5.1 Solving Systems of Linear Equations by Graphing

Essential Question How can you solve a system of linear equations?

1 ACTIVITY: Writing a System of Linear Equations

Work with a partner.

Your family starts a bed-and-breakfast. It spends $500 fixing up a bedroom to rent. The cost for food and utilities is $10 per night. Your family charges $60 per night to rent the bedroom.

a. Write an equation that represents the costs.

Cost, C (in dollars)	=	$10 per night	•	Number of nights, x	+	$500

b. Write an equation that represents the revenue (income).

Revenue, R (in dollars)	=	$60 per night	•	Number of nights, x

c. A set of two (or more) linear equations is called a **system of linear equations**. Write the system of linear equations for this problem.

2 ACTIVITY: Using a Table to Solve a System

Work with a partner. Use the cost and revenue equations from Activity 1 to find how many nights your family needs to rent the bedroom before recovering the cost of fixing up the bedroom. This is the *break-even point*.

a. Copy and complete the table.

x	0	1	2	3	4	5	6	7	8	9	10	11
C												
R												

b. How many nights does your family need to rent the bedroom before breaking even?

Systems of Equations

In this lesson, you will

- write and solve systems of linear equations by graphing.
- solve real-life problems.

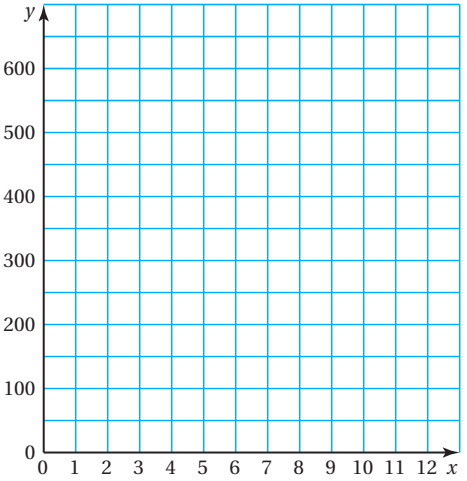
3 ACTIVITY: Using a Graph to Solve a System

Work with a partner.

a. Graph the cost equation from Activity 1.

b. In the same coordinate plane, graph the revenue equation from Activity 1.

c. Find the point of intersection of the two graphs. What does this point represent? How does this compare to the break-even point in Activity 2? Explain.



4 ACTIVITY: Using a Graphing Calculator

Work with a partner. Use a graphing calculator to solve the system.

$y = 10x + 500$ Equation 1
$y = 60x$ Equation 2

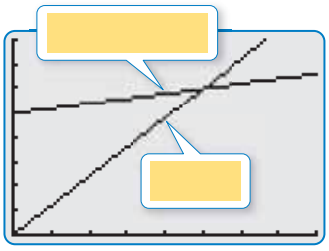
Math Practice

Use Technology to Explore

How do you decide the values for the viewing window of your calculator? What other viewing windows could you use?

a. Enter the equations into your calculator. Then graph the equations. What is an appropriate window?

b. On your graph, how can you determine which line is the graph of which equation? Label the equations on the graph shown.



c. Visually estimate the point of intersection of the graphs.

d. To find the solution, use the *intersect* feature to find the point of intersection. The solution is (,).

What Is Your Answer?

5. IN YOUR OWN WORDS How can you solve a system of linear equations? How can you check your solution?

6. CHOOSE TOOLS Solve one of the systems by using a table, another system by sketching a graph, and the remaining system by using a graphing calculator. Explain why you chose each method.

a. $y = 4.3x + 1.2$
$y = -1.7x - 2.4$

b. $y = x$
$y = -2x + 9$

c. $y = -x - 5$
$y = 3x + 1$

Practice Use what you learned about systems of linear equations to complete Exercises 4–6 on page 206.

5.1 Lesson

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Key Vocabulary
system of linear equations, *p. 204*
solution of a system of linear equations, *p. 204*

A **system of linear equations** is a set of two or more linear equations in the same variables. An example is shown below.

$y = x + 1$ Equation 1
$y = 2x - 7$ Equation 2

A **solution of a system of linear equations** in two variables is an ordered pair that is a solution of each equation in the system. The solution of a system of linear equations is the point of intersection of the graphs of the equations.

Reading
A system of linear equations is also called a *linear system*.

Key Idea

Solving a System of Linear Equations by Graphing

Step 1: Graph each equation in the same coordinate plane.

Step 2: Estimate the point of intersection.

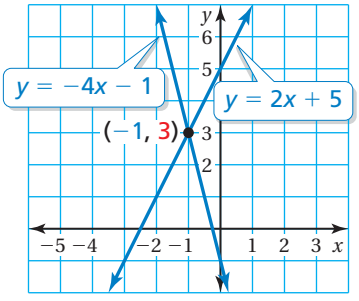
Step 3: Check the point from Step 2 by substituting for x and y in each equation of the original system.

EXAMPLE 1 Solving a System of Linear Equations by Graphing

Solve the system by graphing. $y = 2x + 5$ Equation 1
$y = -4x - 1$ Equation 2

Step 1: Graph each equation.

Step 2: Estimate the point of intersection. The graphs appear to intersect at $(-1, 3)$.



Step 3: Check the point from Step 2.

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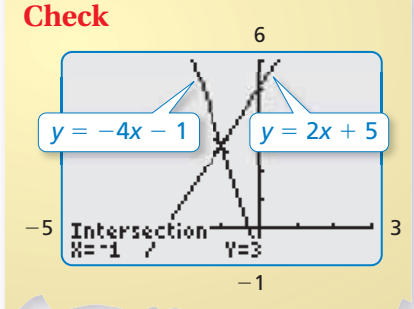


Equation 1
$y = 2x + 5$
$3 \stackrel{?}{=} 2(-1) + 5$
$3 = 3$ ✓

Equation 2
$y = -4x - 1$
$3 \stackrel{?}{=} -4(-1) - 1$
$3 = 3$ ✓

∴ The solution is $(-1, 3)$.

On Your Own

Now You're Ready
Exercises 10–12

Solve the system of linear equations by graphing.

1. $y = x - 1$
$y = -x + 3$

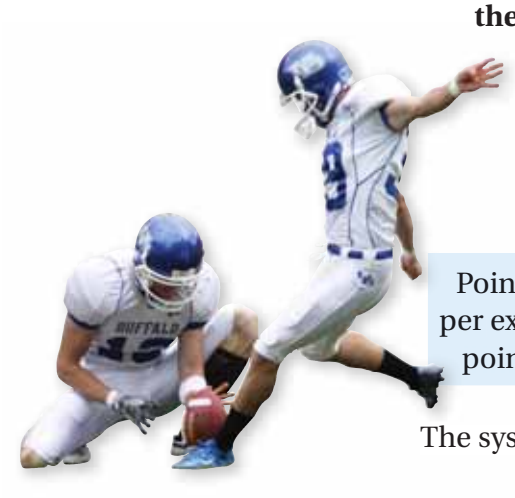
2. $y = -5x + 14$
$y = x - 10$

3. $y = x$
$y = 2x + 1$

Multi-Language Glossary at BigIdeasMath.com

EXAMPLE 2 Real-Life Application

A kicker on a football team scores 1 point for making an extra point and 3 points for making a field goal. The kicker makes a total of 8 extra points and field goals in a game and scores 12 points. Write and solve a system of linear equations to find the number x of extra points and the number y of field goals.



Use a verbal model to write a system of linear equations.

Number of extra points, x	+	Number of field goals, y	=	Total number of kicks

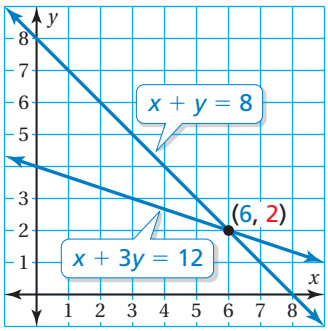
Points per extra point	•	Number of extra points, x	+	Points per field goal	•	Number of field goals, y	=	Total number of points

The system is:

$x + y = 8$ Equation 1

$x + 3y = 12$ Equation 2

Step 1: Graph each equation.



Step 2: Estimate the point of intersection. The graphs appear to intersect at (6, 2).

Step 3: Check your point from Step 2.

Equation 1

$x + y = 8$

$6 + 2 \stackrel{?}{=} 8$

$8 = 8$ ✓

Equation 2

$x + 3y = 12$

$6 + 3(2) \stackrel{?}{=} 12$

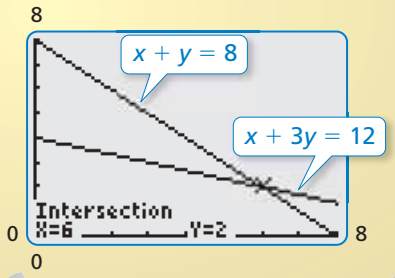
$12 = 12$ ✓

∴ The solution is (6, 2). So, the kicker made 6 extra points and 2 field goals.

Study Tip

It may be easier to graph the equations in a system by rewriting the equations in slope-intercept form.

Check



On Your Own

Now You're Ready
Exercises 13–15

Solve the system of linear equations by graphing.

4. $y = -4x - 7$
$x + y = 2$

5. $x - y = 5$
$-3x + y = -1$

6. $\frac{1}{2}x + y = -6$
$6x + 2y = 8$

7. WHAT IF? The kicker makes a total of 7 extra points and field goals and scores 17 points. Write and solve a system of linear equations to find the numbers of extra points and field goals.

5.1 Exercises

Check It Out
Help with Homework
BigIdeasMath.com

Vocabulary and Concept Check

1. **VOCABULARY** Do the equations $4x - 3y = 5$ and $7y + 2x = -8$ form a system of linear equations? Explain.
2. **WRITING** What does it mean to solve a system of equations?
3. **WRITING** You graph a system of linear equations, and the solution appears to be (3, 4). How can you verify that the solution is (3, 4)?

Practice and Problem Solving

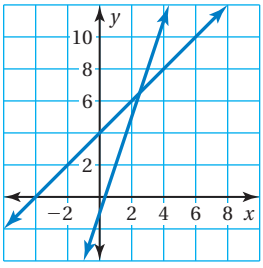
Use a table to find the break-even point. Check your solution.

4. $C = 15x + 150$
   $R = 45x$
5. $C = 24x + 80$
   $R = 44x$
6. $C = 36x + 200$
   $R = 76x$

Match the system of linear equations with the corresponding graph. Use the graph to estimate the solution. Check your solution.

7. $y = 1.5x - 2$
   $y = -x + 13$
8. $y = x + 4$
   $y = 3x - 1$
9. $y = \frac{2}{3}x - 3$
   $y = -2x + 5$

A.



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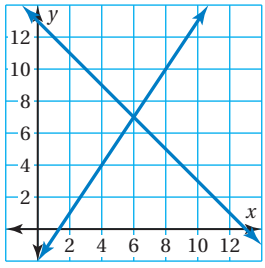
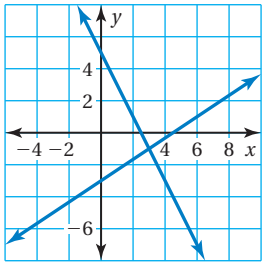


C.



Solve the system of linear equations by graphing.

1 10. $y = 2x + 9$
$y = 6 - x$

11. $y = -x - 4$
$y = \frac{3}{5}x + 4$

12. $y = 2x + 5$
$y = \frac{1}{2}x - 1$

2 13. $x + y = 27$
$y = x + 3$

14. $y - x = 17$
$y = 4x + 2$

15. $x - y = 7$
$0.5x + y = 5$

16. **CARRIAGE RIDES** The cost C (in dollars) for the care and maintenance of a horse and carriage is $C = 15x + 2000$, where x is the number of rides.

   a. Write an equation for the revenue R in terms of the number of rides.

   b. How many rides are needed to break even?

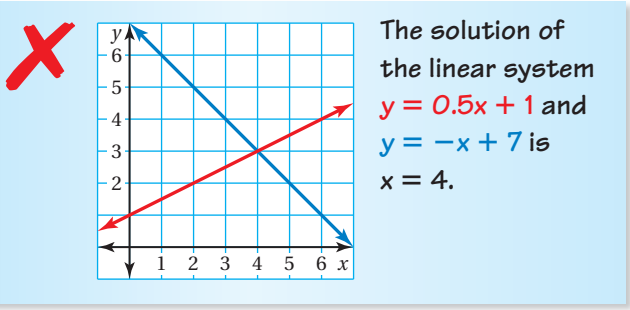
Use a graphing calculator to solve the system of linear equations.

17. $2.2x + y = 12.5$
$1.4x - 4y = 1$

18. $2.1x + 4.2y = 14.7$
$-5.7x - 1.9y = -11.4$

19. $-1.1x - 5.5y = -4.4$
$0.8x - 3.2y = -11.2$

20. ERROR ANALYSIS Describe and correct the error in solving the system of linear equations.



21. REASONING Is it possible for a system of two linear equations to have exactly two solutions? Explain your reasoning.

22. MODELING You have a total of 42 math and science problems for homework. You have 10 more math problems than science problems. How many problems do you have in each subject? Use a system of linear equations to justify your answer.

23. CANOE RACE You and your friend are in a canoe race. Your friend is a half mile in front of you and paddling 3 miles per hour. You are paddling 3.4 miles per hour.

a. You are 8.5 miles from the finish line. How long will it take you to catch up to your friend?

b. You both maintain your paddling rates for the remainder of the race. How far ahead of your friend will you be when you cross the finish line?

24. Critical Thinking Your friend is trying to grow her hair as long as her cousin's hair. The table shows their hair lengths (in inches) in different months.

Month	Friend's Hair (in.)	Cousin's Hair (in.)
3	4	7
8	6.5	9

a. Write a system of linear equations that represents this situation.

b. Will your friend's hair ever be as long as her cousin's hair? If so, in what month?

Fair Game Review What you learned in previous grades & lessons

Solve the equation. Check your solution. *(Section 1.2)*

25. $\frac{3}{4}c - \frac{1}{4}c + 3 = 7$

26. $5(2 - y) + y = -6$

27. $6x - 3(x + 8) = 9$

28. MULTIPLE CHOICE What is the slope of the line that passes through $(-2, -2)$ and $(3, -1)$? *(Section 4.2)*

Ⓐ -5 Ⓑ $-\frac{1}{5}$ Ⓒ $\frac{1}{5}$ Ⓓ 5