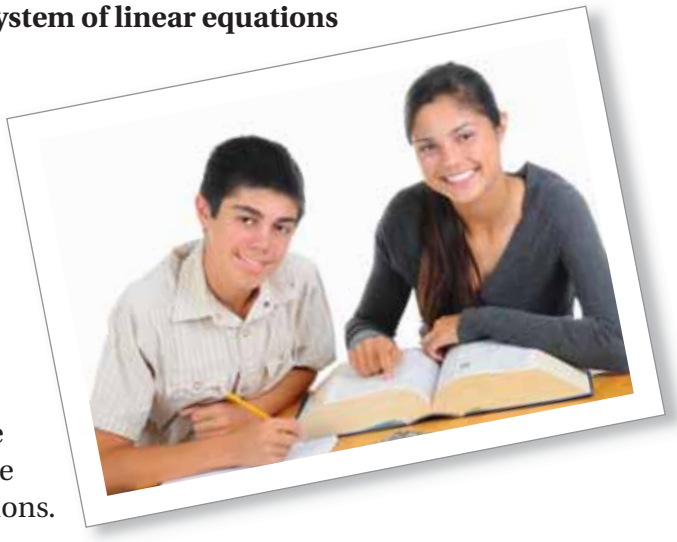
5.3 Solving Systems of Linear Equations by Elimination

Essential Question How can you use elimination to solve a system of linear equations?

1 ACTIVITY: Using Elimination to Solve a System

Work with a partner. Solve each system of linear equations by using two methods.



Method 1: Subtract.

Subtract Equation 2 from Equation 1. What is the result? Explain how you can use the result to solve the system of equations.

Method 2: Add.

Add the two equations. What is the result? Explain how you can use the result to solve the system of equations.

Is the solution the same using both methods?

a. $2x + y = 4$
$2x - y = 0$

b. $3x - y = 4$
$3x + y = 2$

c. $x + 2y = 7$
$x - 2y = -5$

2 ACTIVITY: Using Elimination to Solve a System

Work with a partner.

$2x + y = 2$ Equation 1
$x + 5y = 1$ Equation 2

a. Can you add or subtract the equations to solve the system of linear equations? Explain.

b. Explain what property you can apply to Equation 1 in the system so that the y-coefficients are the same.

c. Explain what property you can apply to Equation 2 in the system so that the x-coefficients are the same.

d. You solve the system in part (b). Your partner solves the system in part (c). Compare your solutions.

e. Use a graphing calculator to check your solution.

Systems of Equations

In this lesson, you will

- write and solve systems of linear equations by elimination.
- solve real-life problems.

3 ACTIVITY: Solving a Secret Code

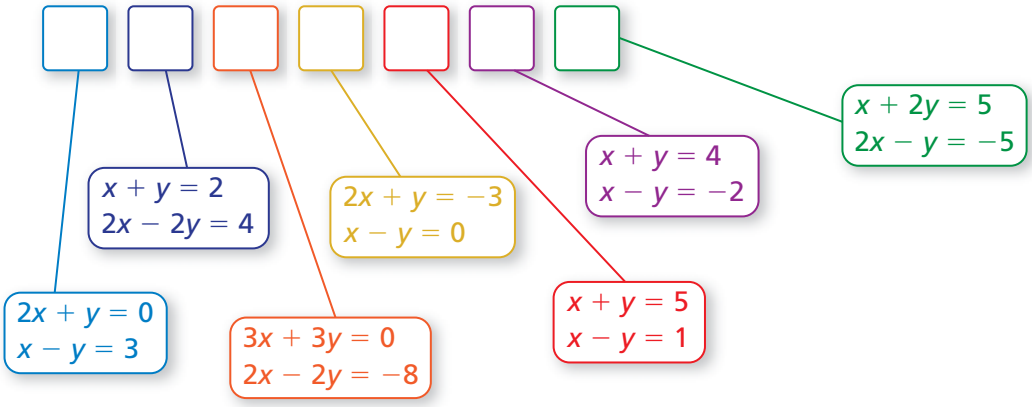
Math Practice

Find Entry Points

What is the first thing you do to solve a system of linear equations? Why?

Work with a partner. Solve the puzzle to find the name of a famous mathematician who lived in Egypt around 350 A.D.

4	B	W	R	M	F	Y	K	N
3	O	J	A	S	I	D	X	Z
2	Q	P	C	E	G	B	T	J
1	M	R	C	Z	N	O	U	W
0	K	X	U	H	L	Y	S	Q
−1	F	E	A	S	W	K	R	M
−2	G	J	Z	N	H	V	D	G
−3	E	L	X	L	F	Q	O	B
	−3	−2	−1	0	1	2	3	4



What Is Your Answer?

4. **IN YOUR OWN WORDS** How can you use elimination to solve a system of linear equations?

5. **STRUCTURE** When can you add or subtract equations in a system to solve the system? When do you have to multiply first? Justify your answers with examples.

6. **LOGIC** In Activity 2, why can you multiply equations in the system by a constant and not change the solution of the system? Explain your reasoning.

Practice Use what you learned about systems of linear equations to complete Exercises 4–6 on page 221.

5.3 Lesson

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Key Idea

Solving a System of Linear Equations by Elimination

Step 1: Multiply, if necessary, one or both equations by a constant so at least 1 pair of like terms has the same or opposite coefficients.

Step 2: Add or subtract the equations to eliminate one of the variables.

Step 3: Solve the resulting equation for the remaining variable.

Step 4: Substitute the value from Step 3 into one of the original equations and solve.

EXAMPLE 1 Solving a System of Linear Equations by Elimination

Solve the system by elimination. $x + 3y = -2$ Equation 1; $x - 3y = 16$ Equation 2

Step 1: The coefficients of the y-terms are already opposites.

Step 2: Add the equations.

$$\begin{array}{rl} x + 3y = -2 & \text{Equation 1} \\ x - 3y = 16 & \text{Equation 2} \\ \hline 2x = 14 & \text{Add the equations.} \end{array}$$

Step 3: Solve for x.

$$\begin{array}{rl} 2x = 14 & \text{Equation from Step 2} \\ x = 7 & \text{Divide each side by 2.} \end{array}$$

Step 4: Substitute 7 for x in one of the original equations and solve for y.

$$\begin{array}{rl} x + 3y = -2 & \text{Equation 1} \\ 7 + 3y = -2 & \text{Substitute 7 for } x. \\ 3y = -9 & \text{Subtract 7 from each side.} \\ y = -3 & \text{Divide each side by 3.} \end{array}$$

∴ The solution is $(7, -3)$.

Study Tip

Because the coefficients of x are the same, you can also solve the system by subtracting in Step 2.

$$\begin{array}{r} x + 3y = -2 \\ x - 3y = 16 \\ \hline 6y = -18 \end{array}$$

So, $y = -3$.

Check

Equation 1

$$x + 3y = -2$$
$$7 + 3(-3) \stackrel{?}{=} -2$$
$$-2 = -2 \checkmark$$

Equation 2

$$x - 3y = 16$$
$$7 - 3(-3) \stackrel{?}{=} 16$$
$$16 = 16 \checkmark$$

On Your Own

Now You're Ready
Exercises 7–12

Solve the system of linear equations by elimination. Check your solution.

1. $2x - y = 9$, $4x + y = 21$
2. $-5x + 2y = 13$, $5x + y = -1$
3. $3x + 4y = -6$, $7x + 4y = -14$

EXAMPLE 2 Solving a System of Linear Equations by Elimination

Solve the system by elimination.

$$-6x + 5y = 25 \quad \text{Equation 1}$$
$$-2x - 4y = 14 \quad \text{Equation 2}$$

Step 1: Multiply Equation 2 by 3.

$$-6x + 5y = 25 \qquad -6x + 5y = 25 \quad \text{Equation 1}$$
$$-2x - 4y = 14 \xrightarrow{\text{Multiply by 3.}} -6x - 12y = 42 \quad \text{Revised Equation 2}$$

Step 2: Subtract the equations.

$$\begin{array}{rrl} -6x + & 5y = & 25 \quad \text{Equation 1} \\ -6x - & 12y = & 42 \quad \text{Revised Equation 2} \\ \hline & 17y = & -17 \quad \text{Subtract the equations.} \end{array}$$

Step 3: Solve for y.

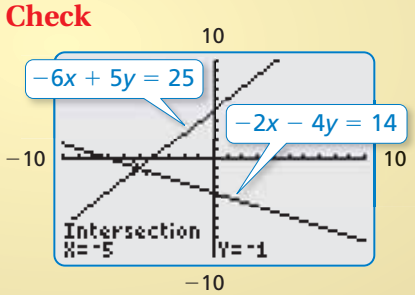
$$17y = -17 \quad \text{Equation from Step 2}$$
$$y = -1 \quad \text{Divide each side by 17.}$$

Step 4: Substitute -1 for y in one of the original equations and solve for x.

$$-2x - 4y = 14 \quad \text{Equation 2}$$
$$-2x - 4(-1) = 14 \quad \text{Substitute } -1 \text{ for } y.$$
$$-2x + 4 = 14 \quad \text{Multiply.}$$
$$-2x = 10 \quad \text{Subtract 4 from each side.}$$
$$x = -5 \quad \text{Divide each side by } -2.$$

∴ The solution is $(-5, -1)$.

Check



Study Tip

In Example 2, notice that you can also multiply Equation 2 by -3 and then add the equations.

On Your Own

Solve the system of linear equations by elimination. Check your solution.

4. $3x + y = 11$
$6x + 3y = 24$

5. $4x - 5y = -19$
$-x - 2y = 8$

6. $5y = 15 - 5x$
$y = -2x + 3$

EXAMPLE 3 Real-Life Application

You buy 8 hostas and 15 daylilies for $193. Your friend buys 3 hostas and 12 daylilies for $117. Write and solve a system of linear equations to find the cost of each daylily.

Use a verbal model to write a system of linear equations.

Number of hostas	•	Cost of each hosta, x	+	Number of daylilies	•	Cost of each daylily, y	=	Total cost

The system is: $8x + 15y = 193$ Equation 1 (You)

$3x + 12y = 117$ Equation 2 (Your friend)

Step 1: To find the cost y of each daylily, eliminate the x-terms. Multiply Equation 1 by 3. Multiply Equation 2 by 8.

$8x + 15y = 193$ **Multiply by 3.** $24x + 45y = 579$ Revised Equation 1

$3x + 12y = 117$ **Multiply by 8.** $24x + 96y = 936$ Revised Equation 2

Step 2: Subtract the revised equations.

$$\begin{array}{rl} 24x + 45y = & 579 \\ \underline{24x + 96y = \ \ 936} & \\ -51y = & -357 \end{array}$$

Revised Equation 1

Revised Equation 2

Subtract the equations.

Step 3: Solving the equation $-51y = -357$ gives $y = 7$.

∴ So, each daylily costs $7.

On Your Own

Now You're Ready
Exercises 16–21

7. A landscaper buys 4 peonies and 9 geraniums for $190. Another landscaper buys 5 peonies and 6 geraniums for $185. Write and solve a system of linear equations to find the cost of each peony.

Summary

Methods for Solving Systems of Linear Equations

Method	When to Use
Graphing *(Lesson 5.1)*	To estimate solutions
Substitution *(Lesson 5.2)*	When one of the variables in one of the equations has a coefficient of 1 or −1
Elimination *(Lesson 5.3)*	When at least 1 pair of like terms has the same or opposite coefficients
Elimination (Multiply First) *(Lesson 5.3)*	When one of the variables cannot be eliminated by adding or subtracting the equations

5.3 Exercises

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Vocabulary and Concept Check

1. **WRITING** Describe how to solve a system of linear equations by elimination.
2. **NUMBER SENSE** When should you use multiplication to solve a system of linear equations by elimination?
3. **WHICH ONE DOESN'T BELONG?** Which system of equations does *not* belong with the other three? Explain your reasoning.

$3x + 3y = 3$
$2x - 3y = 7$

$-2x + y = 6$
$2x - 3y = -10$

$2x + 3y = 11$
$3x - 2y = 10$

$x + y = 5$
$3x - y = 3$

Practice and Problem Solving

Use a method from Activity 1 to solve the system.

4. $x + y = 3$
   $x - y = 1$
5. $-x + 3y = 0$
   $x + 3y = 12$
6. $3x + 2y = 3$
   $3x - 2y = -9$

Solve the system of linear equations by elimination. Check your solution.

7. $x + 3y = 5$
   $-x - y = -3$
8. $x - 2y = -7$
   $3x + 2y = 3$
9. $4x + 3y = -5$
   $-x + 3y = -10$
10. $2x + 7y = 1$
    $2x - 4y = 12$
11. $2x + 5y = 16$
    $3x - 5y = -1$
12. $3x - 2y = 4$
    $6x - 2y = -2$

13. **ERROR ANALYSIS** Describe and correct the error in solving the system of linear equations.

✗

$5x + 2y = 9$ Equation 1
$3x - 2y = -1$ Equation 2
$2x = 10$
$x = 5$

The solution is $(5, -8)$.

14. **RAFFLE TICKETS** You and your friend are selling raffle tickets for a new laptop. You sell 14 more tickets than your friend sells. Together, you and your friend sell 58 tickets.
    a. Write a system of linear equations that represents this situation.
    b. How many tickets does each of you sell?

15. **JOGGING** You can jog around your block twice and the park once in 10 minutes. You can jog around your block twice and the park 3 times in 22 minutes.
    a. Write a system of linear equations that represents this situation.
    b. How long does it take you to jog around the park?

Solve the system of linear equations by elimination. Check your solution.

2 3 **16.** $2x - y = 0$
$3x - 2y = -3$

17. $x + 4y = 1$
$3x + 5y = 10$

18. $-2x + 3y = 7$
$5x + 8y = -2$

19. $3x + 3 = 3y$
$2x - 6y = 2$

20. $2x - 6 = 4y$
$7y = -3x + 9$

21. $5x = 4y + 8$
$3y = 3x - 3$

22. ERROR ANALYSIS Describe and correct the error in solving the system of linear equations.

$x + y = 1$ Equation 1
$5x + 3y = -3$ Equation 2

Multiply by −5.

$-5x + 5y = -5$
$5x + 3y = -3$
$8y = -8$
$y = -1$

The solution is $(2, -1)$.

23. REASONING For what values of a and b should you solve the system by elimination?

a. $4x - y = 3$
$ax + 10y = 6$

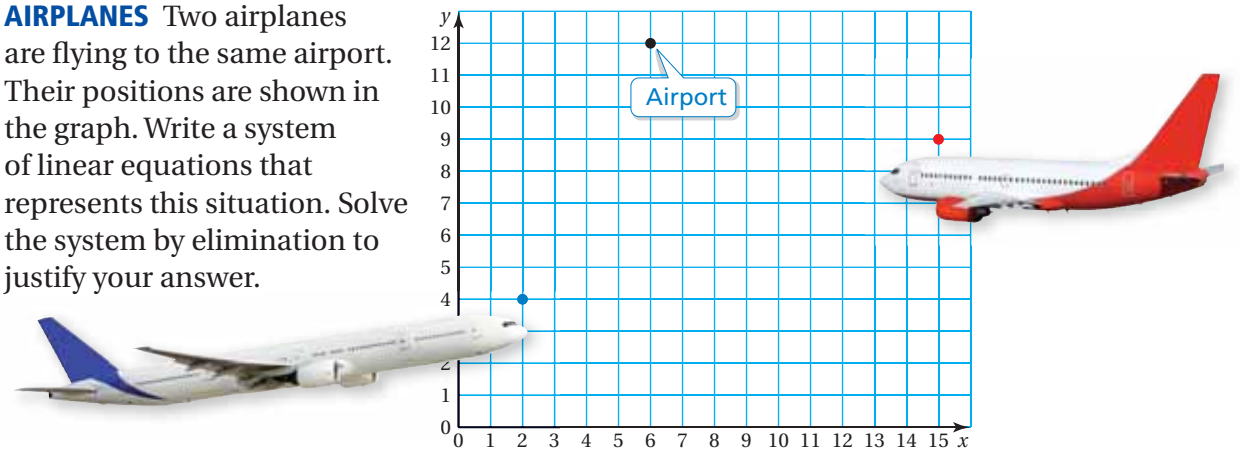
b. $x - 7y = 6$
$-6x + by = 9$

Determine whether the line through the first pair of points intersects the line through the second pair of points. Explain.

24. Line 1: $(-2, 1), (2, 7)$
Line 2: $(-4, -1), (0, 5)$

25. Line 1: $(3, -2), (7, -1)$
Line 2: $(5, 2), (6, -2)$

26. AIRPLANES Two airplanes are flying to the same airport. Their positions are shown in the graph. Write a system of linear equations that represents this situation. Solve the system by elimination to justify your answer.



27. TEST PRACTICE The table shows the number of correct answers on a practice standardized test. You score 86 points on the test, and your friend scores 76 points.

	You	Your Friend
Multiple Choice	23	28
Short Response	10	5

a. Write a system of linear equations that represents this situation.

b. How many points is each type of question worth?

28. **LOGIC** You solve a system of equations in which x represents the number of adult tickets sold and y represents the number of student tickets sold. Can $(-6, 24)$ be the solution of the system? Explain your reasoning.

29. **VACATION** The table shows the activities of two tourists at a vacation resort. You want to go parasailing for 1 hour and horseback riding for 2 hours. How much do you expect to pay?

	Parasailing	Horseback Riding	Total Cost
Tourist 1	2 hours	5 hours	\$205
Tourist 2	3 hours	3 hours	\$240

30. **REASONING** The solution of a system of linear equations is $(2, -4)$. One equation in the system is $2x + y = 0$. Explain how you could find a second equation for the system. Then find a second equation. Solve the system by elimination to justify your answer.

31. **JEWELER** A metal alloy is a mixture of two or more metals. A jeweler wants to make 8 grams of 18-carat gold, which is 75% gold. The jeweler has an alloy that is 90% gold and an alloy that is 50% gold. How much of each alloy should the jeweler use?

STEM VIDEO

32. **PROBLEM SOLVING** A powerboat takes 30 minutes to travel 10 miles downstream. The return trip takes 50 minutes. What is the speed of the current?

33. Critical Thinking Solve the system of equations by elimination.

$$2x - y + 3z = -1$$
$$x + 2y - 4z = -1$$
$$y - 2z = 0$$

Fair Game Review What you learned in previous grades & lessons

Decide whether the two equations are equivalent. *(Section 1.2 and Section 1.3)*

34. $4n + 1 = n - 8$
$3n = -9$

35. $2a + 6 = 12$
$a + 3 = 6$

36. $7v - \frac{3}{2} = 5$
$14v - 3 = 15$

37. **MULTIPLE CHOICE** Which line has the same slope as $y = \frac{1}{2}x - 3$? *(Section 4.4)*

(A) $y = -2x + 4$ (B) $y = 2x + 3$ (C) $y - 2x = 5$ (D) $2y - x = 7$