

9.4 Volumes of Prisms

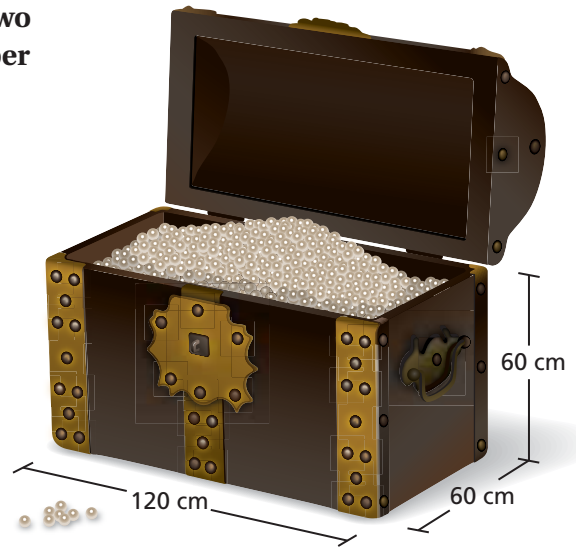
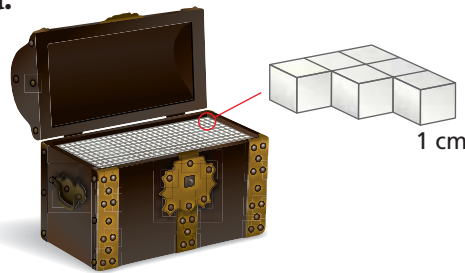
Essential Question How can you find the volume of a prism?

1 ACTIVITY: Pearls in a Treasure Chest

Work with a partner. A treasure chest is filled with valuable pearls. Each pearl is about 1 centimeter in diameter and is worth about \$80.

Use the diagrams below to describe two ways that you can estimate the number of pearls in the treasure chest.

a.



b.



c. Use the method in part (a) to estimate the value of the pearls in the chest.

2 ACTIVITY: Finding a Formula for Volume

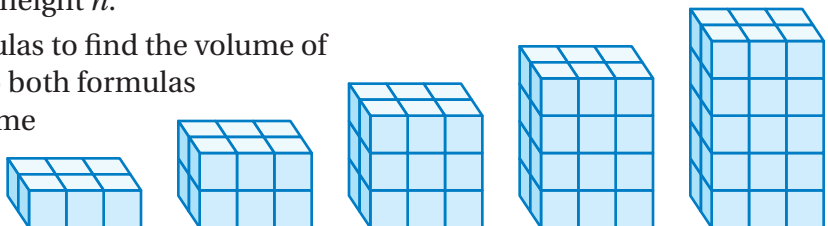
Geometry

In this lesson, you will

- find volumes of prisms.
- solve real-life problems.

Work with a partner. You know that the formula for the volume of a rectangular prism is $V = \ell wh$.

- Write a formula that gives the volume in terms of the area of the base B and the height h .
- Use both formulas to find the volume of each prism. Do both formulas give you the same volume?



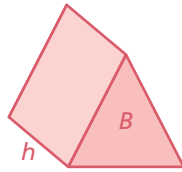
3 ACTIVITY: Finding a Formula for Volume

Math Practice

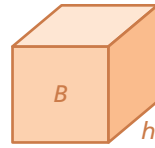
Use a Formula

What are the given quantities? How can you use the quantities to write a formula?

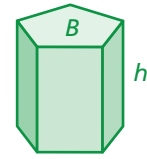
Work with a partner. Use the concept in Activity 2 to find a formula that gives the volume of any prism.



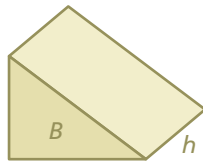
Triangular Prism



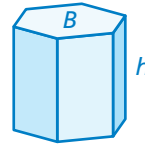
Rectangular Prism



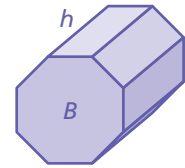
Pentagonal Prism



Triangular Prism



Hexagonal Prism



Octagonal Prism

4 ACTIVITY: Using a Formula



STEM
VIDEO

Work with a partner. A ream of paper has 500 sheets.

- Does a single sheet of paper have a volume? Why or why not?
- If so, explain how you can find the volume of a single sheet of paper.



What Is Your Answer?

- IN YOUR OWN WORDS** How can you find the volume of a prism?
- STRUCTURE** Draw a prism that has a trapezoid as its base. Use your formula to find the volume of the prism.

Practice

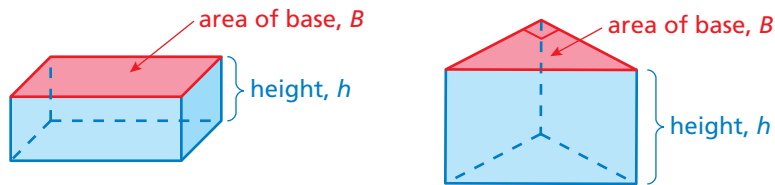
Use what you learned about the volumes of prisms to complete Exercises 4–6 on page 380.

The *volume* of a three-dimensional figure is a measure of the amount of space that it occupies. Volume is measured in cubic units.

Key Idea

Volume of a Prism

Words The volume V of a prism is the product of the area of the base and the height of the prism.



Algebra

$$V = Bh$$

Area of base

Height of prism

Remember



The volume V of a cube with an edge length of s is $V = s^3$.

EXAMPLE 1 Finding the Volume of a Prism

Study Tip



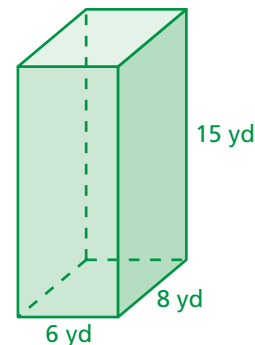
The area of the base of a rectangular prism is the product of the length ℓ and the width w .

You can use $V = \ell wh$ to find the volume of a rectangular prism.

Find the volume of the prism.

$$\begin{aligned} V &= Bh && \text{Write formula for volume.} \\ &= 6(8) \cdot 15 && \text{Substitute.} \\ &= 48 \cdot 15 && \text{Simplify.} \\ &= 720 && \text{Multiply.} \end{aligned}$$

∴ The volume is 720 cubic yards.

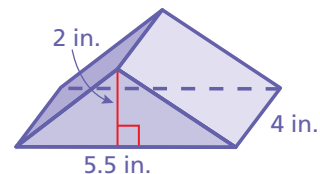


EXAMPLE 2 Finding the Volume of a Prism

Find the volume of the prism.

$$\begin{aligned} V &= Bh && \text{Write formula for volume.} \\ &= \frac{1}{2}(5.5)(2) \cdot 4 && \text{Substitute.} \\ &= 5.5 \cdot 4 && \text{Simplify.} \\ &= 22 && \text{Multiply.} \end{aligned}$$

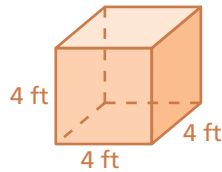
∴ The volume is 22 cubic inches.



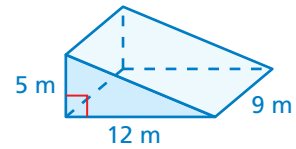
On Your Own

Find the volume of the prism.

1.



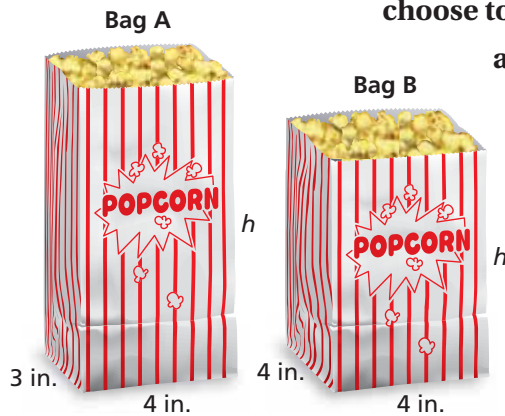
2.



EXAMPLE 3 Real-Life Application

A movie theater designs two bags to hold 96 cubic inches of popcorn.

(a) Find the height of each bag. (b) Which bag should the theater choose to reduce the amount of paper needed? Explain.



a. Find the height of each bag.

Bag A

$$V = Bh$$

$$96 = 4(3)(h)$$

$$96 = 12h$$

$$8 = h$$

∴ The height is 8 inches.

Bag B

$$V = Bh$$

$$96 = 4(4)(h)$$

$$96 = 16h$$

$$6 = h$$

∴ The height is 6 inches.

b. To determine the amount of paper needed, find the surface area of each bag. Do not include the top base.

Bag A

$$S = \ell w + 2\ell h + 2wh$$

$$= 4(3) + 2(4)(8) + 2(3)(8)$$

$$= 12 + 64 + 48$$

$$= 124 \text{ in.}^2$$

Bag B

$$S = \ell w + 2\ell h + 2wh$$

$$= 4(4) + 2(4)(6) + 2(4)(6)$$

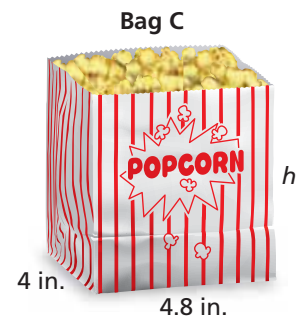
$$= 16 + 48 + 48$$

$$= 112 \text{ in.}^2$$

∴ The surface area of Bag B is less than the surface area of Bag A. So, the theater should choose Bag B.

On Your Own

3. You design Bag C that has a volume of 96 cubic inches. Should the theater in Example 3 choose your bag? Explain.

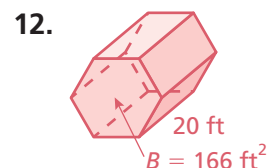
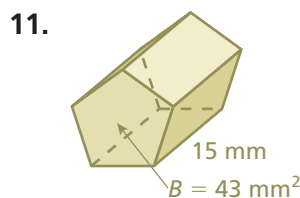
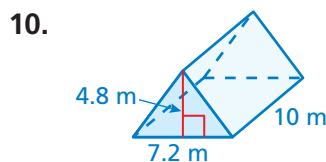
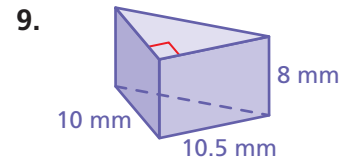
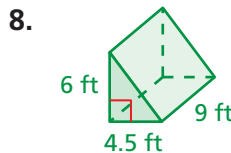
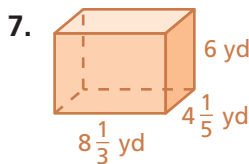
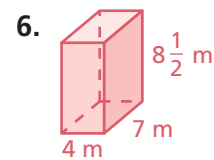
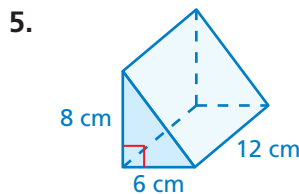
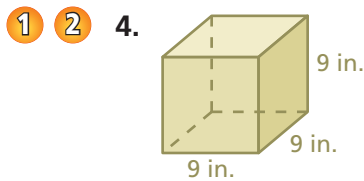


Vocabulary and Concept Check

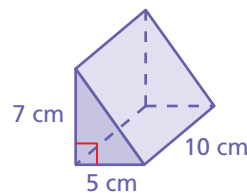
- VOCABULARY** What types of units are used to describe volume?
- VOCABULARY** Explain how to find the volume of a prism.
- CRITICAL THINKING** How are volume and surface area different?

Practice and Problem Solving

Find the volume of the prism.

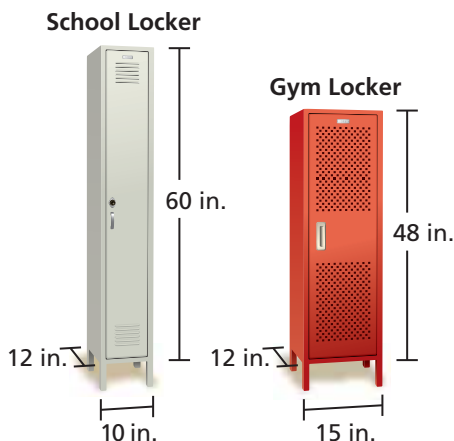


13. **ERROR ANALYSIS** Describe and correct the error in finding the volume of the triangular prism.



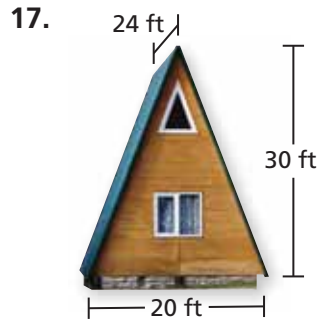
X

$$\begin{aligned} V &= Bh \\ &= 10(5)(7) \\ &= 50 \cdot 7 \\ &= 350 \text{ cm}^3 \end{aligned}$$



14. **LOCKER** Each locker is shaped like a rectangular prism. Which has more storage space? Explain.
15. **CEREAL BOX** A cereal box is 9 inches by 2.5 inches by 10 inches. What is the volume of the box?

Find the volume of the prism.



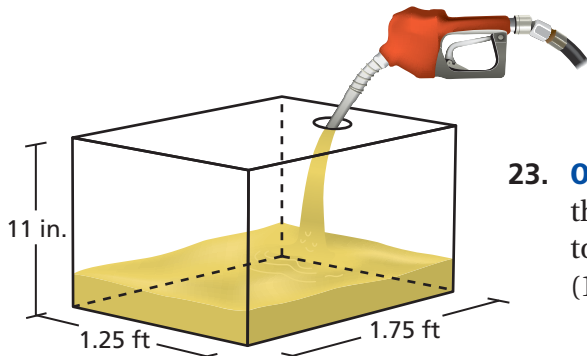
18. **LOGIC** Two prisms have the same volume. Do they *always*, *sometimes*, or *never* have the same surface area? Explain.

19. **CUBIC UNITS** How many cubic inches are in a cubic foot? Use a sketch to explain your reasoning.

20. **CAPACITY** As a gift, you fill the calendar with packets of chocolate candy. Each packet has a volume of 2 cubic inches. Find the maximum number of packets you can fit inside the calendar.



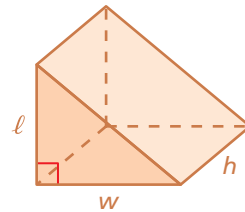
21. **PRECISION** Two liters of water are poured into an empty vase shaped like an octagonal prism. The base area is 100 square centimeters. What is the height of the water? (1 L = 1000 cm³)



22. **GAS TANK** The gas tank is 20% full. Use the current price of regular gasoline in your community to find the cost to fill the tank. (1 gal = 231 in.³)

23. **OPEN-ENDED** You visit an aquarium. One of the tanks at the aquarium holds 450 gallons of water. Draw a diagram to show one possible set of dimensions of the tank. (1 gal = 231 in.³)

24. **Critical Thinking** How many times greater is the volume of a triangular prism when one of its dimensions is doubled? when all three dimensions are doubled?



Fair Game Review What you learned in previous grades & lessons

Find the selling price. (Section 6.6)

25. Cost to store: \$75
Markup: 20%

26. Cost to store: \$90
Markup: 60%

27. Cost to store: \$130
Markup: 85%

28. **MULTIPLE CHOICE** What is the approximate surface area of a cylinder with a radius of 3 inches and a height of 10 inches? (Section 9.3)

(A) 30 in.²

(B) 87 in.²

(C) 217 in.²

(D) 245 in.²