

**Essential Question** Can a system of linear equations have no solution? Can a system of linear equations have many solutions?

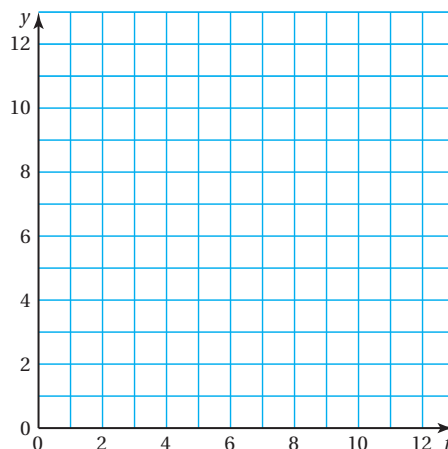
### 1 ACTIVITY: Writing a System of Linear Equations

Work with a partner. Your cousin is 3 years older than you. You can represent your ages by two linear equations.

$$y = t \quad \text{Your age}$$

$$y = t + 3 \quad \text{Your cousin's age}$$

- Graph both equations in the same coordinate plane.
- What is the vertical distance between the two graphs? What does this distance represent?
- Do the two graphs intersect? Explain what this means in terms of your age and your cousin's age.



### 2 ACTIVITY: Using a Table to Solve a System

Work with a partner. You invest \$500 for equipment to make dog backpacks. Each backpack costs you \$15 for materials. You sell each backpack for \$15.



#### Systems of Equations

In this lesson, you will

- solve systems of linear equations with no solution or infinitely many solutions.

- Copy and complete the table for your cost  $C$  and your revenue  $R$ .

$x$	0	1	2	3	4	5	6	7	8	9	10
$C$											
$R$											

- When will you break even? What is wrong?

3

**ACTIVITY: Using a Graph to Solve a Puzzle****Math Practice****Analyze Relationships**

What do you know about the graphs of the two equations? How does this relate to the number of solutions?

Work with a partner. Let  $x$  and  $y$  be two numbers. Here are two clues about the values of  $x$  and  $y$ .

**Words****Equation**

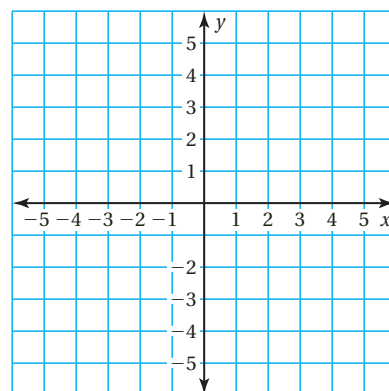
**Clue 1:**  $y$  is 4 more than twice the value of  $x$ .

$$y = 2x + 4$$

**Clue 2:** The difference of  $3y$  and  $6x$  is 12.

$$3y - 6x = 12$$

- Graph both equations in the same coordinate plane.
- Do the two lines intersect? Explain.
- What is the solution of the puzzle?
- Use the equation  $y = 2x + 4$  to complete the table.



$x$	0	1	2	3	4	5	6	7	8	9	10
$y$											

- Does each solution in the table satisfy *both* clues?
- What can you conclude? How many solutions does the puzzle have? How can you describe them?

**What Is Your Answer?**

- IN YOUR OWN WORDS** Can a system of linear equations have no solution? Can a system of linear equations have many solutions? Give examples to support your answers.

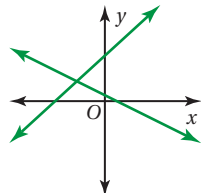
**Practice**

Use what you learned about special systems of linear equations to complete Exercises 3 and 4 on page 228.

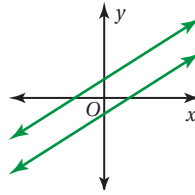
## Key Idea

### Solutions of Systems of Linear Equations

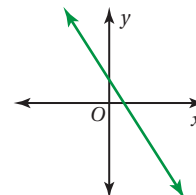
A system of linear equations can have *one solution*, *no solution*, or *infinitely many solutions*.



**One solution**  
The lines intersect.



**No solution**  
The lines are parallel.



**Infinitely many solutions**  
The lines are the same.

### EXAMPLE 1 Solving a System: No Solution

Solve the system.

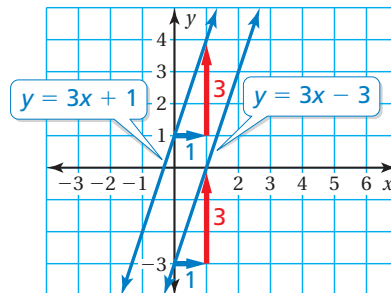
$$y = 3x + 1 \quad \text{Equation 1}$$

$$y = 3x - 3 \quad \text{Equation 2}$$

**Method 1:** Solve by graphing.

Graph each equation. The lines have the same slope and different  $y$ -intercepts. So, the lines are parallel.

Because parallel lines do not intersect, there is no point that is a solution of both equations.



∴ So, the system of linear equations has no solution.

**Method 2:** Solve by substitution.

Substitute  $3x - 3$  for  $y$  in Equation 1.

$$y = 3x + 1 \quad \text{Equation 1}$$

$$3x - 3 = 3x + 1 \quad \text{Substitute } 3x - 3 \text{ for } y.$$

$$-3 = 1 \quad \text{Subtract } 3x \text{ from each side.}$$

∴ The equation  $-3 = 1$  is never true. So, the system of linear equations has no solution.

### On Your Own

Solve the system of linear equations. Check your solution.

- $y = -x + 3$   
 $y = -x + 5$
- $y = -5x - 2$   
 $5x + y = 0$
- $x = 2y + 10$   
 $2x + 3y = -1$

### Study Tip

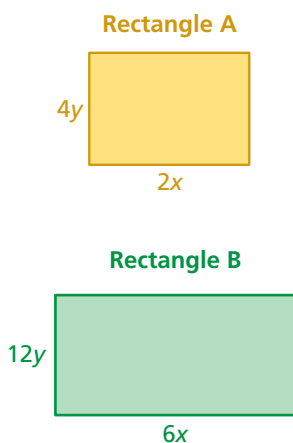
You can solve some linear systems by inspection. In Example 1, notice you can rewrite the system as

$$-3x + y = 1$$

$$-3x + y = -3.$$

This system has no solution because  $-3x + y$  cannot be equal to 1 and  $-3$  at the same time.

## EXAMPLE 2 Solving a System: Infinitely Many Solutions



The perimeter of Rectangle A is 36 units. The perimeter of Rectangle B is 108 units. Write and solve a system of linear equations to find the values of  $x$  and  $y$ .

*Perimeter of Rectangle A*

$$2(2x) + 2(4y) = 36$$

$$4x + 8y = 36 \quad \text{Equation 1}$$

*Perimeter of Rectangle B*

$$2(6x) + 2(12y) = 108$$

$$12x + 24y = 108 \quad \text{Equation 2}$$

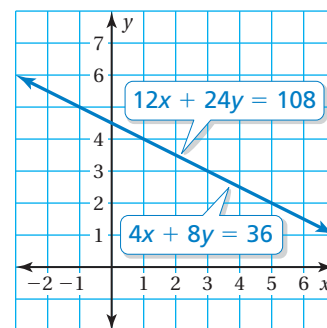
The system is:  $4x + 8y = 36$  Equation 1

$12x + 24y = 108$  Equation 2

**Method 1:** Solve by graphing.

Graph each equation.

The lines have the same slope and the same  $y$ -intercept. So, the lines are the same.



⚡ In this context,  $x$  and  $y$  must be positive. Because the lines are the same, all the points on the line in Quadrant I are solutions of both equations. So, the system of linear equations has infinitely many solutions.

**Method 2:** Solve by elimination.

Multiply Equation 1 by 3 and subtract the equations.

$$\begin{array}{r}
 4x + 8y = 36 \\
 12x + 24y = 108 \\
 \hline
 12x + 24y = 108 \\
 \hline
 0 = 0
 \end{array}$$

Revised Equation 1  
 Equation 2  
 Subtract.

⚡ The equation  $0 = 0$  is always true. In this context,  $x$  and  $y$  must be positive. So, the solutions are all the points on the line  $4x + 8y = 36$  in Quadrant I. The system of linear equations has infinitely many solutions.

### On Your Own

Now You're Ready  
Exercises 11–13

Solve the system of linear equations. Check your solution.

4.  $x + y = 3$

5.  $2x + y = 5$

6.  $2x - 4y = 10$

$x - y = -3$

$4x + 2y = 0$

$-12x + 24y = -60$

7. **WHAT IF?** What happens to the solution in Example 2 if the perimeter of Rectangle A is 54 units? Explain.

## Vocabulary and Concept Check

- WRITING** Describe the difference between the graph of a system of linear equations that has *no solution* and the graph of a system of linear equations that has *infinitely many solutions*.
- REASONING** When solving a system of linear equations algebraically, how do you know when the system has *no solution*? *infinitely many solutions*?

## Practice and Problem Solving

Let  $x$  and  $y$  be two numbers. Find the solution of the puzzle.

3.  $y$  is  $\frac{1}{3}$  more than 4 times the value of  $x$ .

The difference of  $3y$  and  $12x$  is 1.

4.  $\frac{1}{2}$  of  $x$  plus 3 is equal to  $y$ .

$x$  is 6 more than twice the value of  $y$ .

Without graphing, determine whether the system of linear equations has *one solution*, *infinitely many solutions*, or *no solution*. Explain your reasoning.

5.  $y = 5x - 9$

$y = 5x + 9$

6.  $y = 6x + 2$

$y = 3x + 1$

7.  $y = 8x - 2$

$y - 8x = -2$

Solve the system of linear equations. Check your solution.

1 8.  $y = 2x - 2$

$y = 2x + 9$

9.  $y = 3x + 1$

$-x + 2y = -3$

10.  $y = \frac{\pi}{3}x + \pi$

$-\pi x + 3y = -6\pi$

2 11.  $y = -\frac{1}{6}x + 5$

$x + 6y = 30$

12.  $\frac{1}{3}x + y = 1$

$2x + 6y = 6$

13.  $-2x + y = 1.3$

$2(0.5x - y) = 4.6$

14. **ERROR ANALYSIS** Describe and correct the error in solving the system of linear equations.



$y = -2x + 4$

$y = -2x + 6$

The lines have the same slope, so, there are infinitely many solutions.



15. **PIG RACE** In a pig race, your pig gets a head start of 3 feet and is running at a rate of 2 feet per second. Your friend's pig is also running at a rate of 2 feet per second. A system of linear equations that represents this situation is  $y = 2x + 3$  and  $y = 2x$ . Will your friend's pig catch up to your pig? Explain.

16. **REASONING** One equation in a system of linear equations has a slope of  $-3$ . The other equation has a slope of  $4$ . How many solutions does the system have? Explain.
17. **LOGIC** How can you use the slopes and the  $y$ -intercepts of equations in a system of linear equations to determine whether the system has *one solution*, *infinitely many solutions*, or *no solution*? Explain your reasoning.

$$4x + 8y = 64$$

$$8x + 16y = 128$$



18. **MONEY** You and a friend both work two different jobs. The system of linear equations represents the total earnings for  $x$  hours worked at the first job and  $y$  hours worked at the second job. Your friend earns twice as much as you.
- One week, both of you work 4 hours at the first job. How many hours do you and your friend work at the second job?
  - Both of you work the same number of hours at the second job. Compare the number of hours each of you works at the first job.

19. **DOWNLOADS** You download a digital album for \$10. Then you and your friend download the same number of individual songs for \$0.99 each. Write a system of linear equations that represents this situation. Will you and your friend spend the same amount of money? Explain.

20. **REASONING** Does the system shown *always*, *sometimes*, or *never* have no solution when  $a = b$ ?  $a \geq b$ ?  $a < b$ ? Explain your reasoning.

$$y = ax + 1$$

$$y = bx + 4$$

21. **SKIING** The table shows the number of lift tickets and ski rentals sold to two different groups. Is it possible to determine how much each lift ticket costs? Justify your answer.

<b>Group</b>	1	2
<b>Number of Lift Tickets</b>	36	24
<b>Number of Ski Rentals</b>	18	12
<b>Total Cost (dollars)</b>	684	456

22. **Precision** Find the values of  $a$  and  $b$  so the system shown has the solution  $(2, 3)$ . Does the system have any other solutions? Explain.

$$12x - 2by = 12$$

$$3ax - by = 6$$



## Fair Game Review what you learned in previous grades & lessons

Write an equation of the line that passes through the given points. (Section 4.7)

23.  $(0, 0), (2, 6)$

24.  $(0, -3), (3, 3)$

25.  $(-6, 5), (0, 2)$

26. **MULTIPLE CHOICE** What is the solution of  $-2(y + 5) \leq 16$ ? (Skills Review Handbook)

(A)  $y \leq -13$

(B)  $y \geq -13$

(C)  $y \leq -3$

(D)  $y \geq -3$