

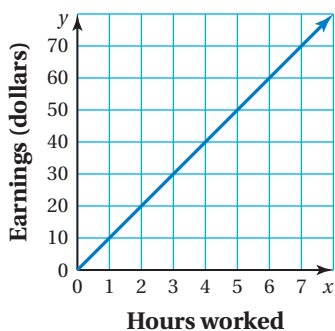
4.3 Graphing Proportional Relationships

Essential Question How can you describe the graph of the equation $y = mx$?

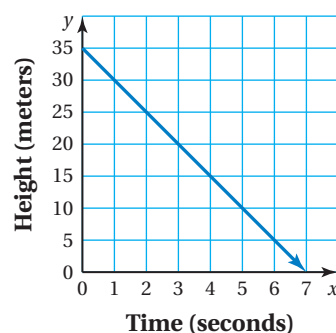
1 ACTIVITY: Identifying Proportional Relationships

Work with a partner. Tell whether x and y are in a proportional relationship. Explain your reasoning.

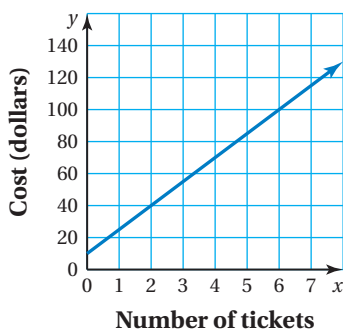
a. **Money**



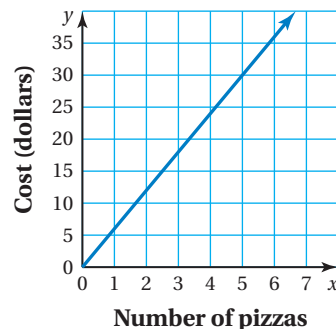
b. **Helicopter**



c. **Tickets**



d. **Pizzas**



e.

Laps, x	1	2	3	4
Time (seconds), y	90	200	325	480

f.

Cups of Sugar, x	$\frac{1}{2}$	1	$1\frac{1}{2}$	2
Cups of Flour, y	1	2	3	4

Graphing Equations

In this lesson, you will

- write and graph proportional relationships.

2 ACTIVITY: Analyzing Proportional Relationships

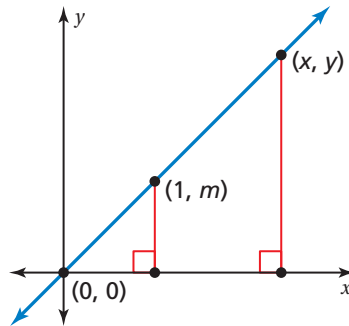
Work with a partner. Use only the proportional relationships in Activity 1 to do the following.

- Find the slope of the line.
- Find the value of y for the ordered pair $(1, y)$.

What do you notice? What does the value of y represent?

3 ACTIVITY: Deriving an Equation

Work with a partner. Let (x, y) represent any point on the graph of a proportional relationship.



- Explain why the two triangles are similar.
- Because the triangles are similar, the corresponding side lengths are proportional. Use the vertical and horizontal side lengths to complete the steps below.

Math Practice

View as Components

What part of the graph can you use to find the side lengths?

$$\frac{\text{yellow box}}{\text{yellow box}} = \frac{m}{1}$$

Ratios of side lengths

$$\frac{\text{yellow box}}{\text{yellow box}} = m$$

Simplify.

$$\text{yellow box} = m \cdot \text{yellow box}$$

Multiplication Property of Equality

What does the final equation represent?

- Use your result in part (b) to write an equation that represents each proportional relationship in Activity 1.

What Is Your Answer?

- IN YOUR OWN WORDS** How can you describe the graph of the equation $y = mx$? How does the value of m affect the graph of the equation?
- Give a real-life example of two quantities that are in a proportional relationship. Write an equation that represents the relationship and sketch its graph.

Practice

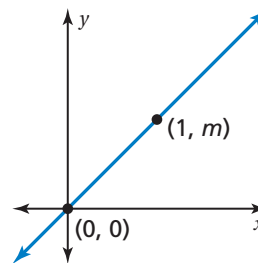
Use what you learned about proportional relationships to complete Exercises 3–6 on page 162.

Key Idea

Direct Variation

Words When two quantities x and y are proportional, the relationship can be represented by the direct variation equation $y = mx$, where m is the constant of proportionality.

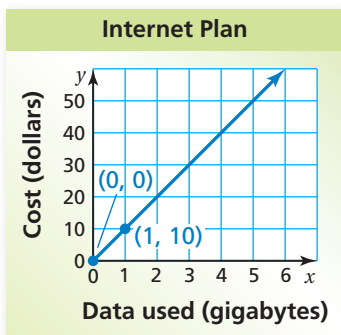
Graph The graph of $y = mx$ is a line with a slope of m that passes through the origin.



Study Tip

In the direct variation equation $y = mx$, m represents the constant of proportionality, the slope, and the unit rate.

EXAMPLE 1 Graphing a Proportional Relationship



The cost y (in dollars) for x gigabytes of data on an Internet plan is represented by $y = 10x$. Graph the equation and interpret the slope.

The equation shows that the slope m is 10. So, the graph passes through $(0, 0)$ and $(1, 10)$.

Plot the points and draw a line through the points. Because negative values of x do not make sense in this context, graph in the first quadrant only.

••• The slope indicates that the unit cost is \$10 per gigabyte.

EXAMPLE 2 Writing and Using a Direct Variation Equation

The weight y of an object on Titan, one of Saturn's moons, is proportional to the weight x of the object on Earth. An object that weighs 105 pounds on Earth would weigh 15 pounds on Titan.

a. Write an equation that represents the situation.

Use the point $(105, 15)$ to find the slope of the line.

$$y = mx \quad \text{Direct variation equation}$$

$$15 = m(105) \quad \text{Substitute 15 for } y \text{ and 105 for } x.$$

$$\frac{1}{7} = m \quad \text{Simplify.}$$

••• So, an equation that represents the situation is $y = \frac{1}{7}x$.

b. How much would a chunk of ice that weighs 3.5 pounds on Titan weigh on Earth?

$$3.5 = \frac{1}{7}x \quad \text{Substitute 3.5 for } y.$$

$$24.5 = x \quad \text{Multiply each side by 7.}$$

••• So, the chunk of ice would weigh 24.5 pounds on Earth.

Study Tip

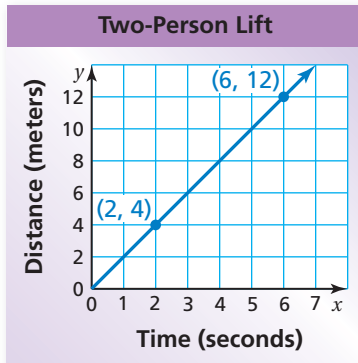
In Example 2, the slope indicates that the weight of an object on Titan is one-seventh its weight on Earth.

On Your Own

Now You're Ready
Exercises 7–8

- WHAT IF?** In Example 1, the cost is represented by $y = 12x$. Graph the equation and interpret the slope.
- In Example 2, how much would a spacecraft that weighs 3500 kilograms on Earth weigh on Titan?

EXAMPLE 3 Comparing Proportional Relationships



The distance y (in meters) that a four-person ski lift travels in x seconds is represented by the equation $y = 2.5x$. The graph shows the distance that a two-person ski lift travels.

- a. Which ski lift is faster?

Interpret each slope as a unit rate.

Four-Person Lift

$$y = 2.5x$$

The slope is 2.5.

The four-person lift travels 2.5 meters per second.

Two-Person Lift

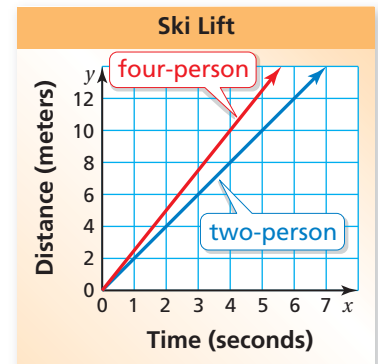
$$\begin{aligned} \text{slope} &= \frac{\text{change in } y}{\text{change in } x} \\ &= \frac{8}{4} = 2 \end{aligned}$$

The two-person lift travels 2 meters per second.

∴ So, the four-person lift is faster than the two-person lift.

- b. Graph the equation that represents the four-person lift in the same coordinate plane as the two-person lift. Compare the steepness of the graphs. What does this mean in the context of the problem?

∴ The graph that represents the four-person lift is steeper than the graph that represents the two-person lift. So, the four-person lift is faster.



On Your Own

Now You're Ready
Exercise 9

- The table shows the distance y (in meters) that a T-bar ski lift travels in x seconds. Compare its speed to the ski lifts in Example 3.

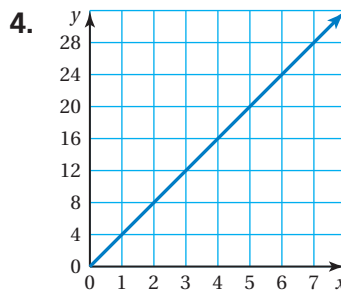
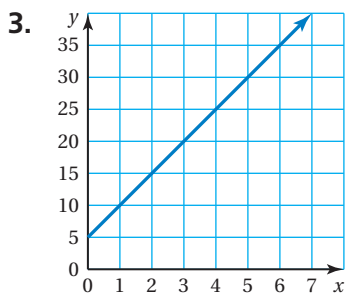
x (seconds)	1	2	3	4
y (meters)	$2\frac{1}{4}$	$4\frac{1}{2}$	$6\frac{3}{4}$	9

Vocabulary and Concept Check

- VOCABULARY** What point is on the graph of every direct variation equation?
- REASONING** Does the equation $y = 2x + 3$ represent a proportional relationship? Explain.

Practice and Problem Solving

Tell whether x and y are in a proportional relationship. Explain your reasoning. If so, write an equation that represents the relationship.



5.

x	3	6	9	12
y	1	2	3	4

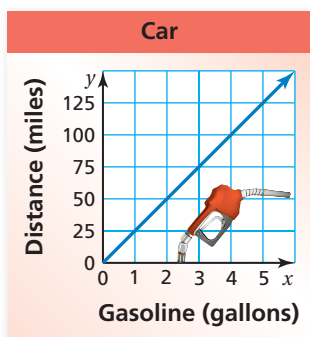
6.

x	2	5	8	10
y	4	8	13	23

- 1 7. **TICKETS** The amount y (in dollars) that you raise by selling x fundraiser tickets is represented by the equation $y = 5x$. Graph the equation and interpret the slope.

- 2 8. **KAYAK** The cost y (in dollars) to rent a kayak is proportional to the number x of hours that you rent the kayak. It costs \$27 to rent the kayak for 3 hours.

- Write an equation that represents the situation.
- Interpret the slope.
- How much does it cost to rent the kayak for 5 hours?



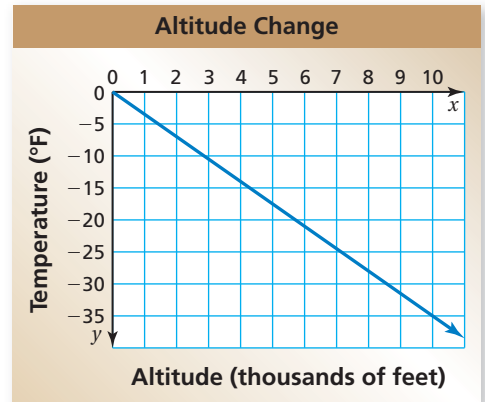
- 3 9. **MILEAGE** The distance y (in miles) that a truck travels on x gallons of gasoline is represented by the equation $y = 18x$. The graph shows the distance that a car travels.
- Which vehicle gets better gas mileage? Explain how you found your answer.
 - How much farther can the vehicle you chose in part (a) travel than the other vehicle on 8 gallons of gasoline?

10. **BIOLOGY** Toenails grow about 13 millimeters per year. The table shows fingernail growth.

Weeks	1	2	3	4
Fingernail Growth (millimeters)	0.7	1.4	2.1	2.8

- a. Do fingernails or toenails grow faster? Explain.
- b. In the same coordinate plane, graph equations that represent the growth rates of toenails and fingernails. Compare the steepness of the graphs. What does this mean in the context of the problem?
11. **REASONING** The quantities x and y are in a proportional relationship. What do you know about the ratio of y to x for any point (x, y) on the line?

12. **PROBLEM SOLVING** The graph relates the temperature change y (in degrees Fahrenheit) to the altitude change x (in thousands of feet).



- a. Is the relationship proportional? Explain.
- b. Write an equation of the line. Interpret the slope.
- c. You are at the bottom of a mountain where the temperature is 74°F . The top of the mountain is 5500 feet above you. What is the temperature at the top of the mountain?

13. **Critical Thinking** Consider the distance equation $d = rt$, where d is the distance (in feet), r is the rate (in feet per second), and t is the time (in seconds).

- a. You run 6 feet per second. Are distance and time proportional? Explain. Graph the equation.
- b. You run for 50 seconds. Are distance and rate proportional? Explain. Graph the equation.
- c. You run 300 feet. Are rate and time proportional? Explain. Graph the equation.
- d. One of these situations represents *inverse variation*. Which one is it? Why do you think it is called inverse variation?



Fair Game Review what you learned in previous grades & lessons

Graph the linear equation. (Section 4.1)

14. $y = -\frac{1}{2}x$

15. $y = 3x - \frac{3}{4}$

16. $y = -\frac{x}{3} - \frac{3}{2}$

17. **MULTIPLE CHOICE** What is the value of x ? (Section 3.3)

(A) 110

(B) 135

(C) 315

(D) 522

