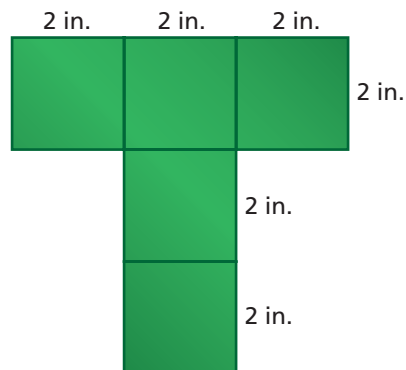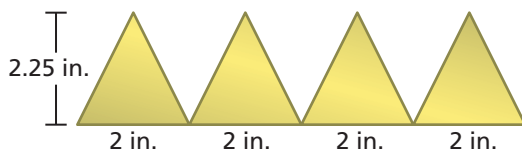**Essential Question** How can you find the volume of a pyramid?

1   ACTIVITY: Finding a Formula Experimentally

Work with a partner.

- **Draw the two nets on cardboard and cut them out.**

2.25 in.

2 in.   2 in.   2 in.   2 in.

2 in.   2 in.   2 in.

2 in.

2 in.

2 in.

2 in.

- **Fold and tape the nets to form an open square box and an open pyramid.**

- **Both figures should have the same size square base and the same height.**

- **Fill the pyramid with pebbles. Then pour the pebbles into the box. Repeat this until the box is full. How many pyramids does it take to fill the box?**

- **Use your result to find a formula for the volume of a pyramid.**
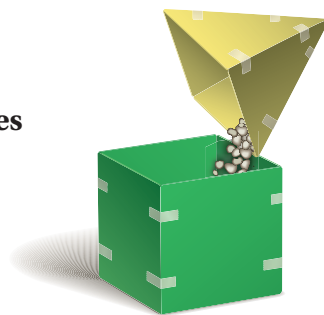
2   ACTIVITY: Comparing Volumes

Work with a partner. You are an archaeologist studying two ancient pyramids. What factors would affect how long it took to build each pyramid? Given similar conditions, which pyramid took longer to build? Explain your reasoning.

Geometry

In this lesson, you will
- find volumes of pyramids.
- solve real-life problems.

The Sun Pyramid in Mexico
Height: about 246 ft
Base: about 738 ft by 738 ft

Cheops Pyramid in Egypt
Height: about 480 ft
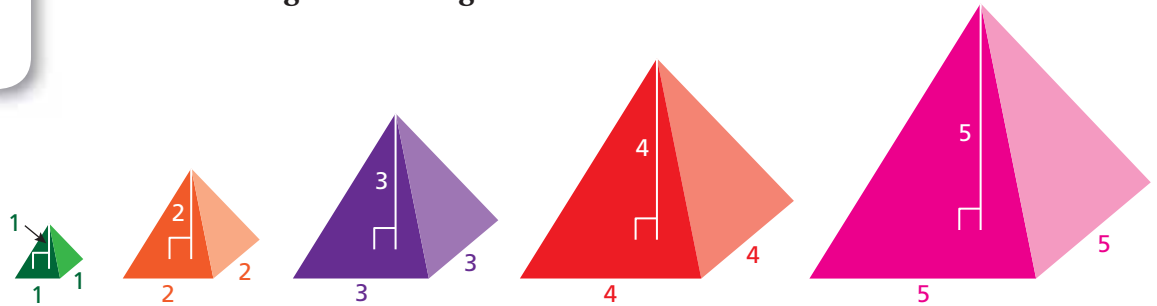Base: about 755 ft by 755 ft

3 ACTIVITY: Finding and Using a Pattern

Work with a partner.

- **Find the volumes of the pyramids.**
- **Organize your results in a table.**
- **Describe the pattern.**
- **Use your pattern to find the volume of a pyramid with a base length and a height of 20.**

4 ACTIVITY: Breaking a Prism into Pyramids

Work with a partner. The rectangular prism can be cut to form three pyramids. Show that the sum of the volumes of the three pyramids is equal to the volume of the prism.

a.   **b.**   **c.**
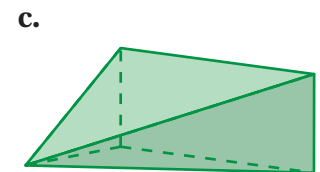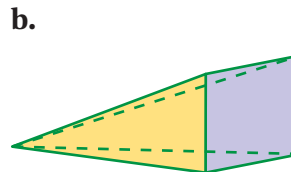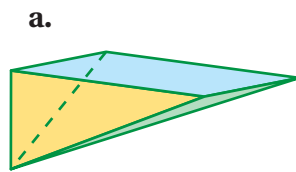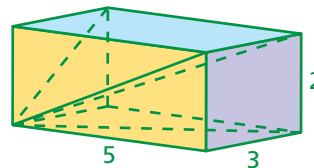
What Is Your Answer?

5. **IN YOUR OWN WORDS** How can you find the volume of a pyramid?

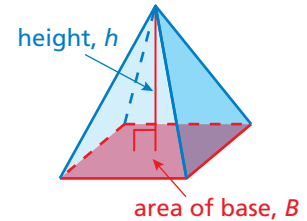6. **STRUCTURE** Write a general formula for the volume of a pyramid.

Practice

Use what you learned about the volumes of pyramids to complete Exercises 4–6 on page 386.

Check It Out
Lesson Tutorials
BigIdeasMath.com

Key Idea

Volume of a Pyramid

Study Tip

The *height* of a pyramid is the perpendicular distance from the base to the vertex.

Words The volume V of a pyramid is one-third the product of the area of the base and the height of the pyramid.

height, h

Area of base

Algebra $V = \dfrac{1}{3}Bh$

Height of pyramid

area of base, B

EXAMPLE 1 Finding the Volume of a Pyramid

Find the volume of the pyramid.

$V = \dfrac{1}{3}Bh$ Write formula for volume.

$= \dfrac{1}{3}(48)(9)$ Substitute.

$= 144$ Multiply.

9 mm

$B = 48$ mm²
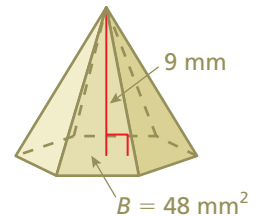
∴ The volume is 144 cubic millimeters.
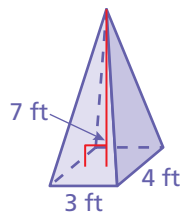
EXAMPLE 2 Finding the Volume of a Pyramid

Find the volume of the pyramid.

Study Tip

The area of the base of a rectangular pyramid is the product of the length ℓ and the width w.

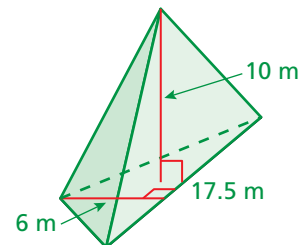You can use $V = \dfrac{1}{3}\ell w h$ to find the volume of a rectangular pyramid.

a.

7 ft

4 ft

3 ft

$V = \dfrac{1}{3}Bh$

$= \dfrac{1}{3}(4)(3)(7)$

$= 28$

∴ The volume is 28 cubic feet.

b.

10 m

17.5 m

6 m

$V = \dfrac{1}{3}Bh$

$= \dfrac{1}{3}\left(\dfrac{1}{2}\right)(17.5)(6)(10)$

$= 175$

∴ The volume is 175 cubic meters.

Find the volume of the pyramid.

1.

$B = 21 \text{ ft}^2$

2.

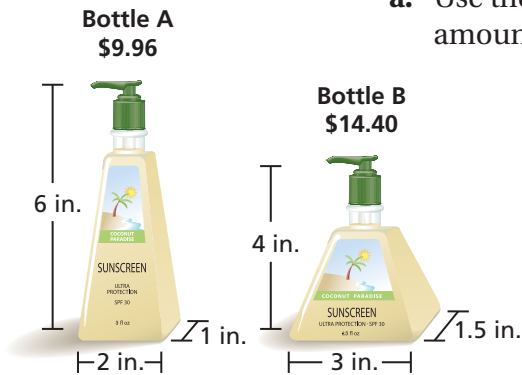7 in.
8 in.
10 in.

3.

11 cm
7 cm
18 cm
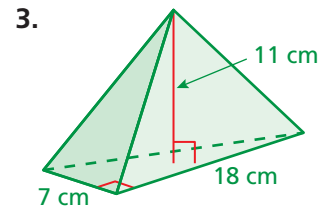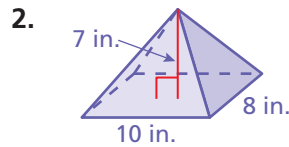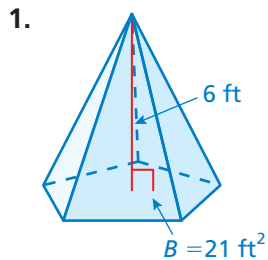
EXAMPLE ③ Real-Life Application

a. The volume of sunscreen in Bottle B is about how many times the volume in Bottle A?

b. Which is the better buy?

a. Use the formula for the volume of a pyramid to estimate the amount of sunscreen in each bottle.

Bottle A
$9.96

Bottle B
$14.40

6 in.
1 in.
2 in.

4 in.
1.5 in.
3 in.

Bottle A	*Bottle B*
$V = \dfrac{1}{3}Bh$	$V = \dfrac{1}{3}Bh$
$= \dfrac{1}{3}(2)(1)(6)$	$= \dfrac{1}{3}(3)(1.5)(4)$
$= 4 \text{ in.}^3$	$= 6 \text{ in.}^3$

∴ So, the volume of sunscreen in Bottle B is about $\dfrac{6}{4} = 1.5$ times the volume in Bottle A.

b. Find the unit cost for each bottle.

Bottle A	*Bottle B*
$\dfrac{\text{cost}}{\text{volume}} = \dfrac{\$9.96}{4 \text{ in.}^3}$	$\dfrac{\text{cost}}{\text{volume}} = \dfrac{\$14.40}{6 \text{ in.}^3}$
$= \dfrac{\$2.49}{1 \text{ in.}^3}$	$= \dfrac{\$2.40}{1 \text{ in.}^3}$

∴ The unit cost of Bottle B is less than the unit cost of Bottle A. So, Bottle B is the better buy.

On Your Own

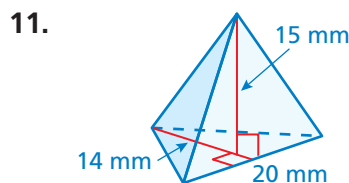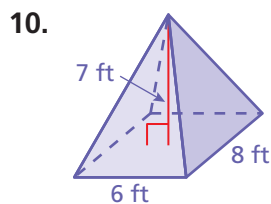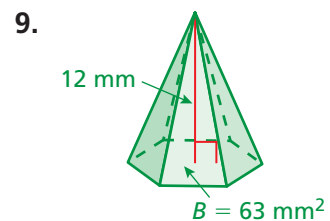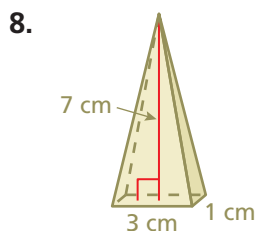**4.** Bottle C is on sale for $13.20. Is Bottle C a better buy than Bottle B in Example 3? Explain.
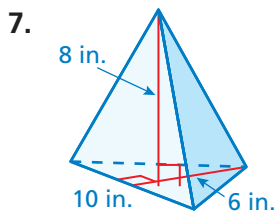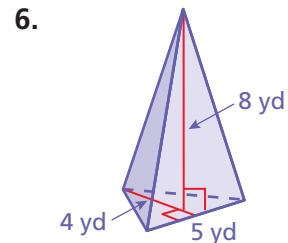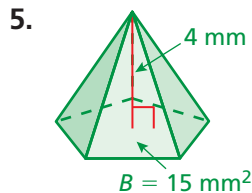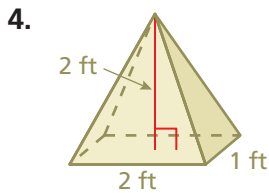
Bottle C

3 in.
2 in.
3 in.
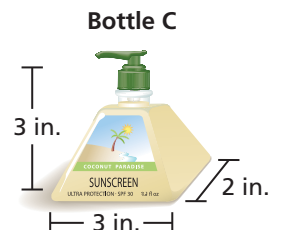
✓ Vocabulary and Concept Check

1. **WRITING** How is the formula for the volume of a pyramid different from the formula for the volume of a prism?

2. **OPEN-ENDED** Describe a real-life situation that involves finding the volume of a pyramid.

3. **REASONING** A triangular pyramid and a triangular prism have the same base and height. The volume of the prism is how many times the volume of the pyramid?
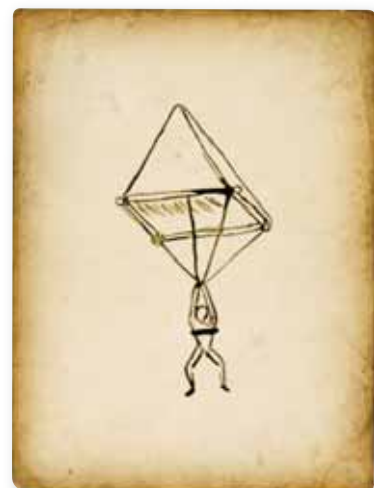
Practice and Problem Solving

Find the volume of the pyramid.

4.

2 ft
1 ft
2 ft

5.

4 mm
$B = 15$ mm²

6.

8 yd
4 yd
5 yd

7.

8 in.
10 in.
6 in.

8.

7 cm
3 cm
1 cm

9.

12 mm
$B = 63$ mm²
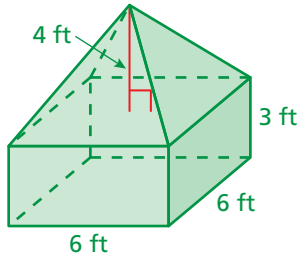
10.

7 ft
8 ft
6 ft

11.

15 mm
14 mm
20 mm

12. PARACHUTE In 1483, Leonardo da Vinci designed a parachute. It is believed that this was the first parachute ever designed. In a notebook, he wrote, "If a man is provided with a length of gummed linen cloth with a length of 12 yards on each side and 12 yards high, he can jump from any great height whatsoever without injury." Find the volume of air inside Leonardo's parachute.
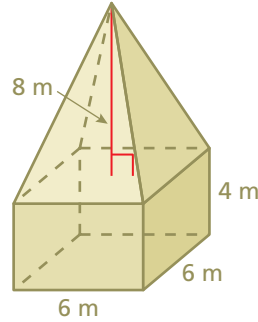
Not drawn to scale
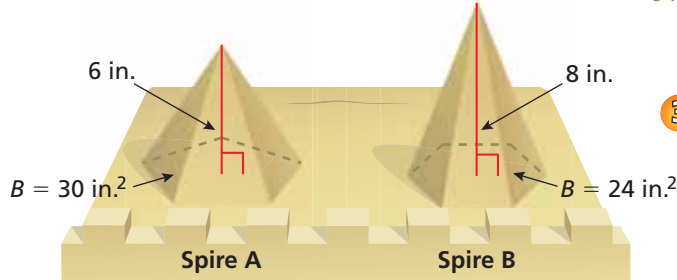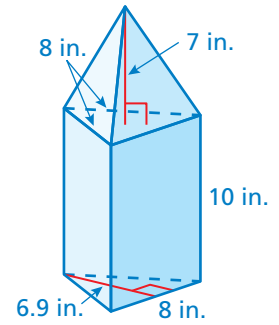
Find the volume of the composite solid.

13.

4 ft, 3 ft, 6 ft, 6 ft

14.

8 m, 4 m, 6 m, 6 m

15.

8 in., 7 in., 10 in., 6.9 in., 8 in.

6 in.   8 in.

$B = 30$ in.² $B = 24$ in.²

Spire A   Spire B

③ 16. SPIRE Which sand-castle spire has a greater volume? How much more sand do you need to make the spire with the greater volume?

**17. PAPERWEIGHT** How much glass is needed to manufacture 1000 paperweights? Explain your reasoning.

4 in.   3 in.   3 in.

Paperweight

18. PROBLEM SOLVING Use the photo of the tepee.

a. What is the shape of the base? How can you tell?

b. The tepee's height is about 10 feet. Estimate the volume of the tepee.
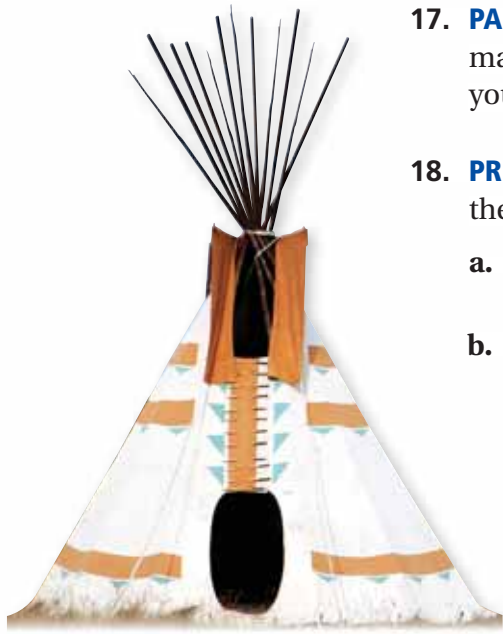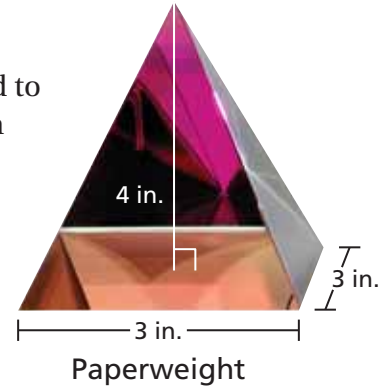
19. OPEN-ENDED A pyramid has a volume of 40 cubic feet and a height of 6 feet. Find one possible set of dimensions of the rectangular base.

20. Reasoning Do the two solids have the same volume? Explain.

z, x, y, $3z$, x, y
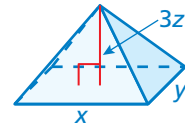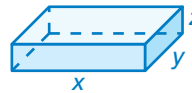
Fair Game Review What you learned in previous grades & lessons

For the given angle measure, find the measure of a supplementary angle and the measure of a complementary angle, if possible. *(Section 7.2)*

21. 27°

22. 82°

23. 120°

24. MULTIPLE CHOICE The circumference of a circle is 44 inches. Which estimate is closest to the area of the circle? *(Section 8.3)*

Ⓐ 7 in.²   Ⓑ 14 in.²   Ⓒ 154 in.²   Ⓓ 484 in.²