher hourly wage?

Nighttime Employee

$y = 7.5x + 30$

The slope is 7.5.

The nighttime employee earns \$7.50 per hour.

Daytime Employee

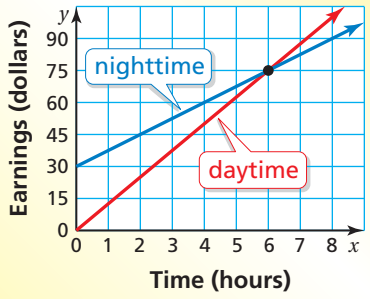
$$\frac{\text{change in earnings}}{\text{change in time}} = \frac{\$12.50}{1 \text{ hour}}$$

The daytime employee earns \$12.50 per hour.

∴ So, the daytime employee has a higher hourly wage.

b. Write a linear function that relates the daytime employee's earnings to the number of hours worked. In the same coordinate plane, graph the linear functions that represent the earnings of the two employees. Interpret the graphs.

Employee Earnings



Use a verbal model to write a linear function that represents the earnings of the daytime employee.

$$\text{Earnings} = \text{Hourly wage} \cdot \text{Hours worked}$$

$$y = 12.5x$$

∴ The graph shows that the daytime employee has a higher hourly wage but does not earn more money than the nighttime employee until each person has worked more than 6 hours.

On Your Own

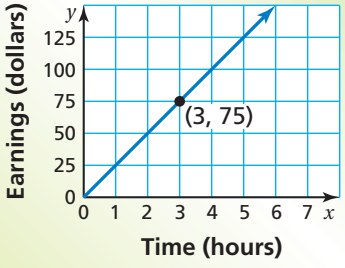
Now You're Ready
Exercise 14

4. Manager A earns \$15 per hour and receives a \$50 bonus. The graph shows the earnings of Manager B.

a. Which manager has a higher hourly wage?

b. After how many hours does Manager B earn more money than Manager A?

Earnings of Manager B



6.3 Exercises

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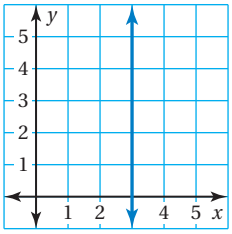
Vocabulary and Concept Check

1. **STRUCTURE** Is $y = mx + b$ a linear function when $b = 0$? Explain.

2. **WRITING** Explain why the vertical line does not represent a linear function.



Practice and Problem Solving

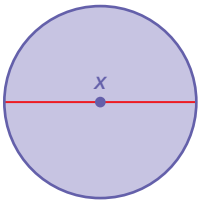
The table shows a familiar linear pattern from geometry. Write a function that relates y to x. What do the variables x and y represent? Graph the function.

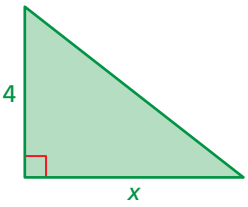
3.

x	1	2	3	4	5
y	π	2π	3π	4π	5π

4.

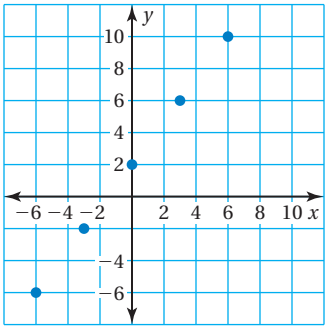
x	1	2	3	4	5
y	2	4	6	8	10



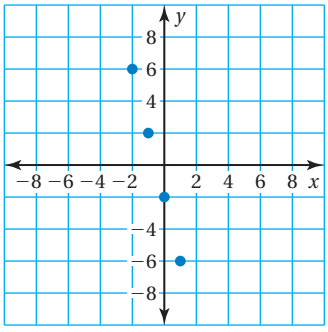


Use the graph or table to write a linear function that relates y to x.

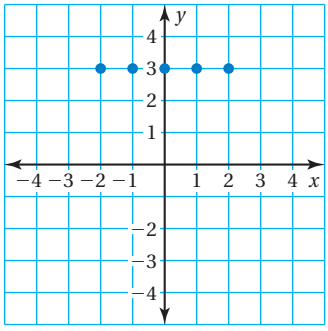
1 2 5.



6.



7.



8.

x	−2	−1	0	1
y	−4	−2	0	2

9.

x	−8	−4	0	4
y	2	1	0	−1

10.

x	−3	0	3	6
y	3	5	7	9

3 11. **MOVIES** The table shows the cost y (in dollars) of renting x movies.

Number of Movies, x	0	1	2	4
Cost, y	0	3	6	12

a. Which variable is independent? dependent?

b. Write a linear function that relates y to x. Interpret the slope.

c. Graph the linear function.

d. How much does it cost to rent three movies?

12. BIKE JUMPS A *bunny hop* is a bike trick in which the rider brings both tires off the ground without using a ramp. The table shows the height y (in inches) of a bunny hop on a bike that weighs x pounds.

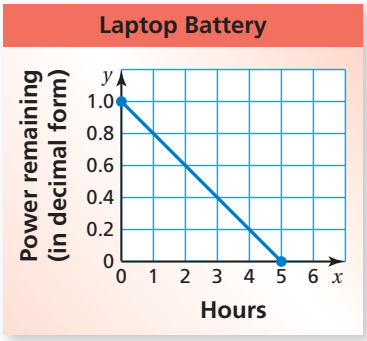
Weight (pounds), x	19	21	23
Height (inches), y	10.2	9.8	9.4

a. Write a linear function that relates the height of a bunny hop to the weight of the bike.

b. Graph the linear function.

c. What is the height of a bunny hop on a bike that weighs 21.5 pounds?

13. BATTERY The graph shows the percent y (in decimal form) of battery power remaining x hours after you turn on a laptop computer.



a. Write a linear function that relates y to x.

b. Interpret the slope, the x-intercept, and the y-intercept.

c. After how many hours is the battery power at 75%?

4 **14. RACE** You and a friend race each other. You give your friend a 50-foot head start. The distance y (in feet) your friend runs after x seconds is represented by the linear function $y = 14x + 50$. The table shows the distances you run.

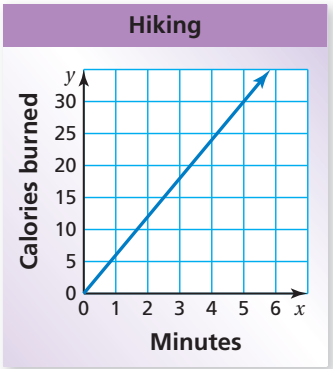
Time (seconds), x	2	4	6	8
Distance (feet), y	38	76	114	152

a. Who runs at a faster rate? What is that rate?

b. Write a linear function that relates your distance to the number of seconds. In the same coordinate plane, graph the linear functions that represent the distances of you and your friend.

c. For what distances will you win the race? Explain.

15. CALORIES The number of calories burned y after x minutes of kayaking is represented by the linear function $y = 4.5x$. The graph shows the calories burned by hiking.



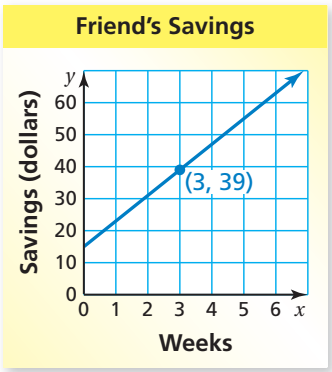
a. Which activity burns more calories per minute?

b. How many more calories are burned by doing the activity in part (a) than the other activity for 45 minutes?

16. SAVINGS You and your friend are saving money to buy bicycles that cost $175 each. The amount y (in dollars) you save after x weeks is represented by the equation $y = 5x + 45$. The graph shows your friend's savings.

a. Who has more money to start? Who saves more per week?

b. Who can buy a bicycle first? Explain.



17. REASONING Can the graph of a linear function be a horizontal line? Explain your reasoning.

Years of Education, x	Annual Salary, y
0	28
2	40
4	52
6	64
10	88

18. SALARY The table shows a person's annual salary y (in thousands of dollars) after x years of education beyond high school.

a. Graph the data. Then describe the pattern.

b. What is the annual salary of the person after 8 years of education beyond high school?

c. Find the annual salary of a person with 30 years of education beyond high school. Do you think this situation makes sense? Explain.

19. Problem Solving The Heat Index is calculated using the relative humidity and the temperature. For every 1 degree increase in the temperature from 94°F to 98°F at 75% relative humidity, the Heat Index rises 4°F.

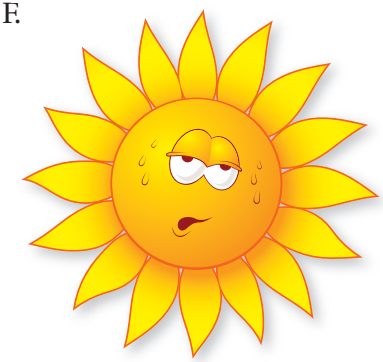
STEM VIDEO

a. On a summer day, the relative humidity is 75%, the temperature is 94°F, and the Heat Index is 122°F. Construct a table that relates the temperature t to the Heat Index H. Start the table at 94°F and end it at 98°F.

b. Identify the independent and dependent variables.

c. Write a linear function that represents this situation.

d. Estimate the Heat Index when the temperature is 100°F.



Fair Game Review What you learned in previous grades & lessons

Solve the equation. *(Section 1.1)*

20. $b - 1.6 \div 4 = -3$

21. $w + |-2.8| = 4.3$

22. $\frac{3}{4} = y - \frac{1}{5}(8)$

23. MULTIPLE CHOICE Which of the following describes the translation from the red figure to the blue figure? *(Section 2.2)*

Ⓐ $(x - 6, y + 5)$ Ⓑ $(x - 5, y + 6)$

Ⓒ $(x + 6, y - 5)$ Ⓓ $(x + 5, y - 6)$