**Essential Question** How can you find the circumference of a circle?

Archimedes was a Greek mathematician, physicist, engineer, and astronomer.

Archimedes discovered that in any circle the ratio of circumference to diameter is always the same. Archimedes called this ratio pi, or π (a letter from the Greek alphabet).

$$\pi = \frac{\text{circumference}}{\text{diameter}}$$

In Activities 1 and 2, you will use the same strategy Archimedes used to approximate π.
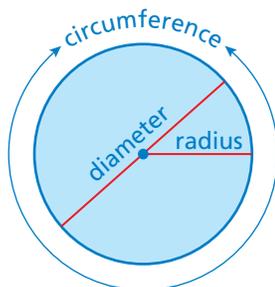
1 ACTIVITY: Approximating Pi

Work with a partner. Copy the table. Record your results in the table.

- **Measure the perimeter of the large square in millimeters.**

- **Measure the diameter of the circle in millimeters.**

- **Measure the perimeter of the small square in millimeters.**

- **Calculate the ratios of the two perimeters to the diameter.**

- **The average of these two ratios is an approximation of π.**
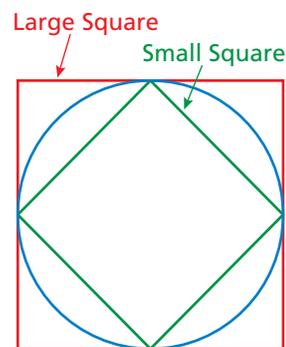
Geometry

In this lesson, you will
- describe a circle in terms of radius and diameter.
- understand the concept of pi.
- find circumferences of circles and perimeters of semicircles.

Sides	Large Perimeter	Diameter of Circle	Small Perimeter	$\dfrac{\text{Large Perimeter}}{\text{Diameter}}$	$\dfrac{\text{Small Perimeter}}{\text{Diameter}}$	Average of Ratios
4						
6						
8						
10						

A page from *Sir Cumference and the First Round Table* by Cindy Neuschwander

2 ACTIVITY: Approximating Pi

Math Practice

Make Conjectures

How can you use the results of the activity to find an approximation of pi?

Continue your approximation of pi. Complete the table from Activity 1 using a hexagon (6 sides), an octagon (8 sides), and a decagon (10 sides).

Large Hexagon

a. Small Hexagon

Large Octagon

b. Small Octagon

Large Decagon

c. Small Decagon
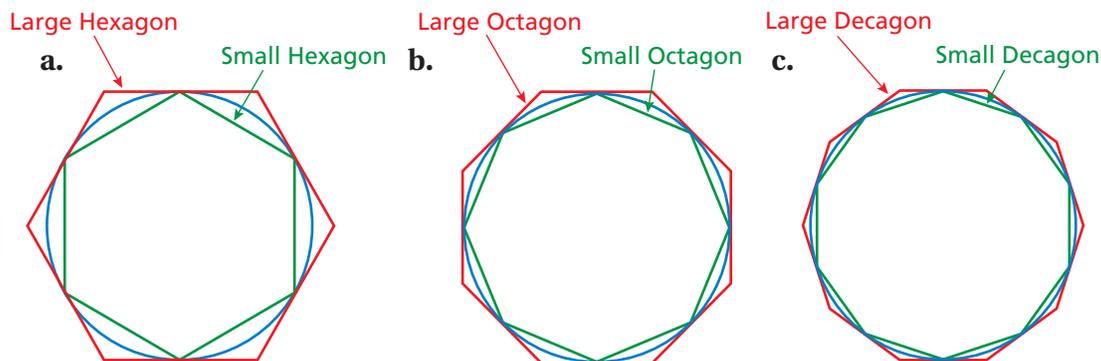
d. From the table, what can you conclude about the value of π? Explain your reasoning.

e. Archimedes calculated the value of π using polygons with 96 sides. Do you think his calculations were more or less accurate than yours?

What Is Your Answer?

3. IN YOUR OWN WORDS Now that you know an approximation for pi, explain how you can use it to find the circumference of a circle. Write a formula for the circumference C of a circle whose diameter is d.

4. CONSTRUCTION Use a compass to draw three circles. Use your formula from Question 3 to find the circumference of each circle.

Practice

Use what you learned about circles and circumference to complete Exercises 9–11 on page 321.

8.1 Lesson

Key Vocabulary 🔊
circle, *p. 318*
center, *p. 318*
radius, *p. 318*
diameter, *p. 318*
circumference, *p. 319*
pi, *p. 319*
semicircle, *p. 320*

A **circle** is the set of all points in a plane that are the same distance from a point called the **center**.

circle

center

The **radius** is the distance from the center to any point on the circle.

The **diameter** is the distance across the circle through the center.
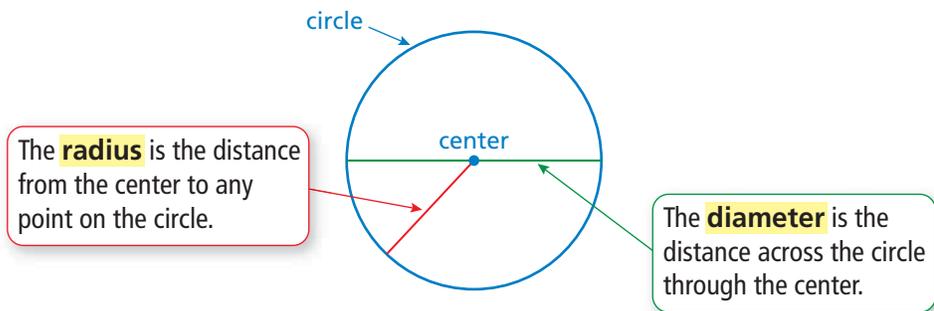
🔑 Key Idea

Radius and Diameter

Words   The diameter d of a circle is twice the radius r. The radius r of a circle is one-half the diameter d.

Algebra   **Diameter:** $d = 2r$ **Radius:** $r = \dfrac{d}{2}$

EXAMPLE 1   Finding a Radius and a Diameter

a. **The diameter of a circle is 20 feet. Find the radius.**

20 ft
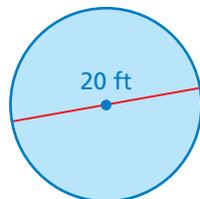
$$r = \frac{d}{2} \qquad \text{Radius of a circle}$$

$$= \frac{20}{2} \qquad \text{Substitute 20 for } d.$$

$$= 10 \qquad \text{Divide.}$$

∴ The radius is 10 feet.

b. **The radius of a circle is 7 meters. Find the diameter.**

7 m
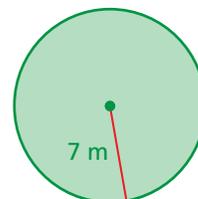
$$d = 2r \qquad \text{Diameter of a circle}$$

$$= 2(7) \qquad \text{Substitute 7 for } r.$$

$$= 14 \qquad \text{Multiply.}$$

∴ The diameter is 14 meters.

🔴 On Your Own

Now You're Ready
Exercises 3–8

1. The diameter of a circle is 16 centimeters. Find the radius.

2. The radius of a circle is 9 yards. Find the diameter.

🔊 Multi-Language Glossary at BigIdeasMath✓com

The distance around a circle is called the **circumference**. The ratio $\frac{\text{circumference}}{\text{diameter}}$ is the same for *every* circle and is represented by the Greek letter π, called **pi**. The value of π can be approximated as 3.14 or $\frac{22}{7}$.

Key Idea

**Circumference of a Circle**

Words The circumference C of a circle is equal to π times the diameter d or π times twice the radius r.

Algebra $C = \pi d$ or $C = 2\pi r$
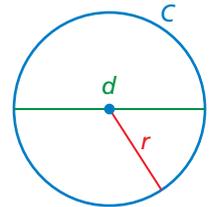
EXAMPLE **2** **Finding Circumferences of Circles**

a. Find the circumference of the flying disc. Use 3.14 for π.

$$C = 2\pi r \qquad \text{Write formula for circumference.}$$

$$\approx 2 \cdot 3.14 \cdot 5 \qquad \text{Substitute 3.14 for } \pi \text{ and 5 for } r.$$

$$= 31.4 \qquad \text{Multiply.}$$

The circumference is about 31.4 inches.

**b. Find the circumference of the watch face. Use $\frac{22}{7}$ for π.**

$$C = \pi d \qquad \text{Write formula for circumference.}$$

$$\approx \frac{22}{7} \cdot 28 \qquad \text{Substitute } \frac{22}{7} \text{ for } \pi \text{ and 28 for } d.$$

$$= 88 \qquad \text{Multiply.}$$

The circumference is about 88 millimeters.

On Your Own

Find the circumference of the object. Use 3.14 or $\frac{22}{7}$ for π.

3.
2 cm

4.
14 ft

5.
9 in.
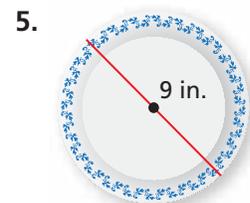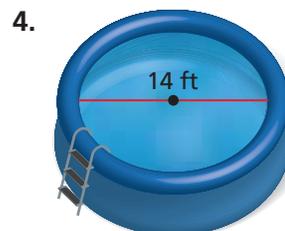
EXAMPLE 3

Estimating a Diameter

C = 31.4 in.

The circumference of the roll of caution tape decreases 10.5 inches after a construction worker uses some of the tape. Which is the best estimate of the diameter of the roll after the decrease?

  (A) 5 inches   **(B)** 7 inches   **(C)** 10 inches   **(D)** 12 inches

After the decrease, the circumference of the roll is
$31.4 - 10.5 = 20.9$ inches.

$C = \pi d$	Write formula for circumference.
$20.9 \approx 3.14 \cdot d$	Substitute 20.9 for C and 3.14 for π.
$21 \approx 3d$	Round 20.9 up to 21. Round 3.14 down to 3.
$7 = d$	Divide each side by 3.

⋰ The correct answer is **(B)**.

On Your Own

6. **WHAT IF?** The circumference of the roll of tape decreases 5.25 inches. Estimate the diameter of the roll after the decrease.

EXAMPLE 4

Finding the Perimeter of a Semicircular Region

A **semicircle** is one-half of a circle. Find the perimeter of the semicircular region.

The straight side is 6 meters long. The distance around the curved part is one-half the circumference of a circle with a diameter of 6 meters.

6 m
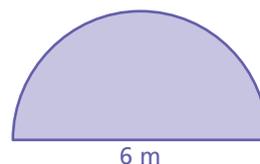
$\dfrac{C}{2} = \dfrac{\pi d}{2}$	Divide the circumference by 2.
$\approx \dfrac{3.14 \cdot 6}{2}$	Substitute 3.14 for π and 6 for d.
$= 9.42$	Simplify.

⋰ So, the perimeter is about $6 + 9.42 = 15.42$ meters.

On Your Own

Now You're Ready
Exercises 15 and 16

Find the perimeter of the semicircular region.

7.

2 ft

8.

7 cm

9.

15 in.
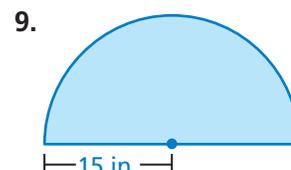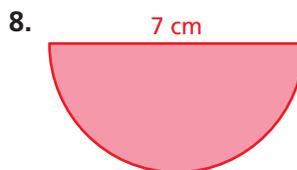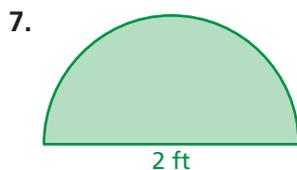
✓ Vocabulary and Concept Check

1. **VOCABULARY** What is the relationship between the radius and the diameter of a circle?

2. **WHICH ONE DOESN'T BELONG?** Which phrase does *not* belong with the other three? Explain your reasoning.

|	
the distance around a circle	π times twice the radius
π times the diameter	the distance from the center to any point on the circle

Practice and Problem Solving

Find the radius of the button.

1. 3.

5 cm

4.

28 mm

5.

$3\frac{1}{2}$ in.
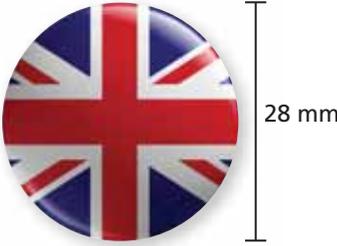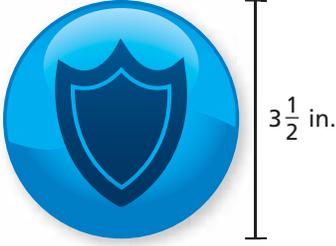
Find the diameter of the object.

6.

6 cm

7.

2 in.

8.

0.8 ft
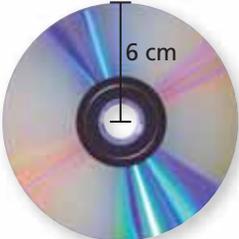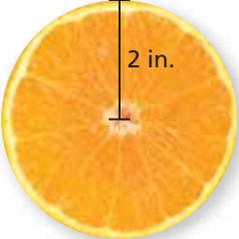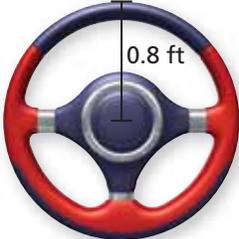
Find the circumference of the pizza. Use 3.14 or $\frac{22}{7}$ for π.

2. 9.

10 in.

10.

7 in.

11.

18 in.

12. **CHOOSE TOOLS** Choose a real-life circular object. Explain why you might need to know its circumference. Then find the circumference.

13. SINKHOLE A circular sinkhole has a circumference of 75.36 meters. A week later, it has a circumference of 150.42 meters.

    a. Estimate the diameter of the sinkhole each week.

   **b.** How many times greater is the diameter of the sinkhole now compared to the previous week?

**14. REASONING** Consider the circles *A*, *B*, *C*, and *D*.

A 8 ft   *B* 10 in.   *C* 2 ft   *D* 50 in.
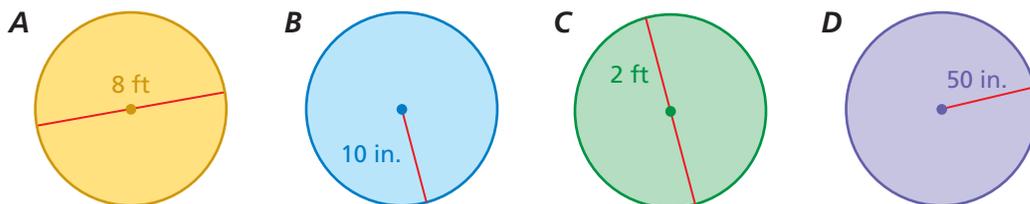
    a. Without calculating, which circle has the greatest circumference?

    b. Without calculating, which circle has the least circumference?

**Find the perimeter of the window.**

④ **15.**

3 ft

16.

20 cm
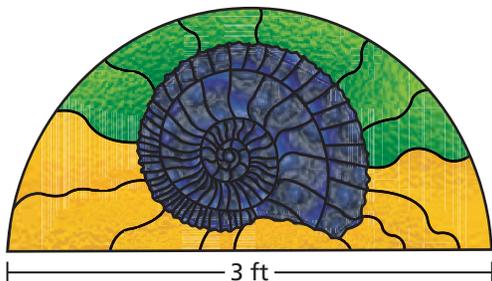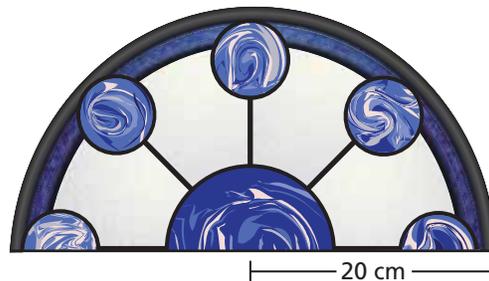
Find the circumferences of both circles.

17.

5 cm
5 cm

18.

9 ft
2.5 ft

19.

22 m
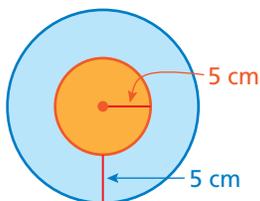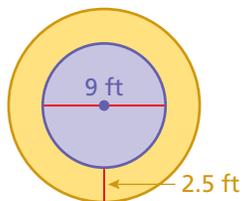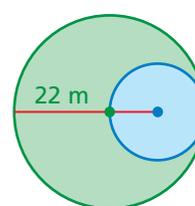
**20. STRUCTURE** Because the ratio $\dfrac{\text{circumference}}{\text{diameter}}$ is the same for every circle, is the ratio $\dfrac{\text{circumference}}{\text{radius}}$ the same for every circle? Explain.

**21. WIRE** A wire is bent to form four semicircles. How long is the wire?

32 cm   32 cm   32 cm   32 cm
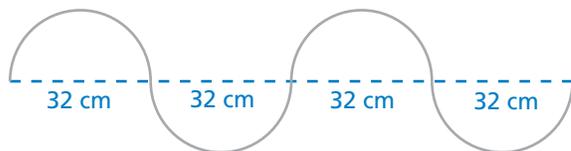
22. CRITICAL THINKING Explain how to draw a circle with a circumference of π^2 inches. Then draw the circle.

23. AROUND THE WORLD "Lines" of latitude on Earth are actually circles. The Tropic of Cancer is the northernmost line of latitude at which the Sun appears directly overhead at noon. The Tropic of Cancer has a radius of 5854 kilometers.

To qualify for an around-the-world speed record, a pilot must cover a distance no less than the circumference of the Tropic of Cancer, cross all meridians, and land on the same airfield where he started.
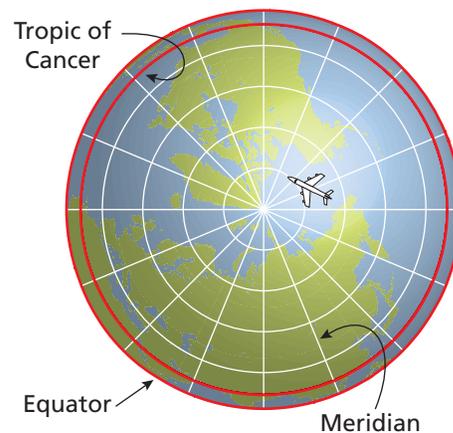
a. What is the minimum distance that a pilot must fly to qualify for an around-the-world speed record?

b. **RESEARCH** Estimate the time it would take for a pilot to qualify for the speed record.
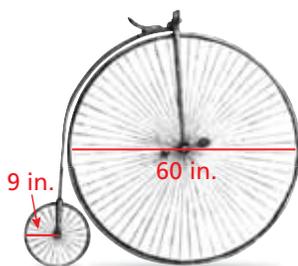
24. PROBLEM SOLVING Bicycles in the late 1800s looked very different than they do today.

a. How many rotations does each tire make after traveling 600 feet? Round your answers to the nearest whole number.

b. Would you rather ride a bicycle made with two large wheels or two small wheels? Explain.

25. **Logic** The length of the minute hand is 150% of the length of the hour hand.

a. What distance will the tip of the minute hand move in 45 minutes? Explain how you found your answer.

b. In 1 hour, how much farther does the tip of the minute hand move than the tip of the hour hand? Explain how you found your answer.
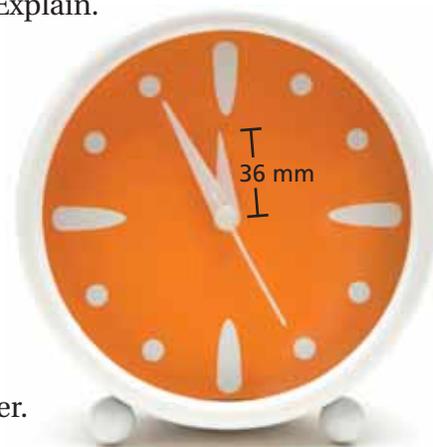
![pencil icon] **Fair Game Review** *What you learned in previous grades & lessons*

Find the perimeter of the polygon. *(Skills Review Handbook)*

26.

4 ft
7 ft

27.

6 m 5 m
9 m

28.

16 in.
12 in. 12 in.
25 in.
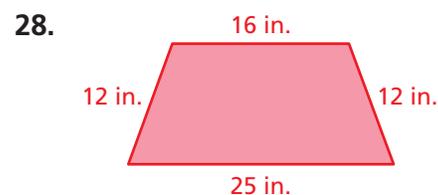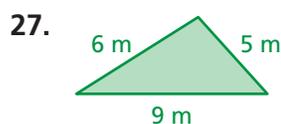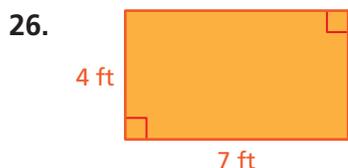
29. MULTIPLE CHOICE What is the median of the data set? *(Skills Review Handbook)*

12, 25, 16, 9, 5, 22, 27, 20

Ⓐ 7 　　　　 Ⓑ 16 　　　　 Ⓒ 17 　　　　 Ⓓ 18